Games with names

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What this talk is about

Generation of new resources (*names*)
is a pervasive feature in computation

We present games and related formalisms
for the semantics of new resources

Ingredients

Game Semantics

Nominal Sets

Fresh-Register Automata

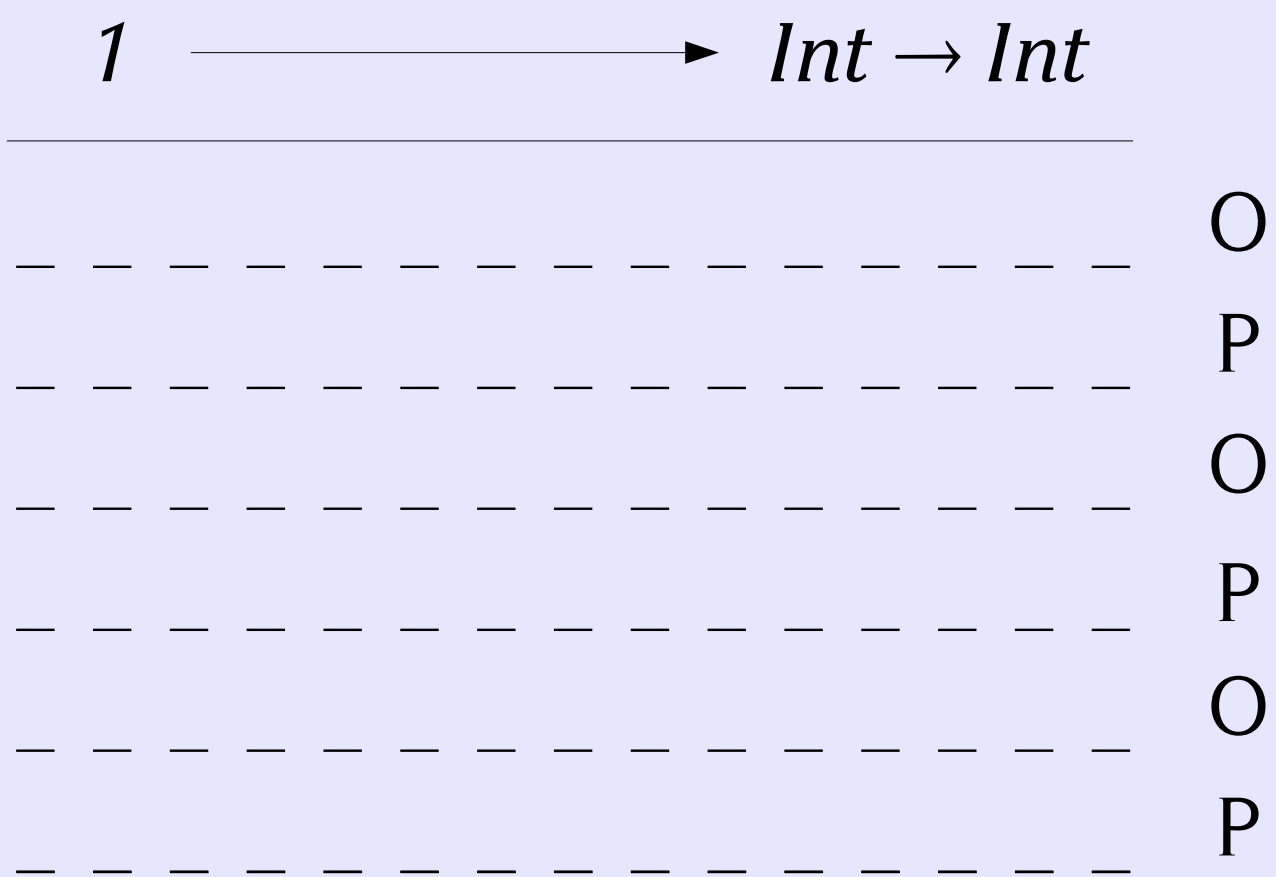
Game Semantics

Game Semantics

- Computation is modelled as a 2-player game between:
  - *Opponent* (the environment)
  - *Proponent* (the program)

Example

$\vdash \lambda x.x+1 : \texttt{int} \to \texttt{int}$



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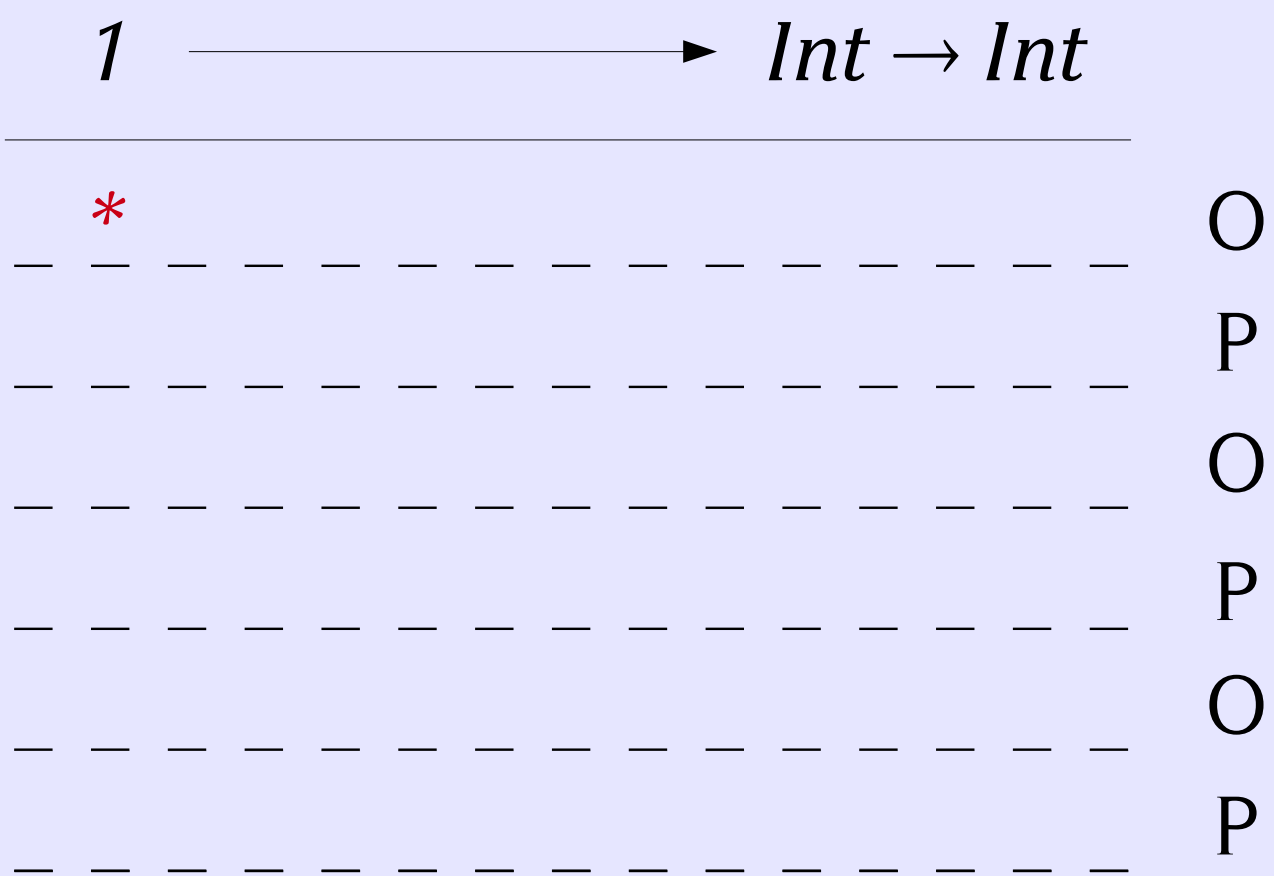

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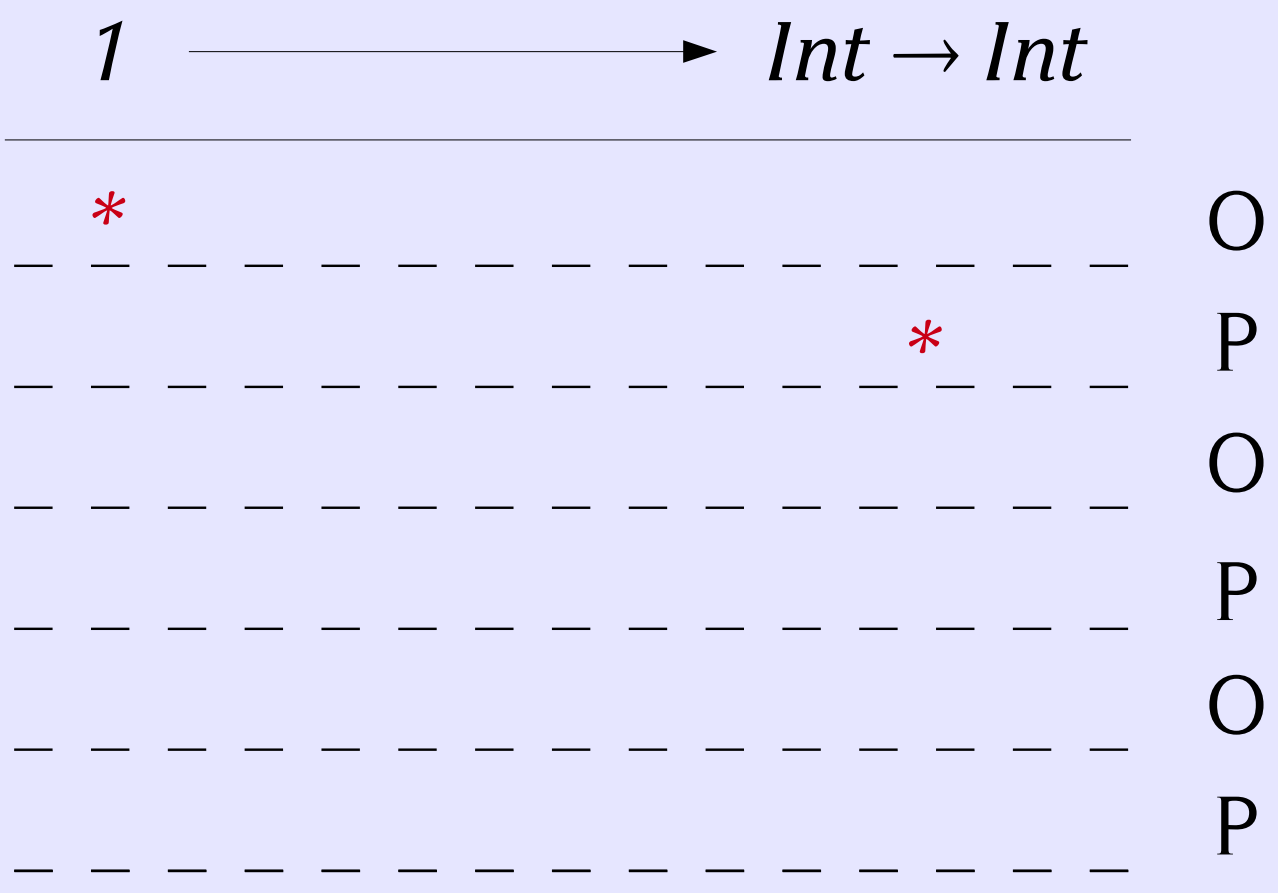

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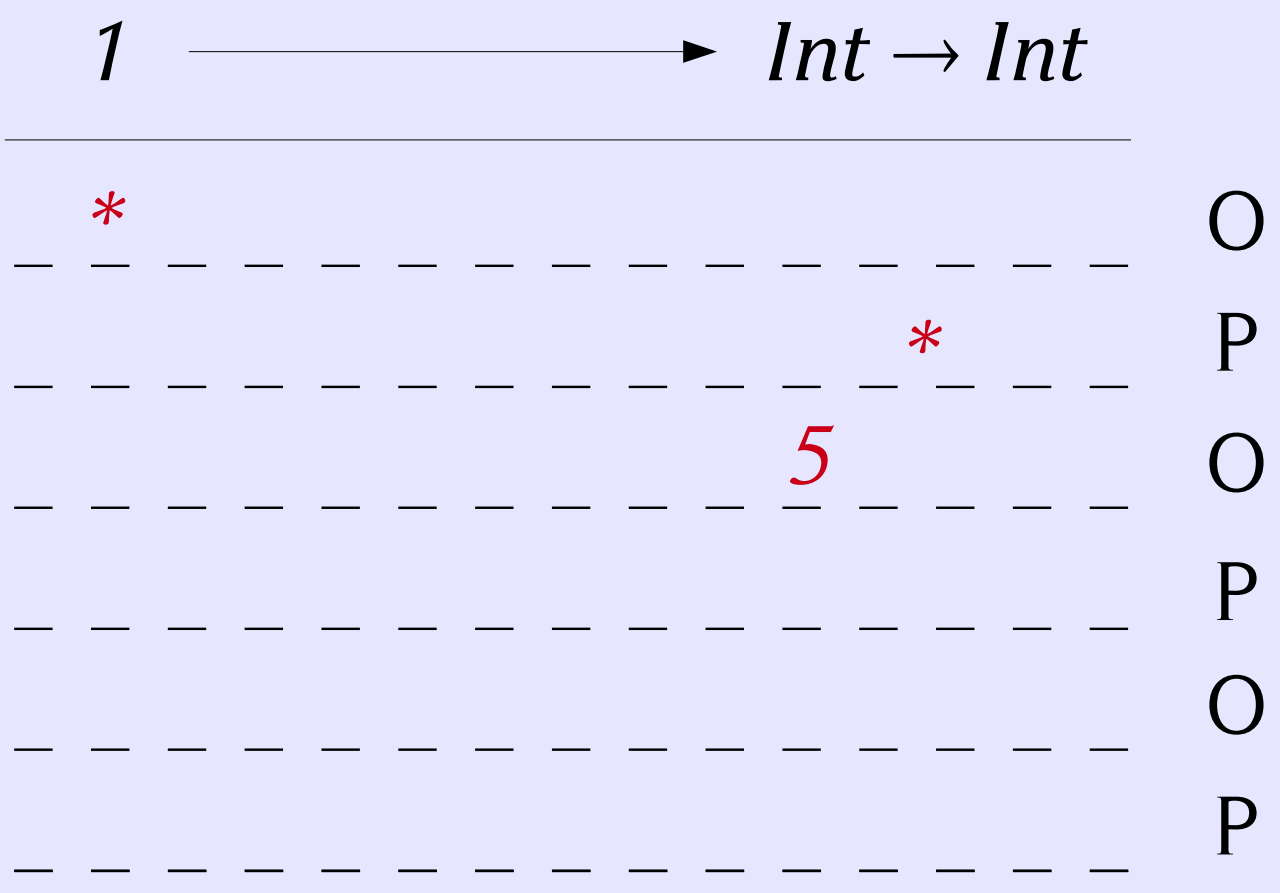

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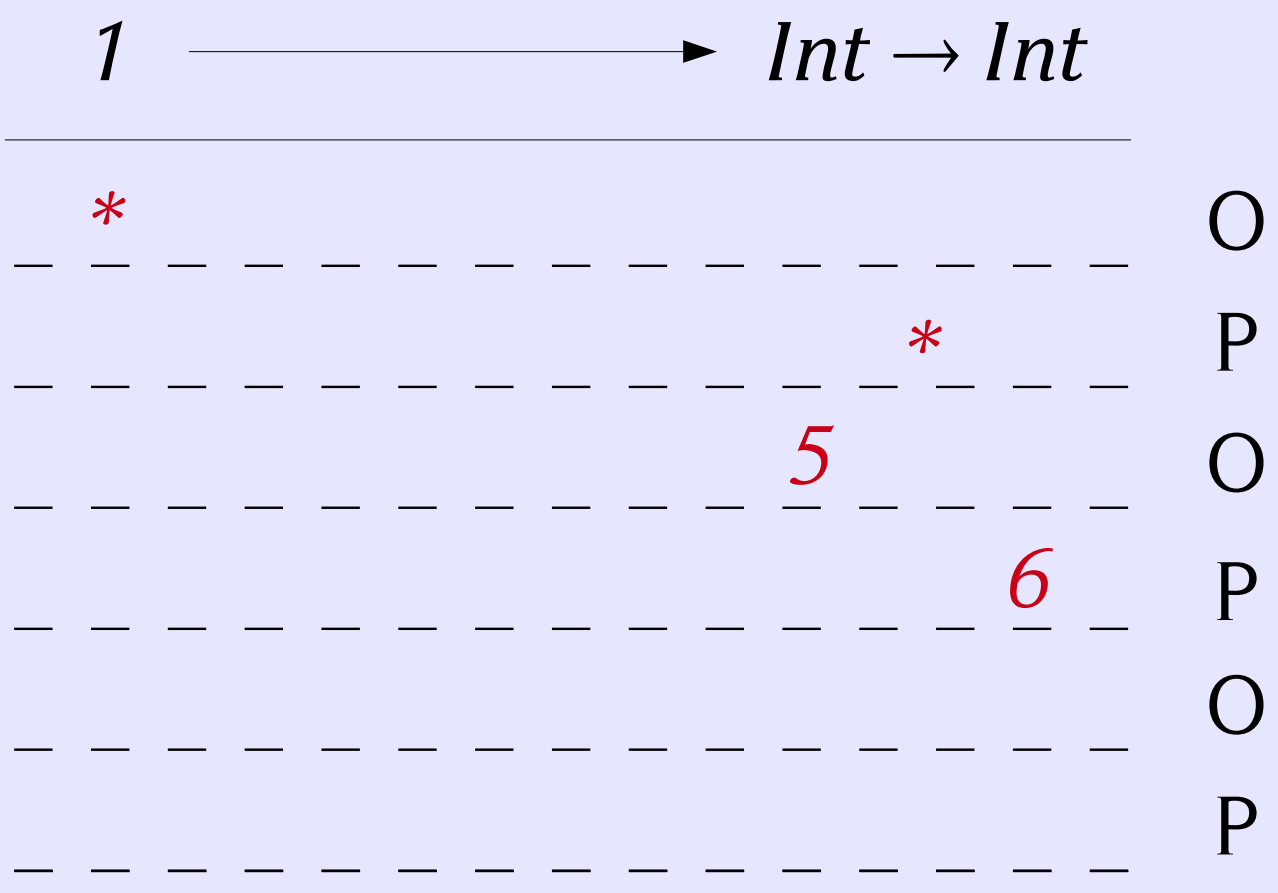

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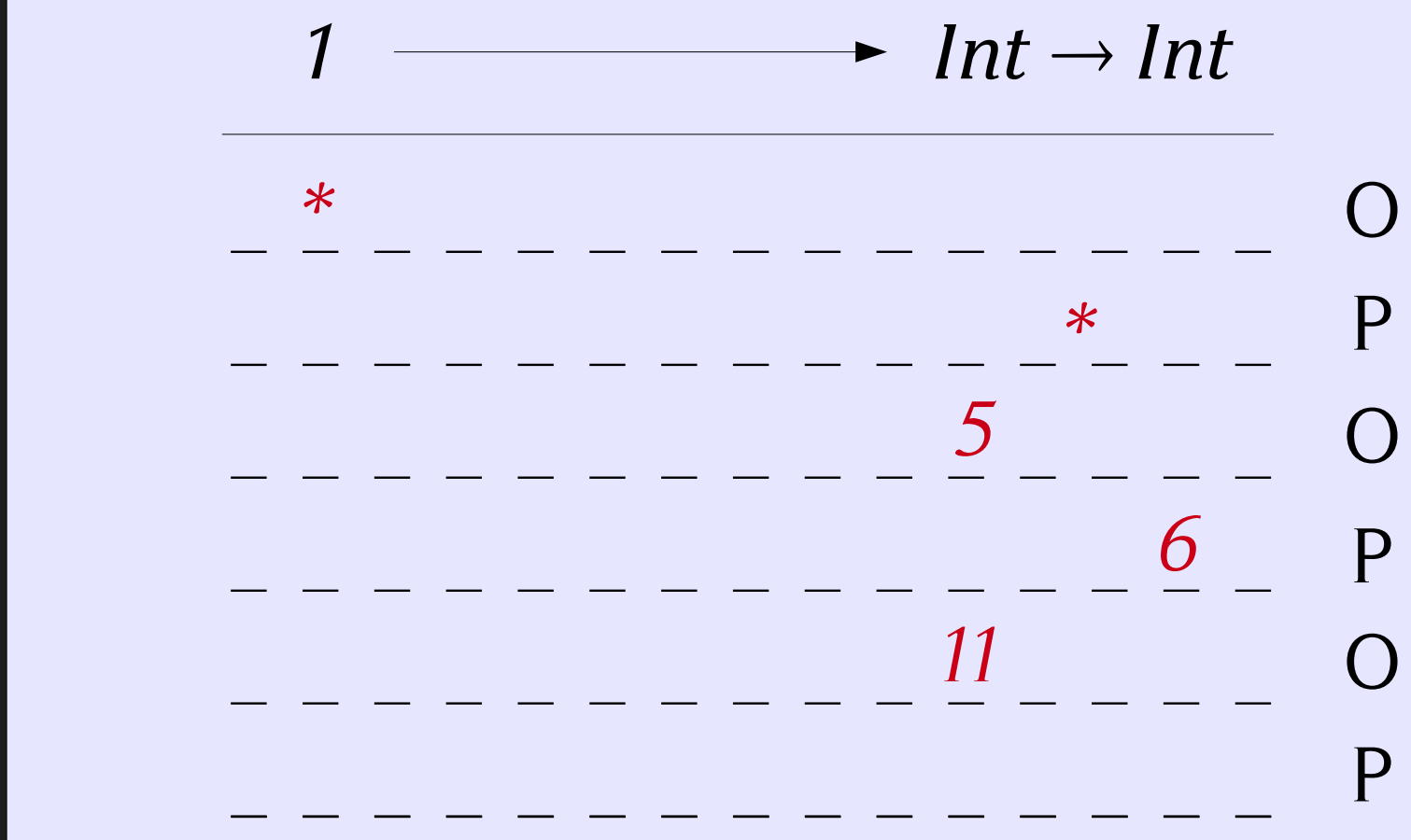

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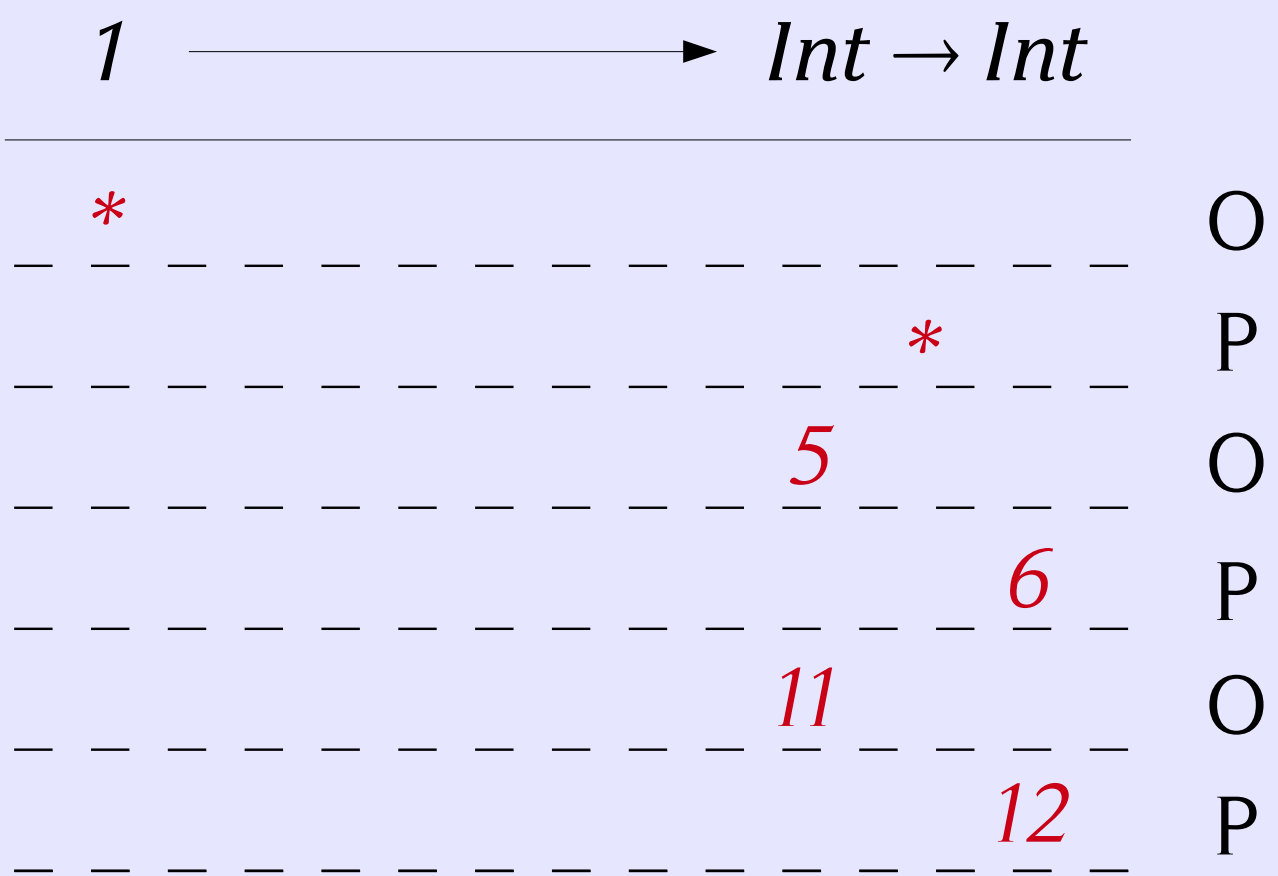

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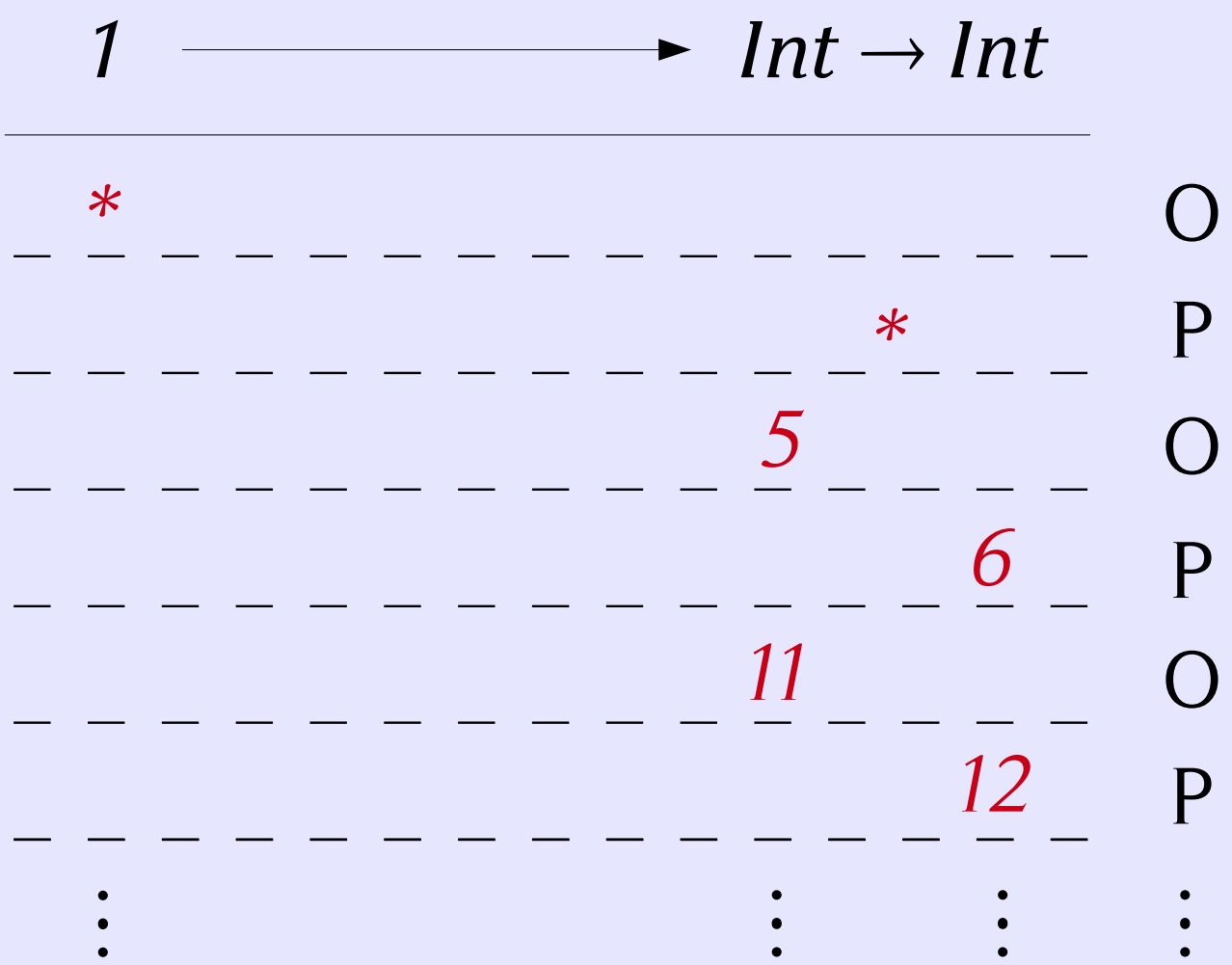

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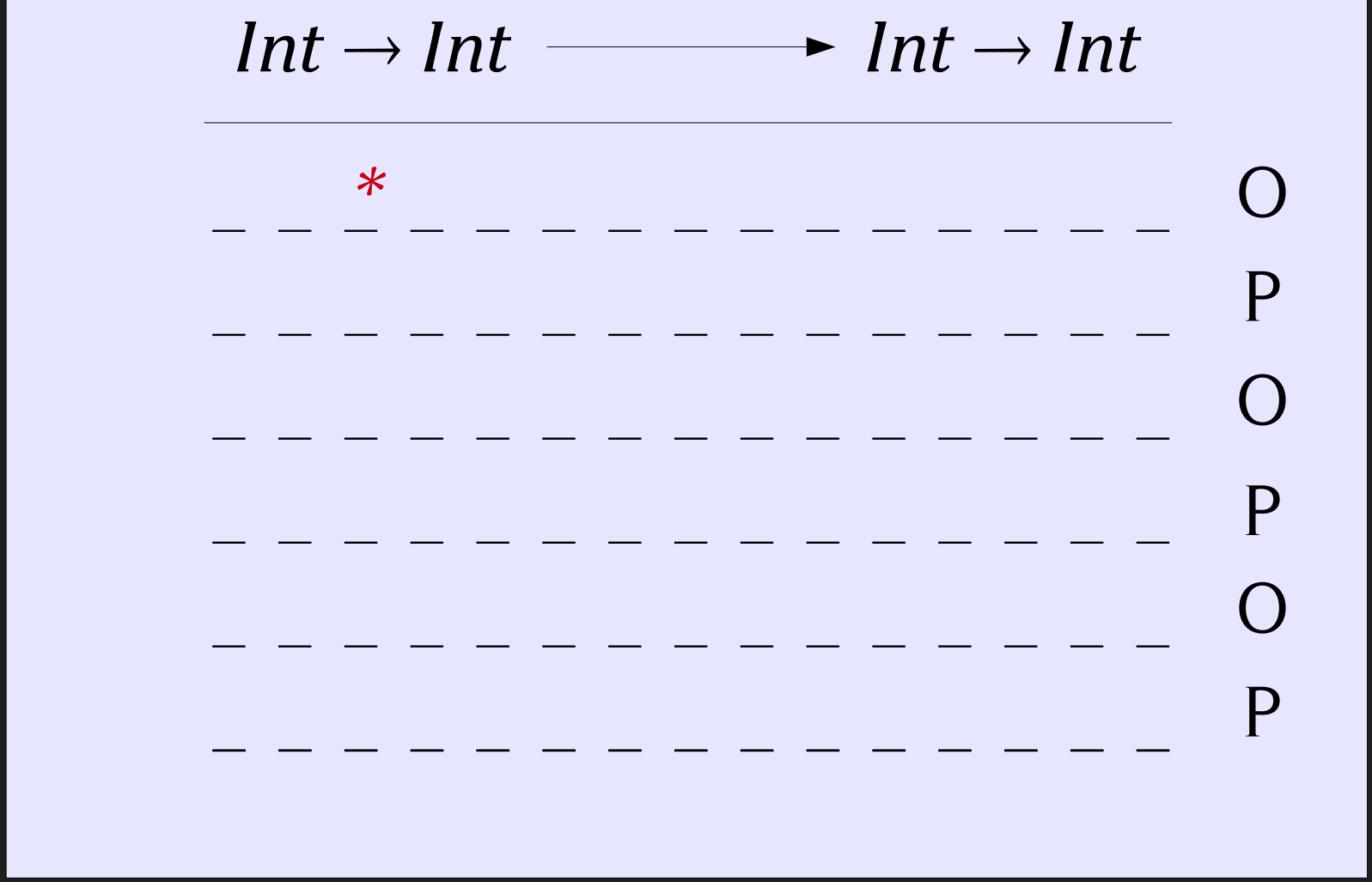
$\texttt{f : int} \rightarrow \texttt{int} \vdash \lambda\texttt{x.f(x)+1 : int} \rightarrow \texttt{int}$



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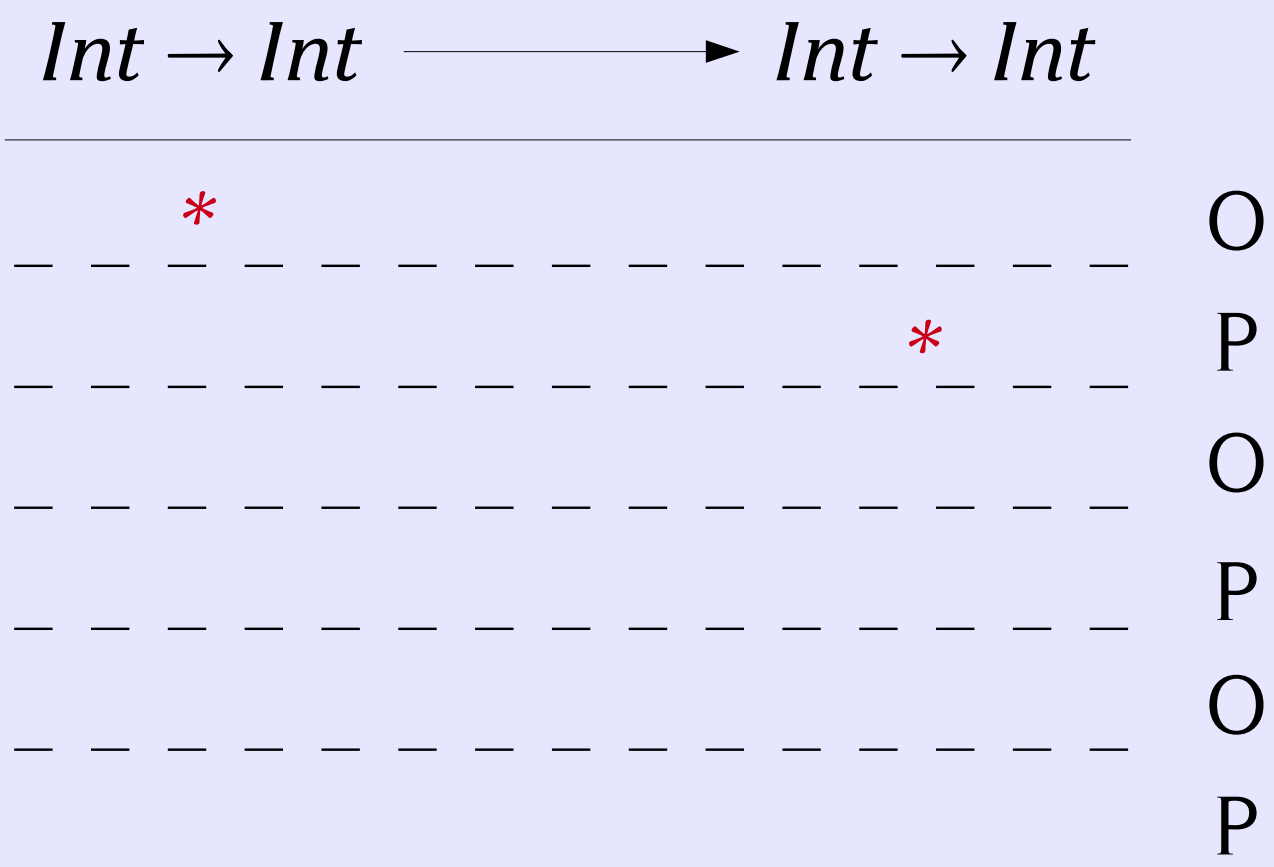

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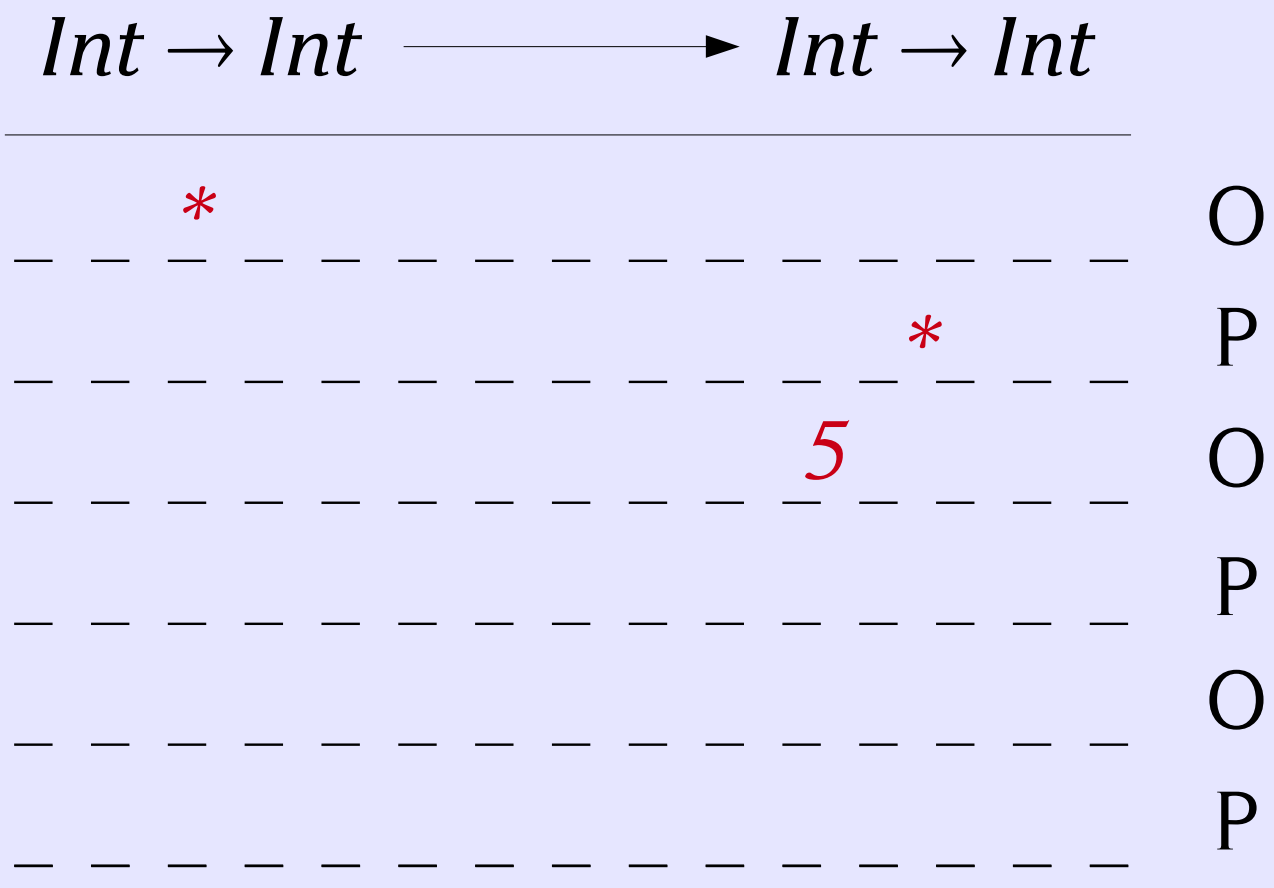

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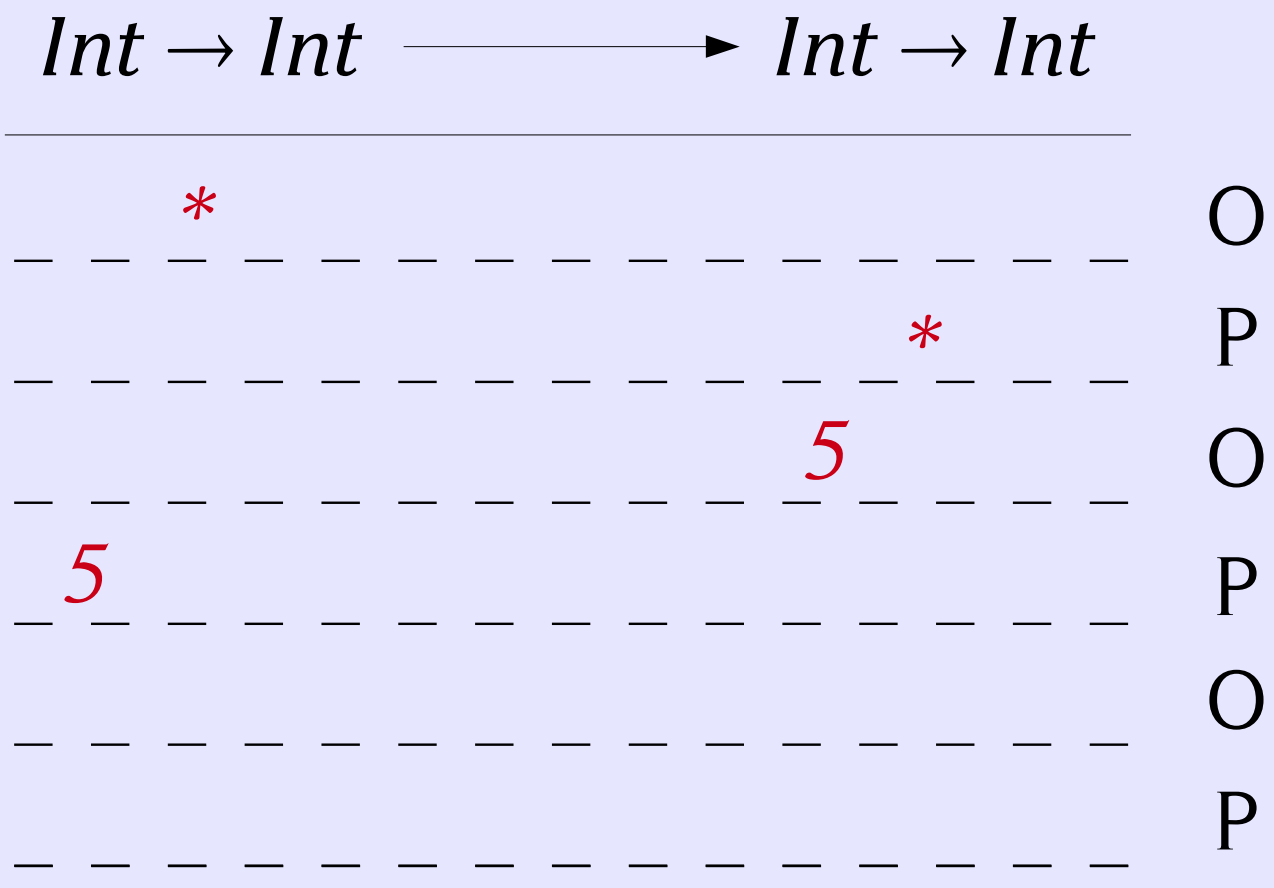

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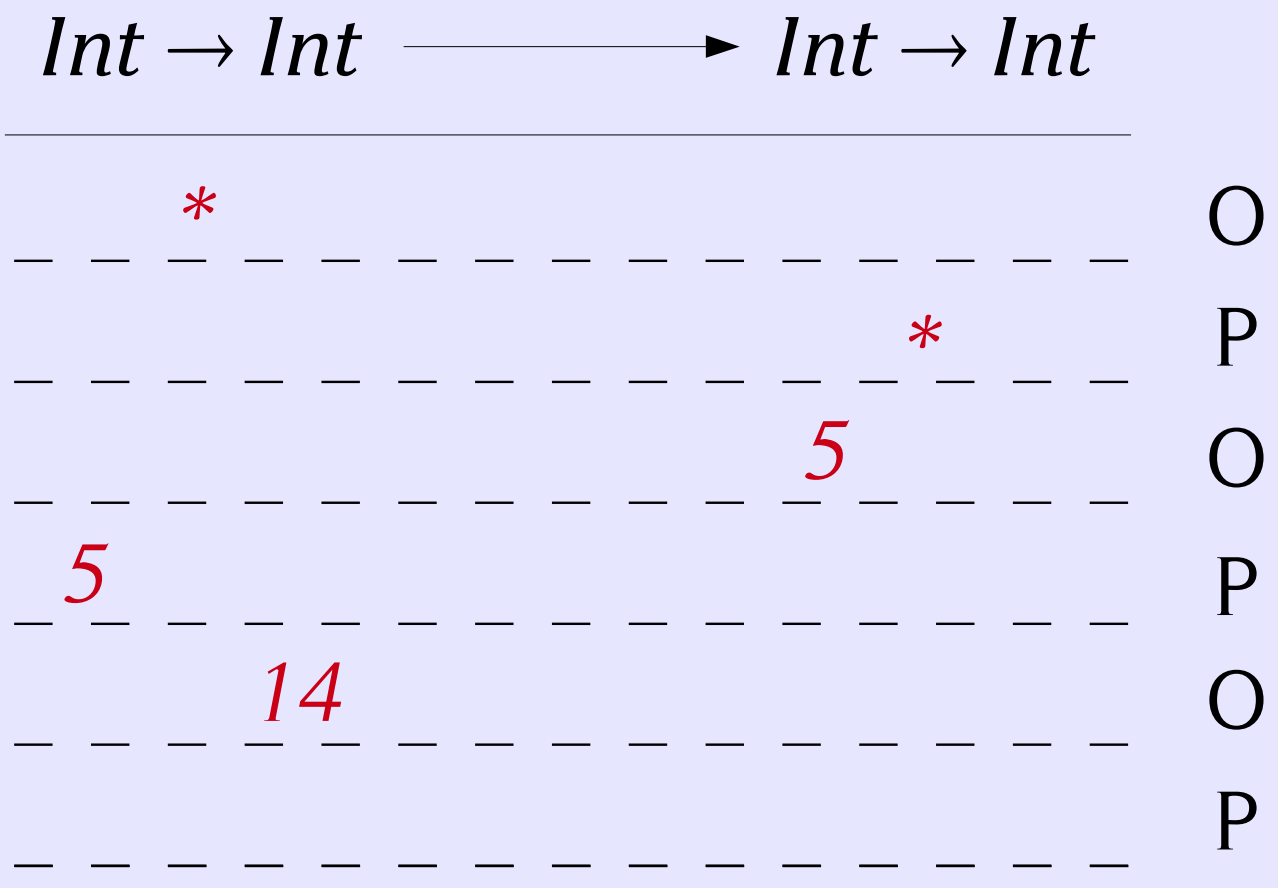

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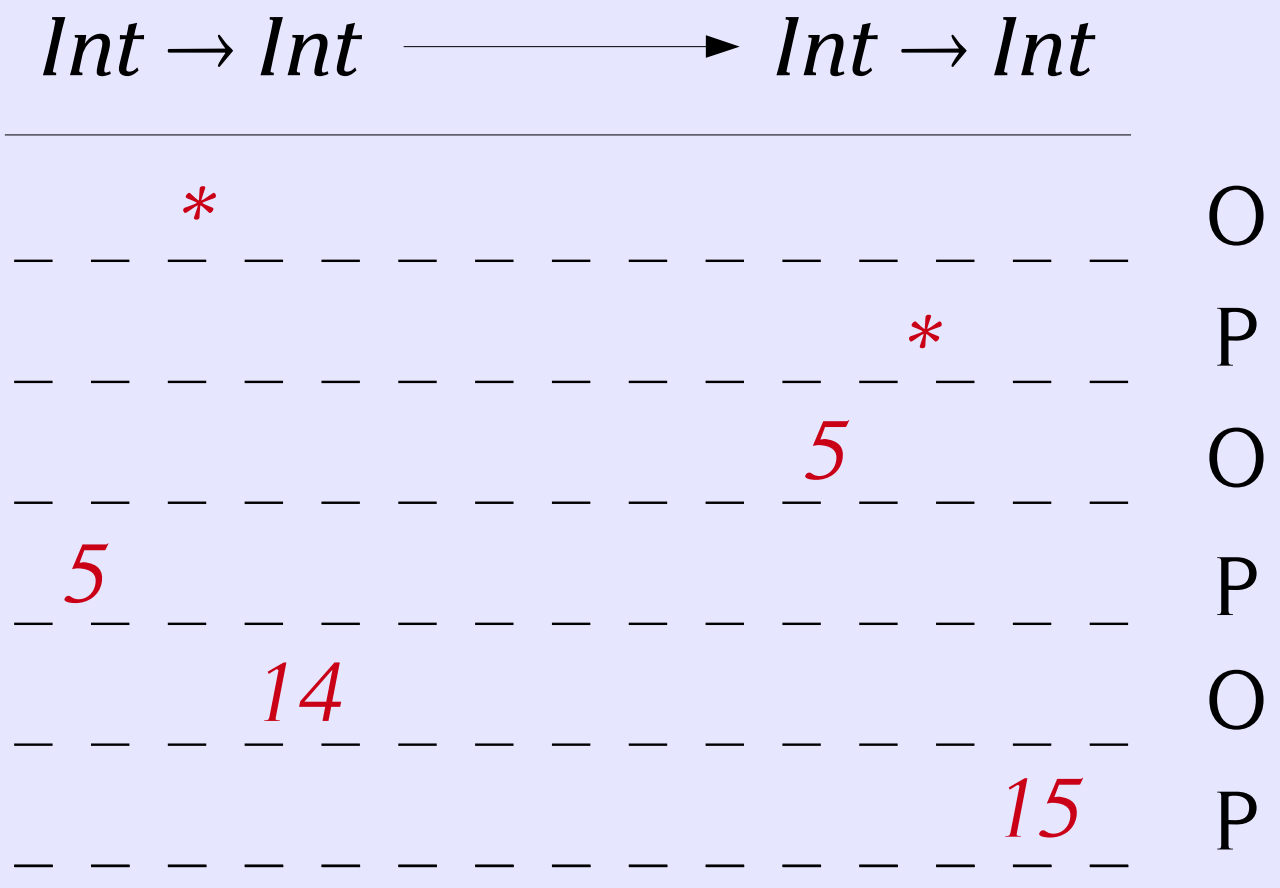

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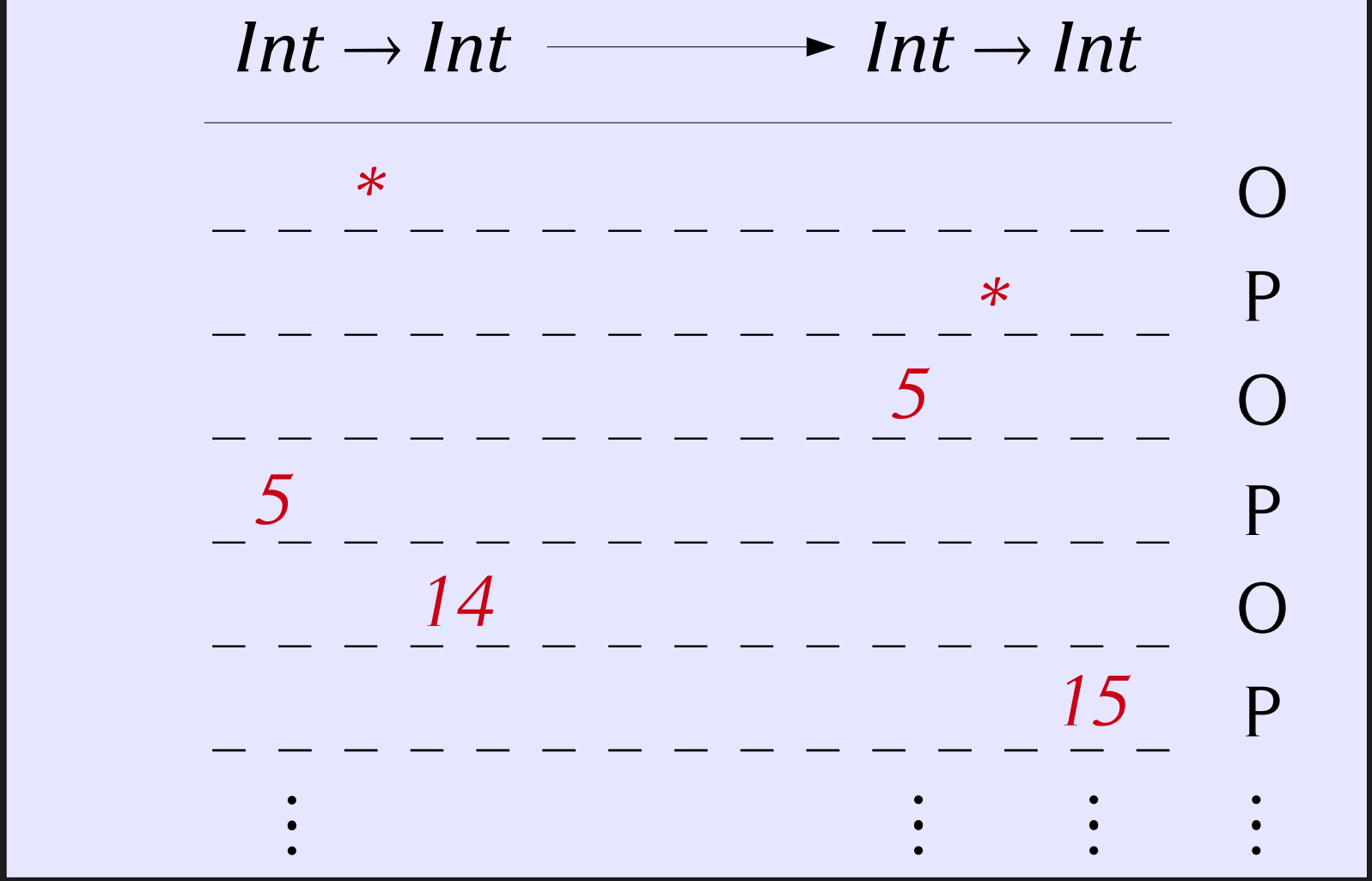

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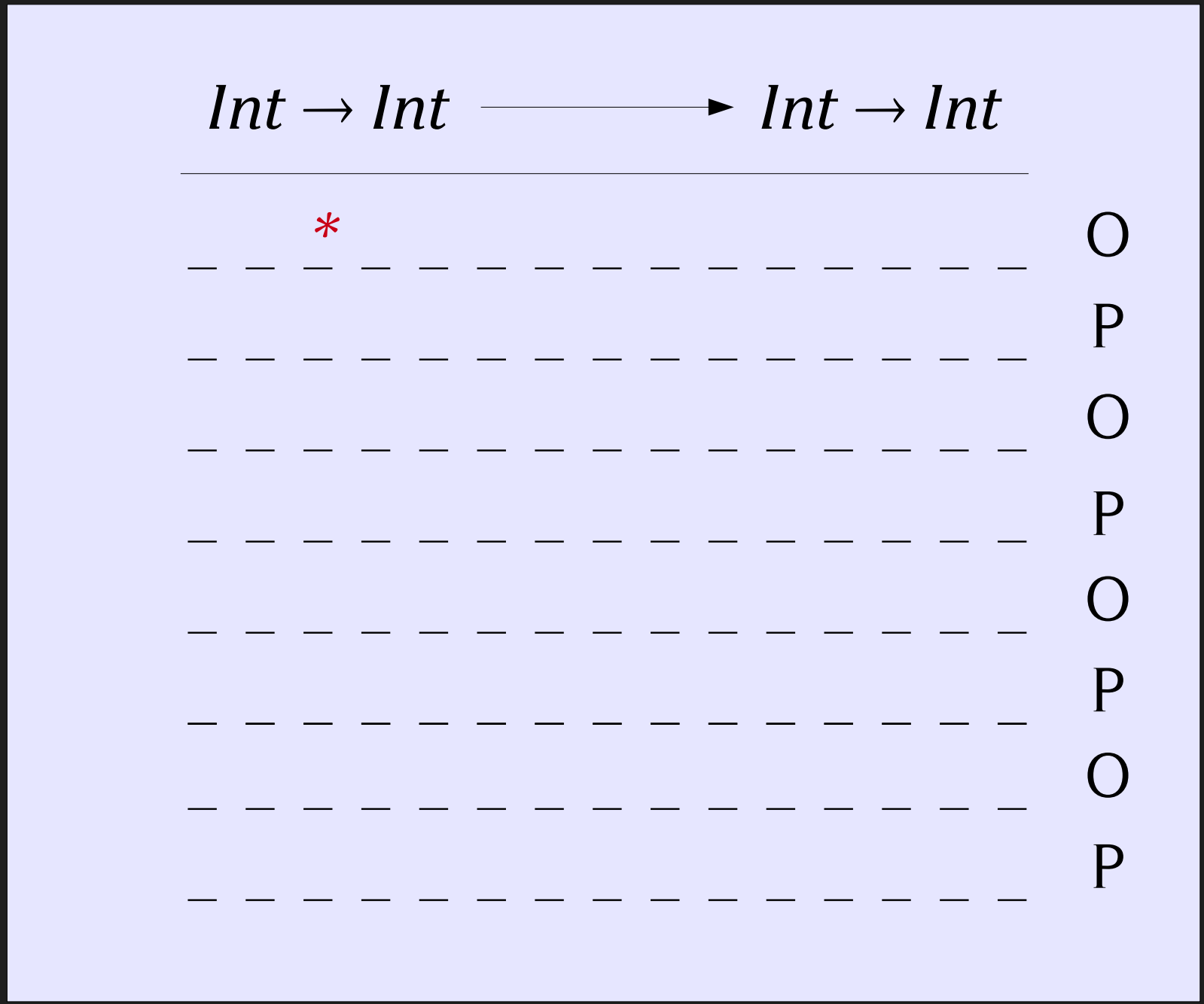

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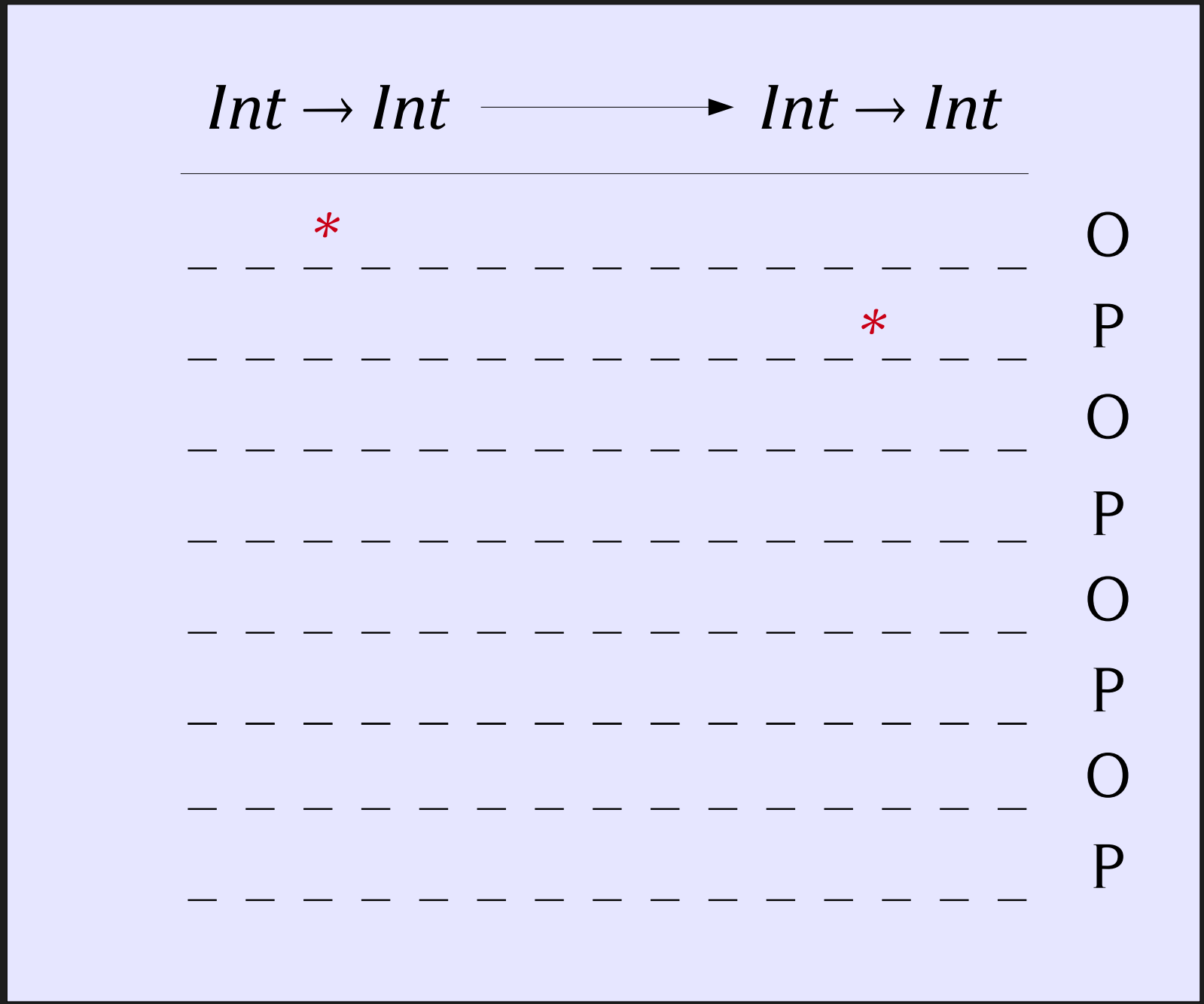

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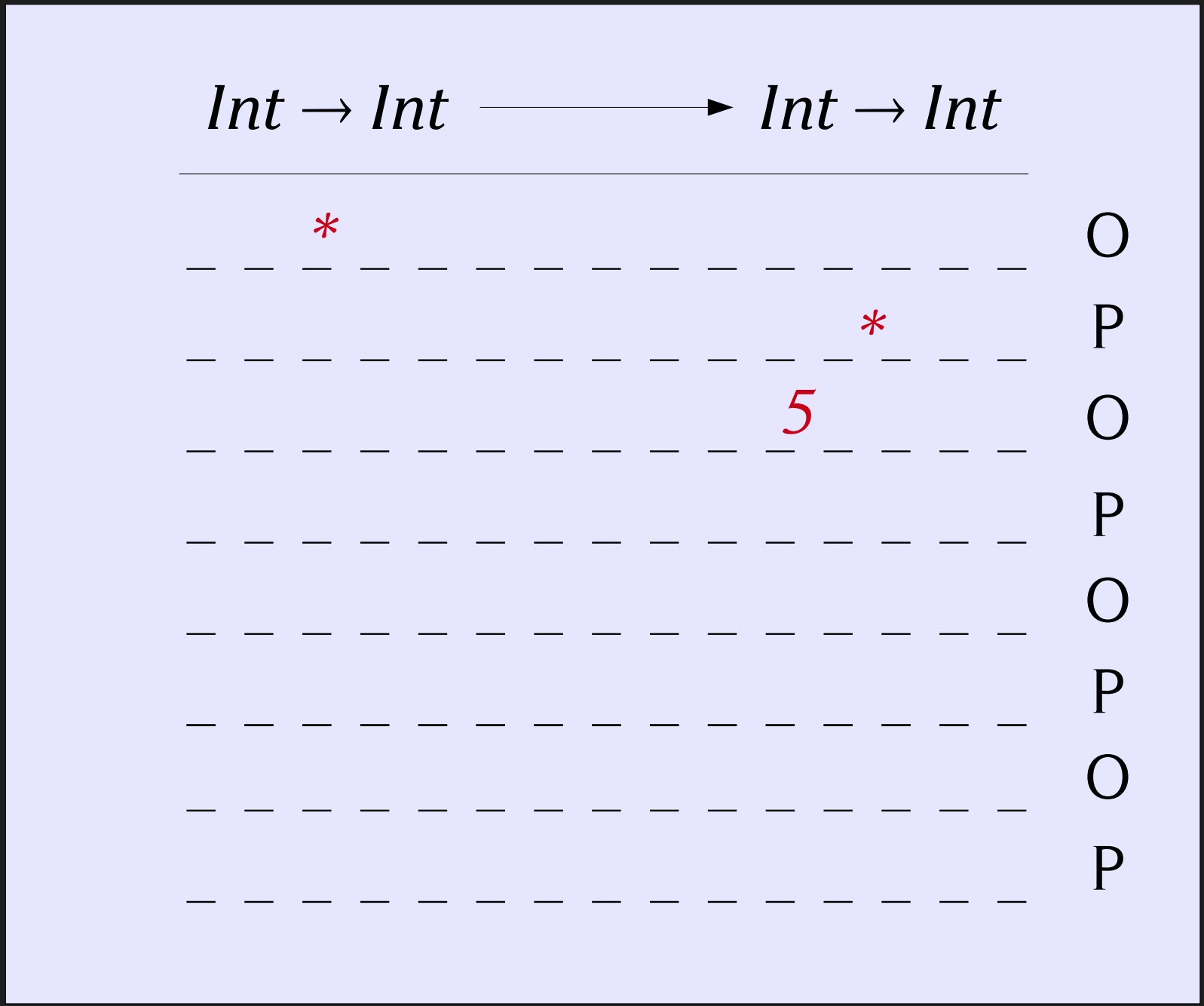

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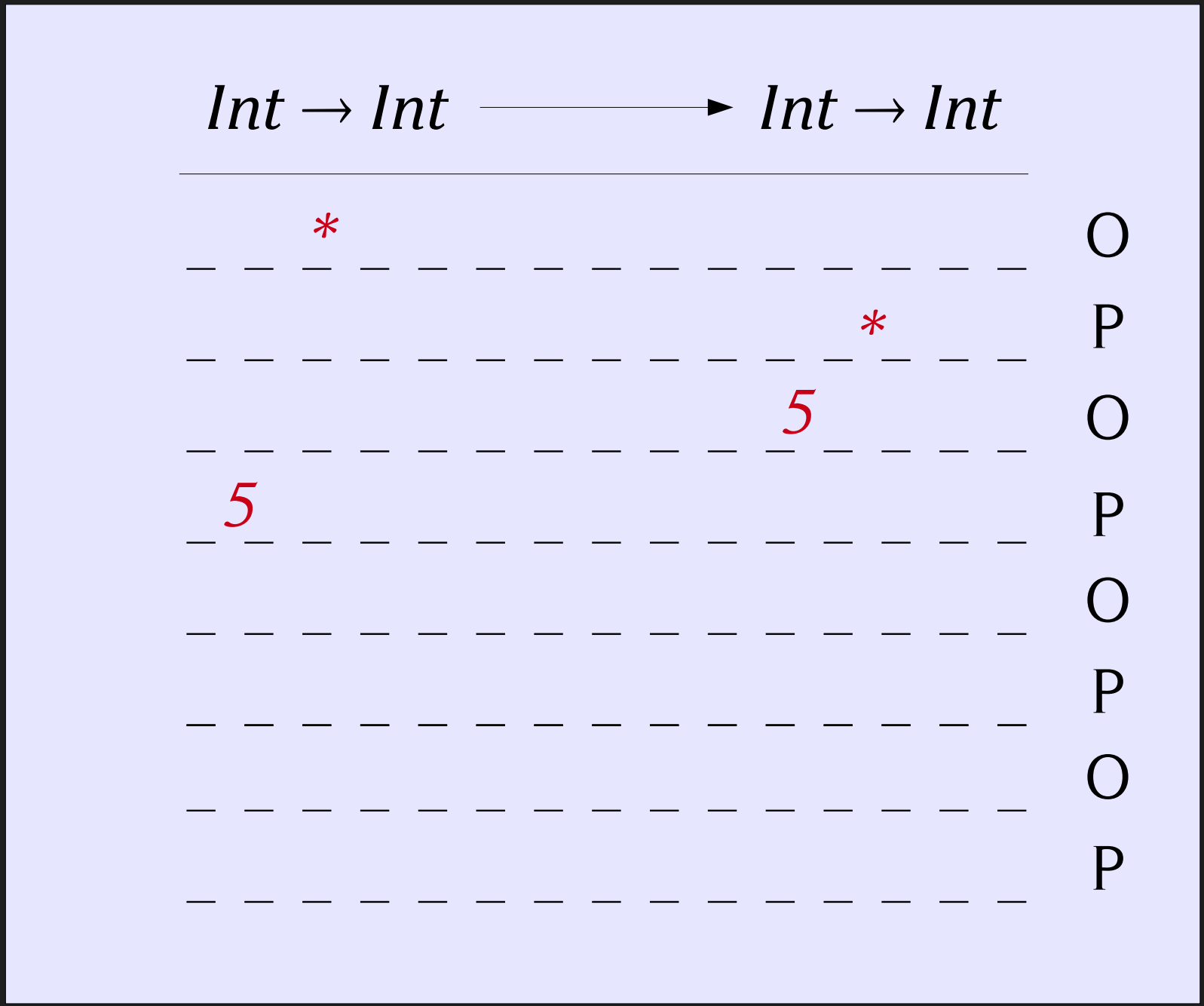

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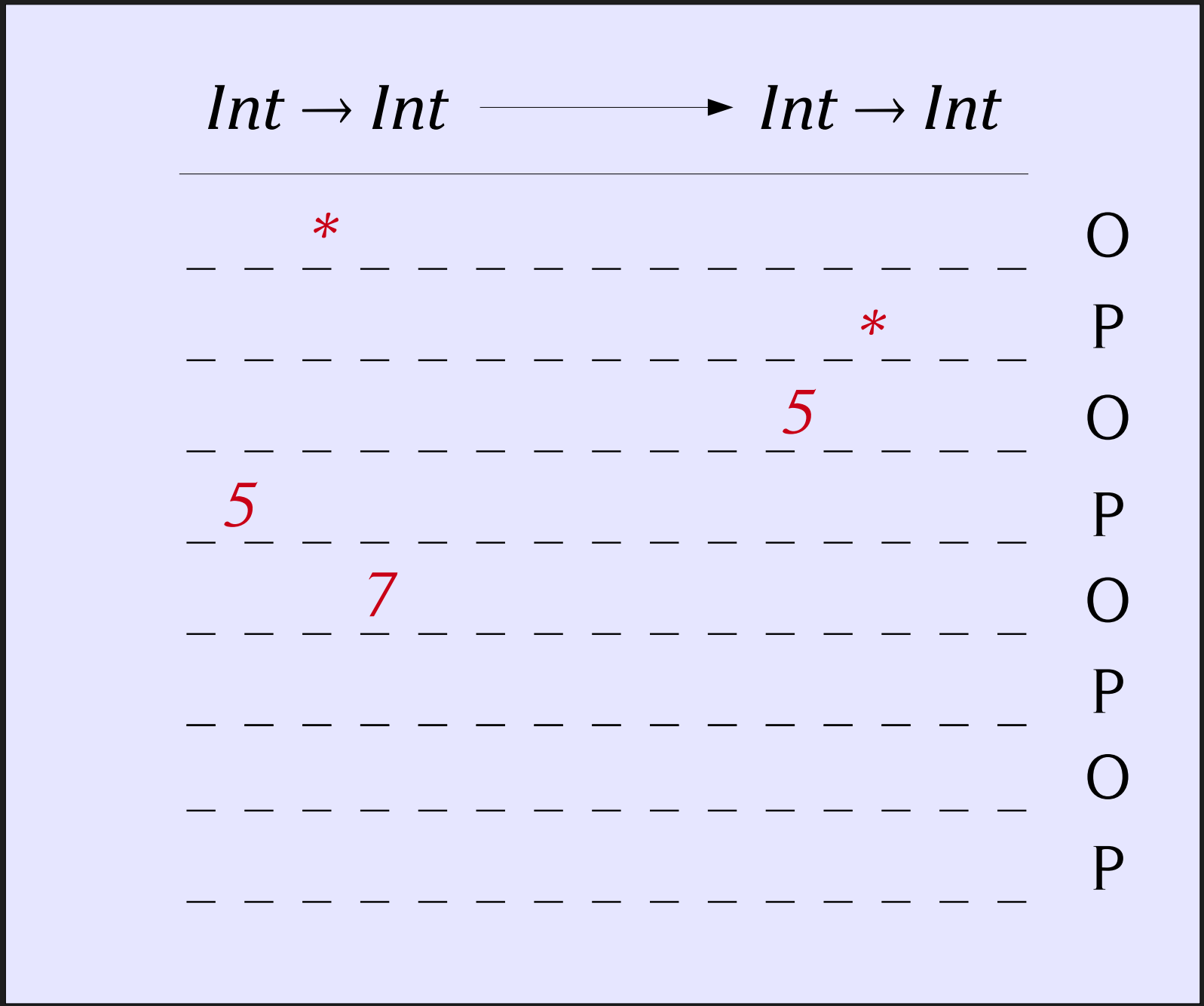

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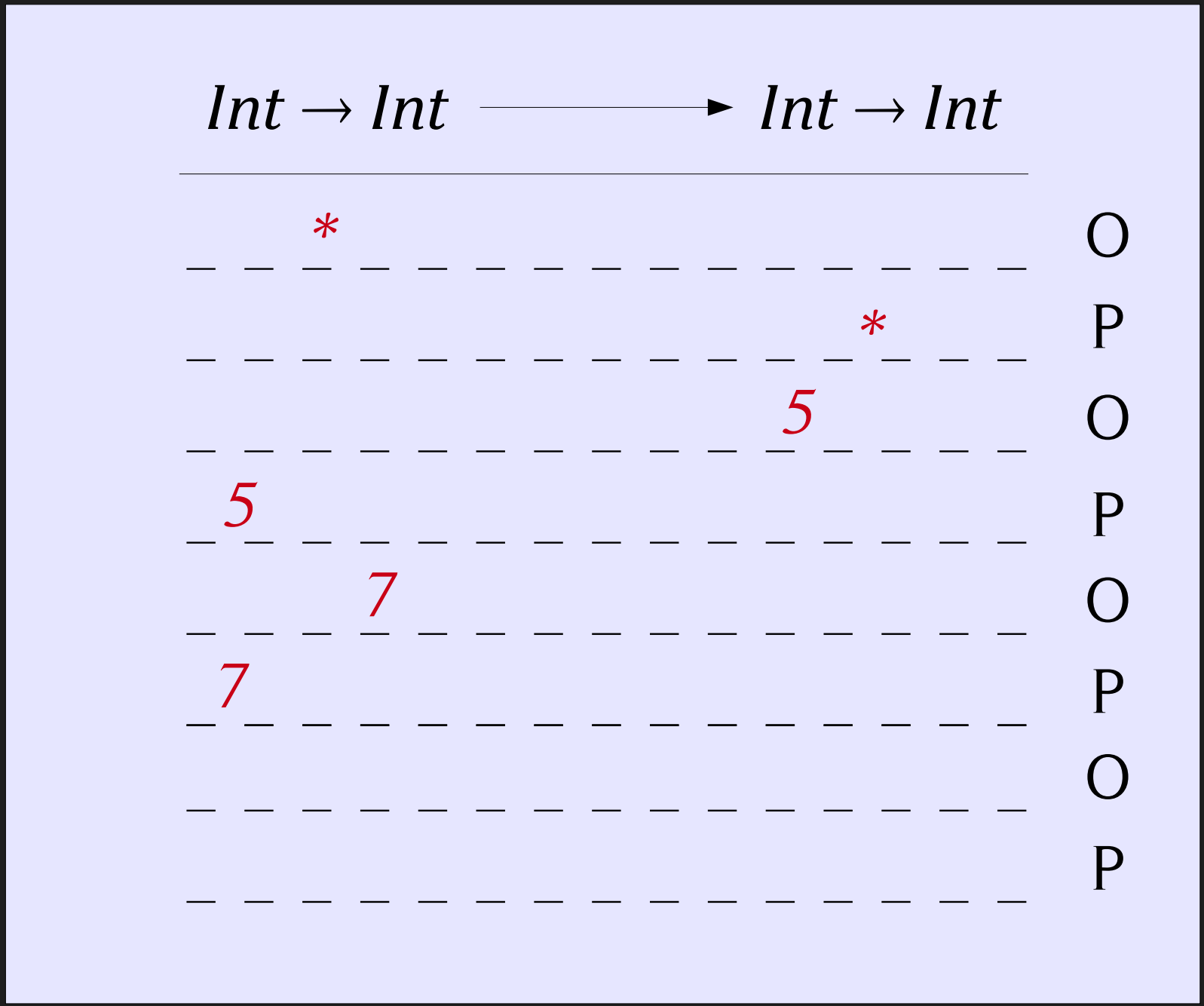

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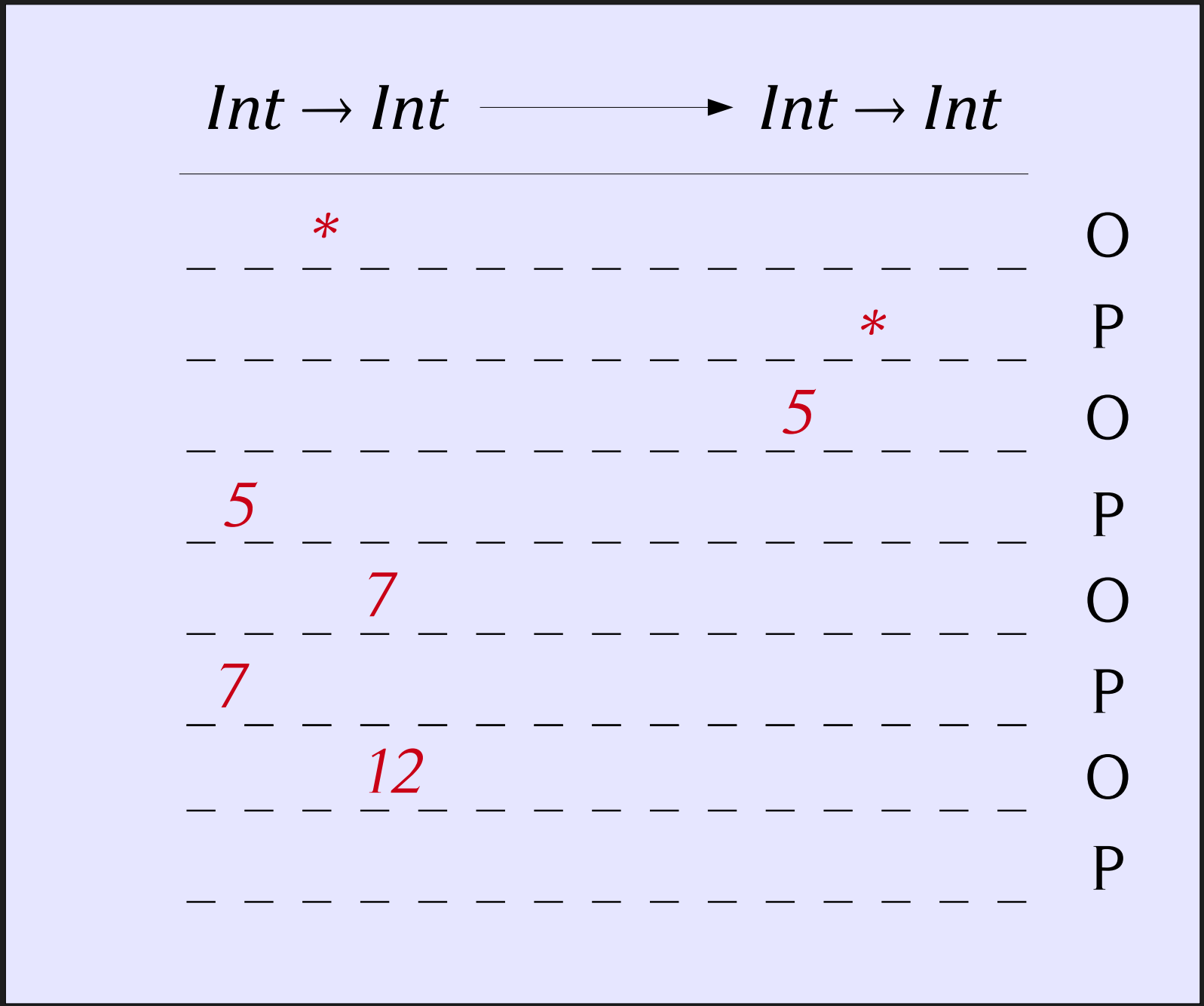

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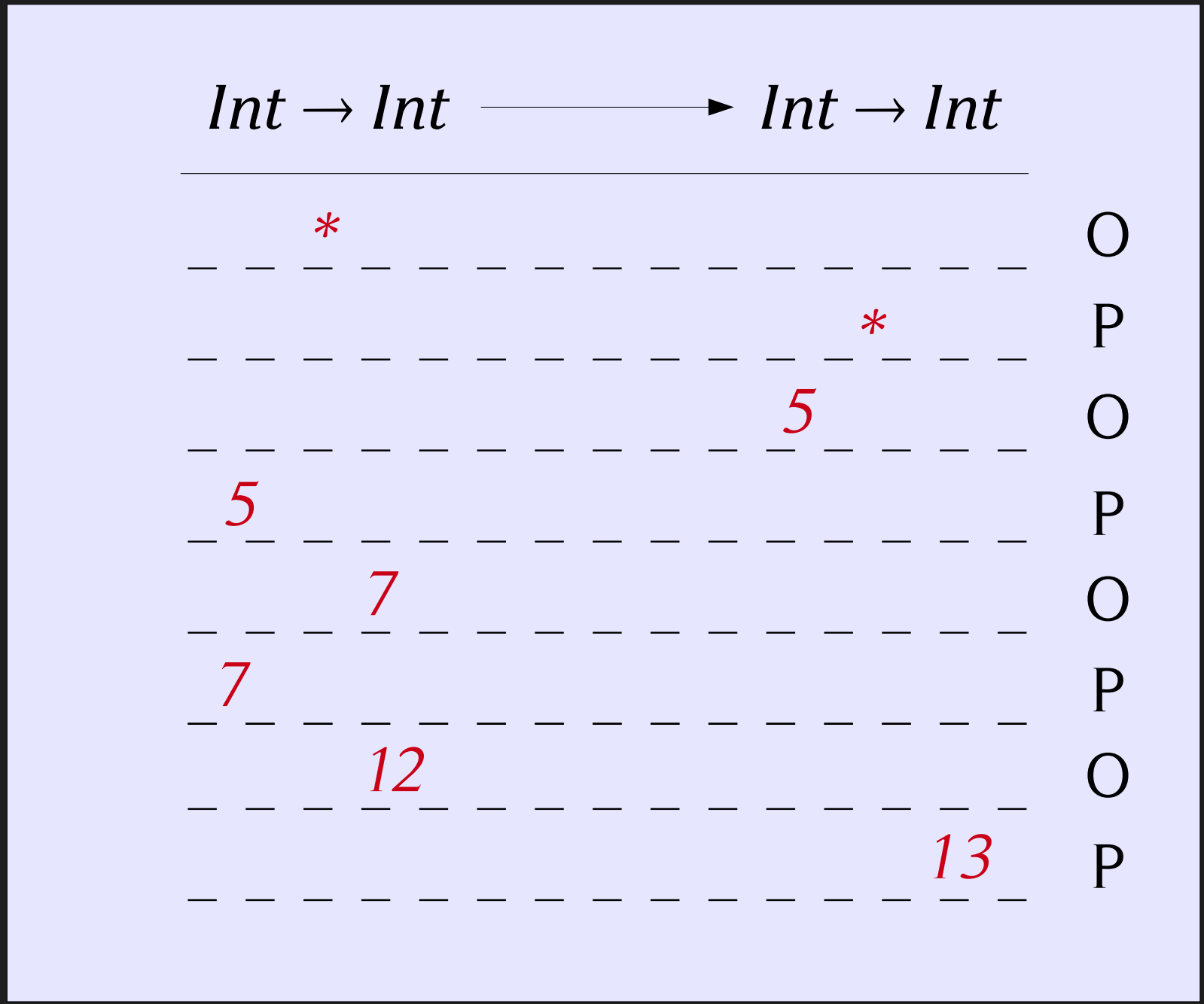

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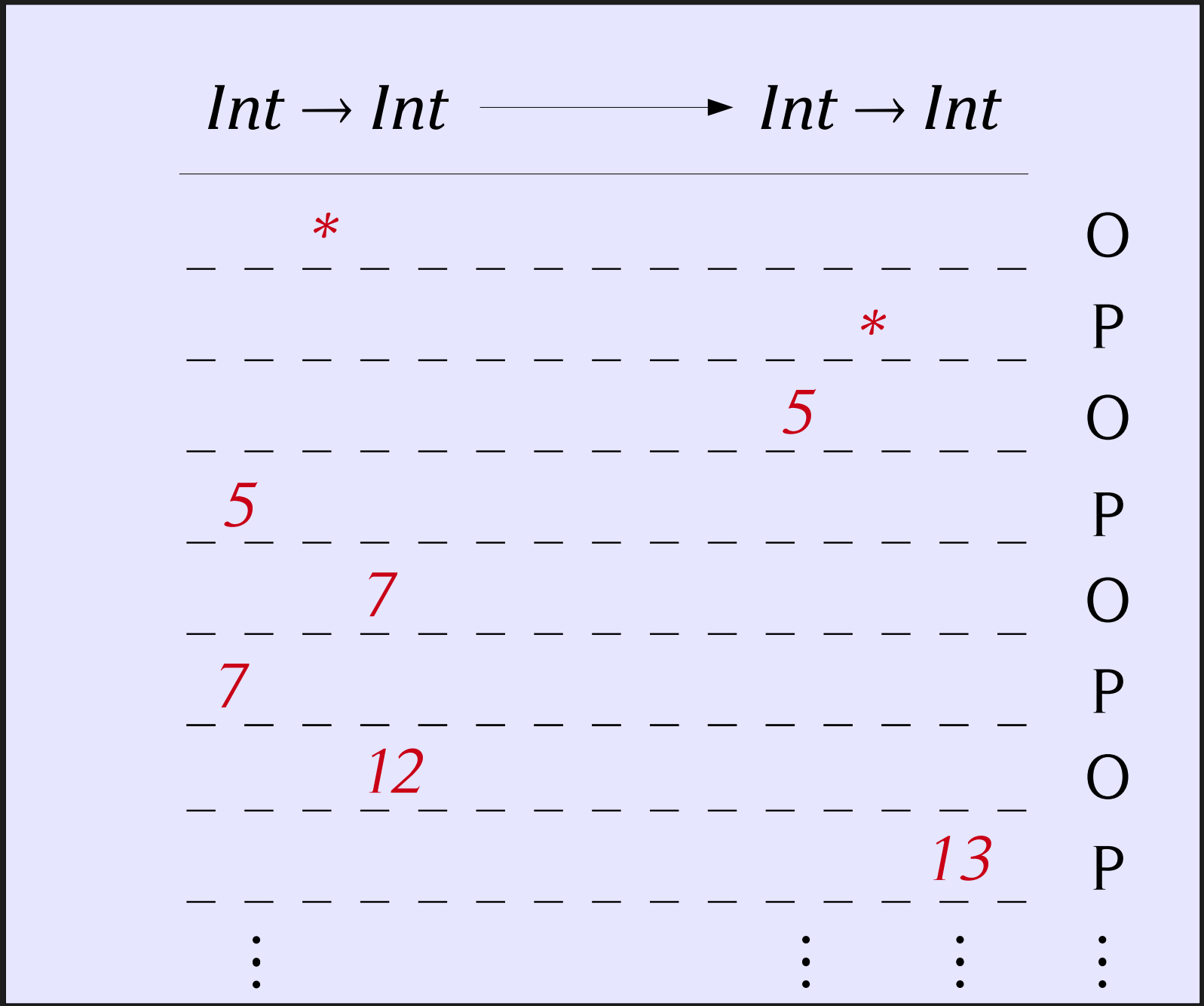

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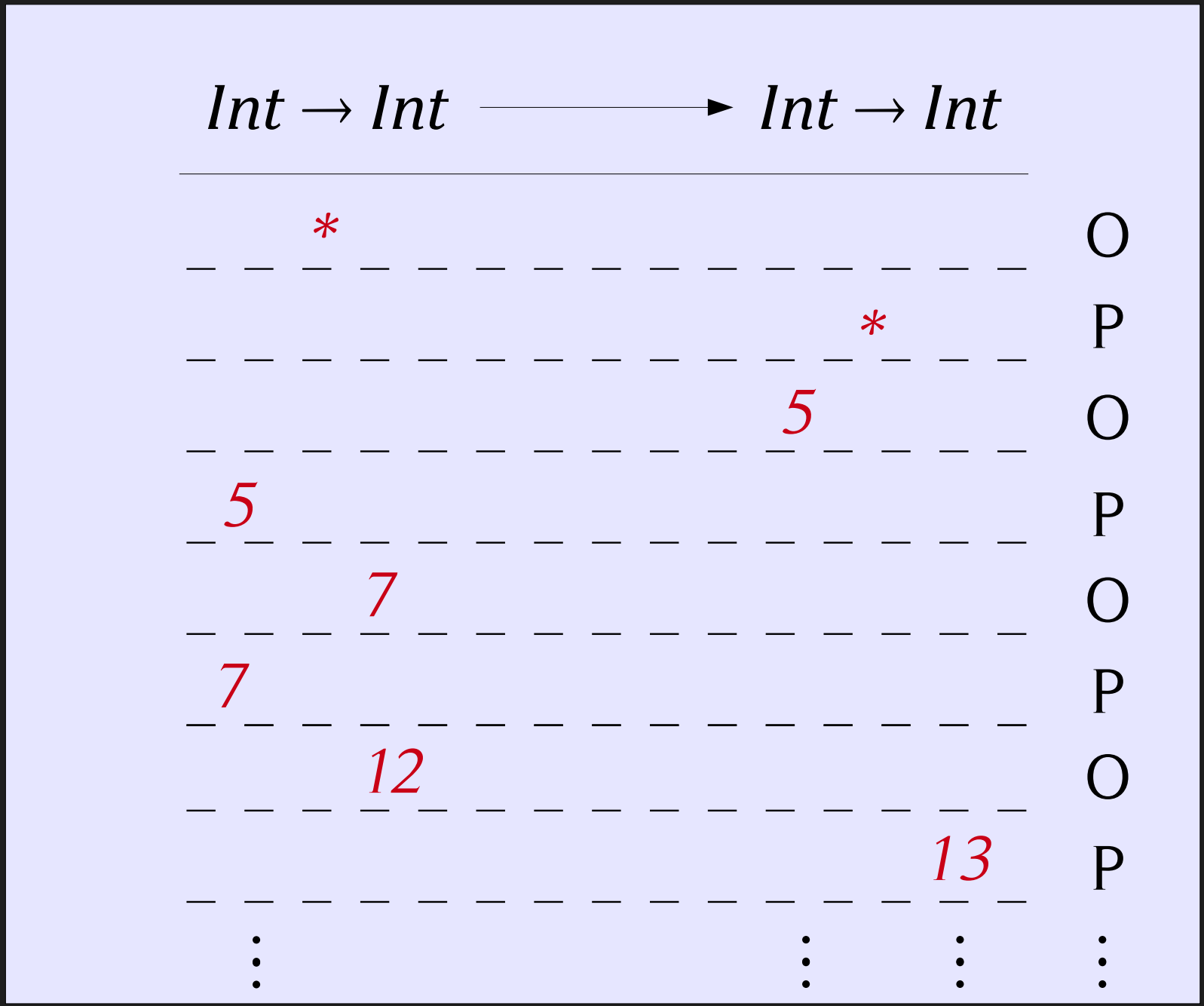

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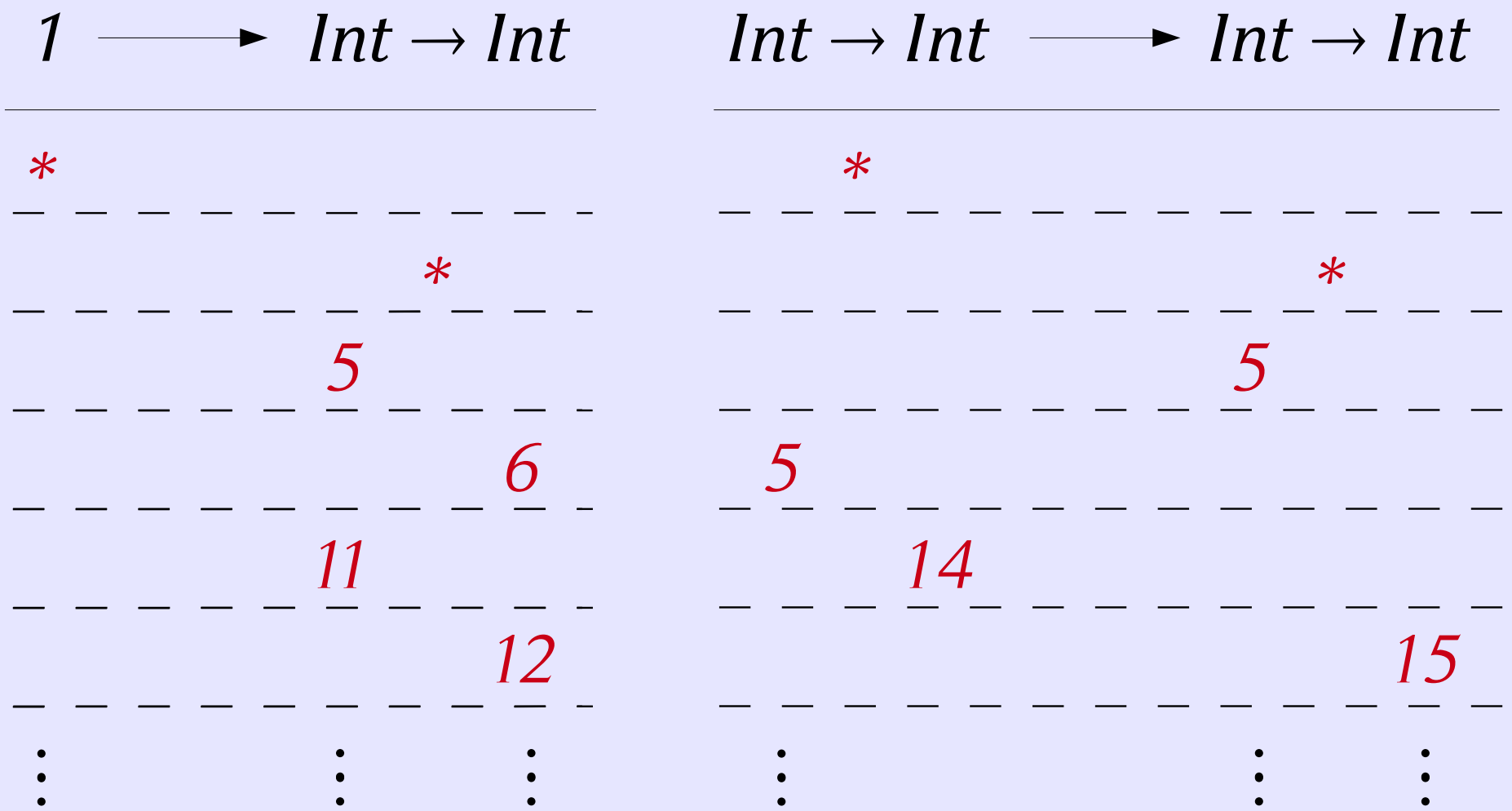

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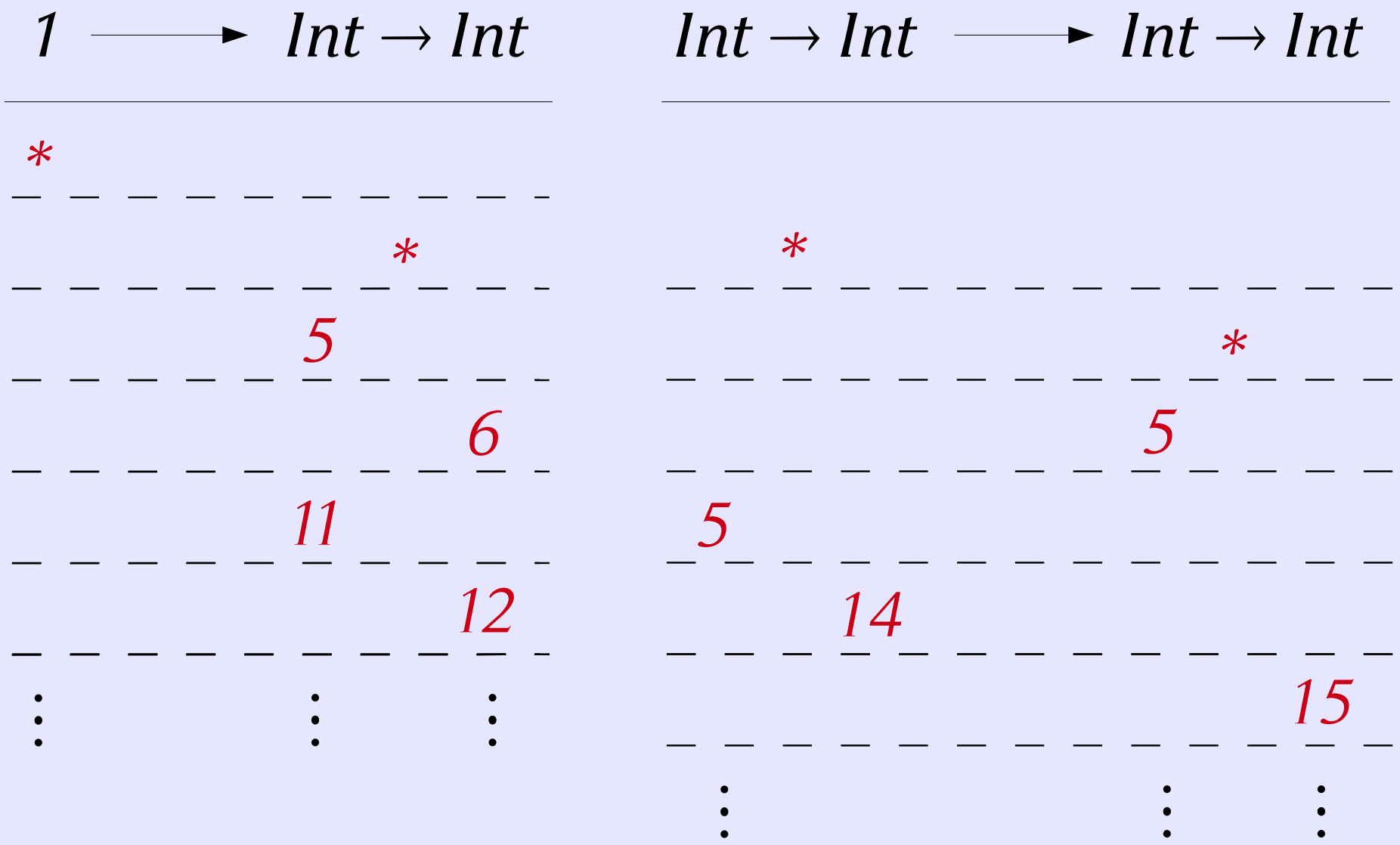

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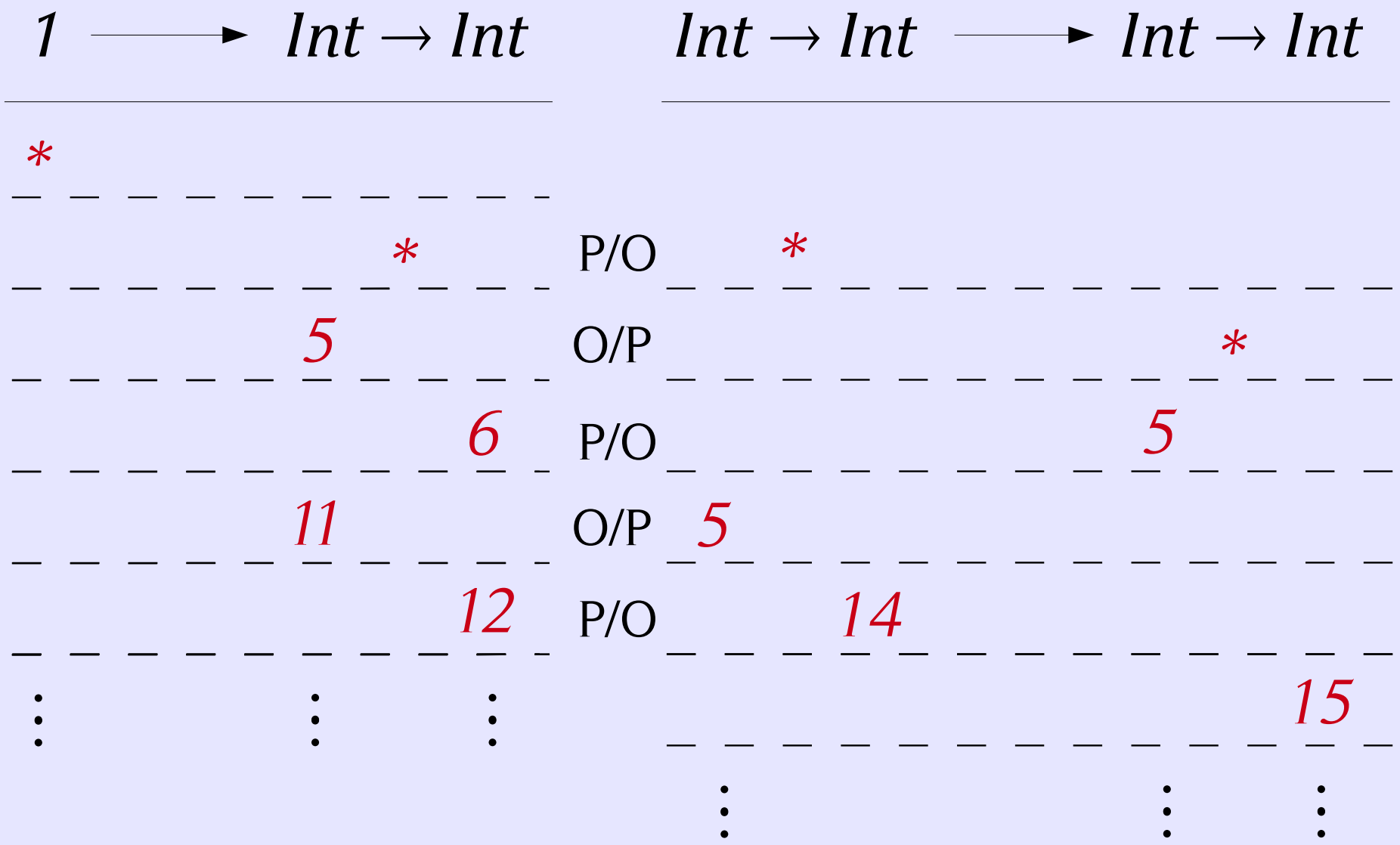

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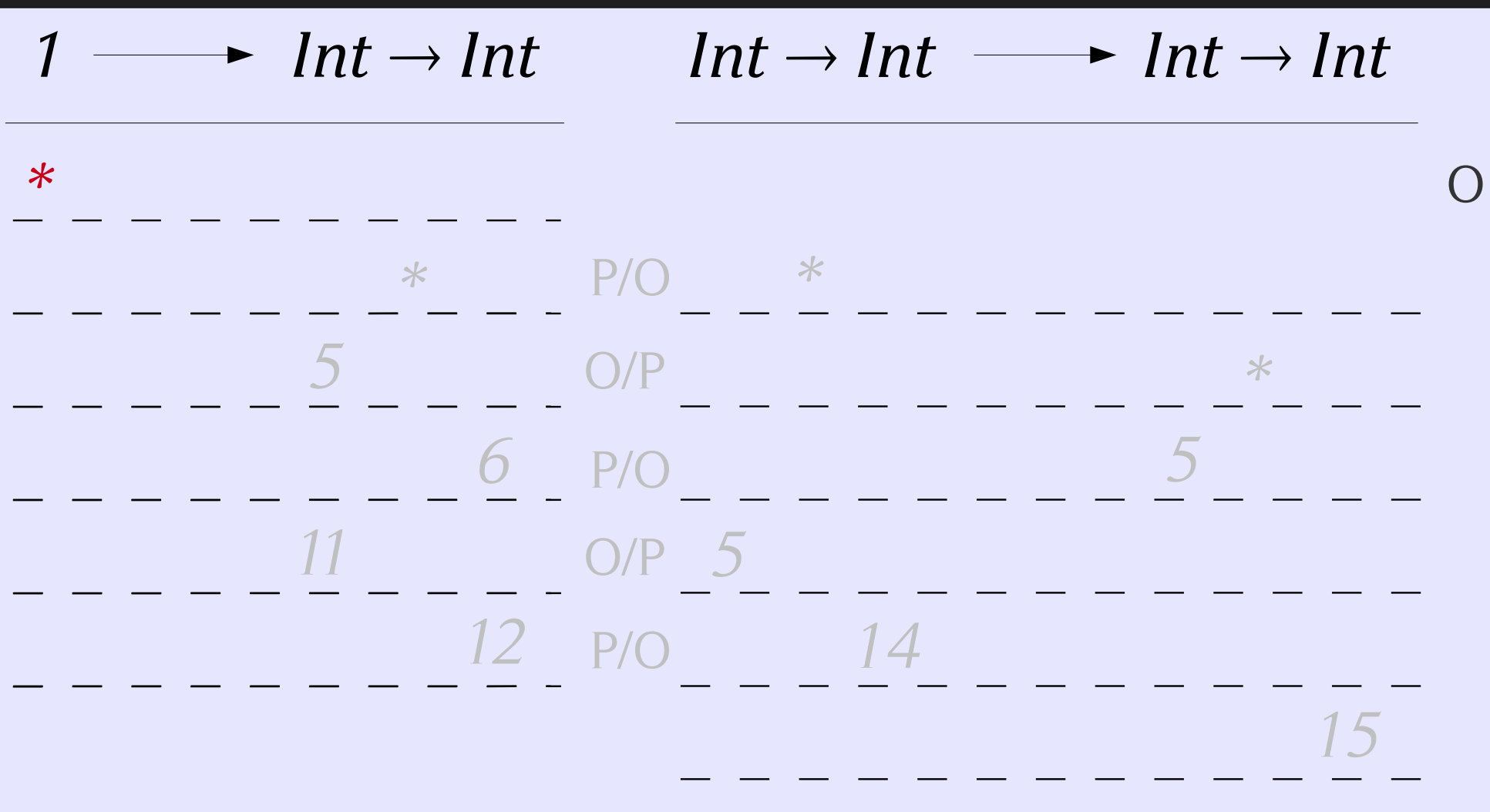

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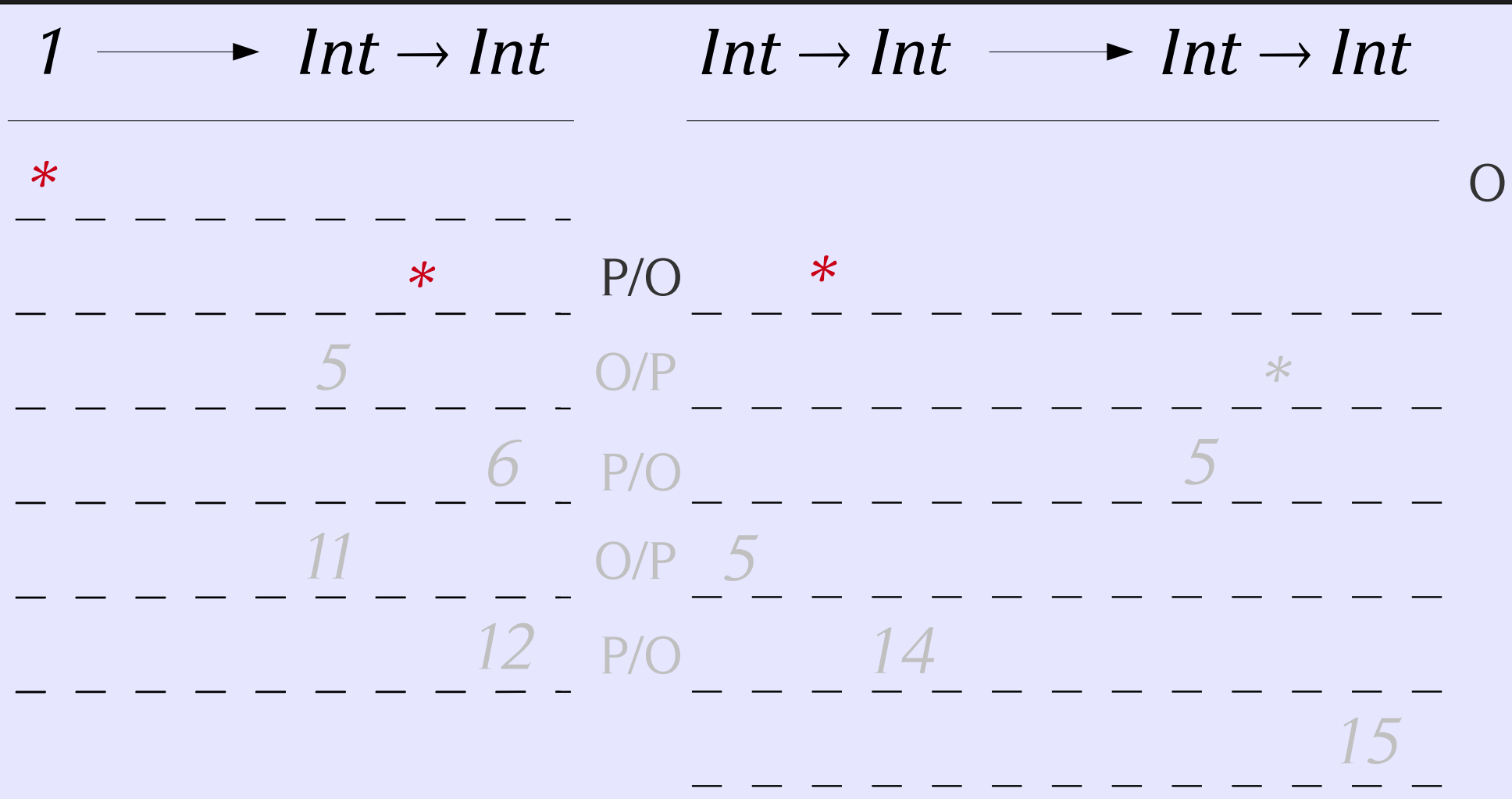

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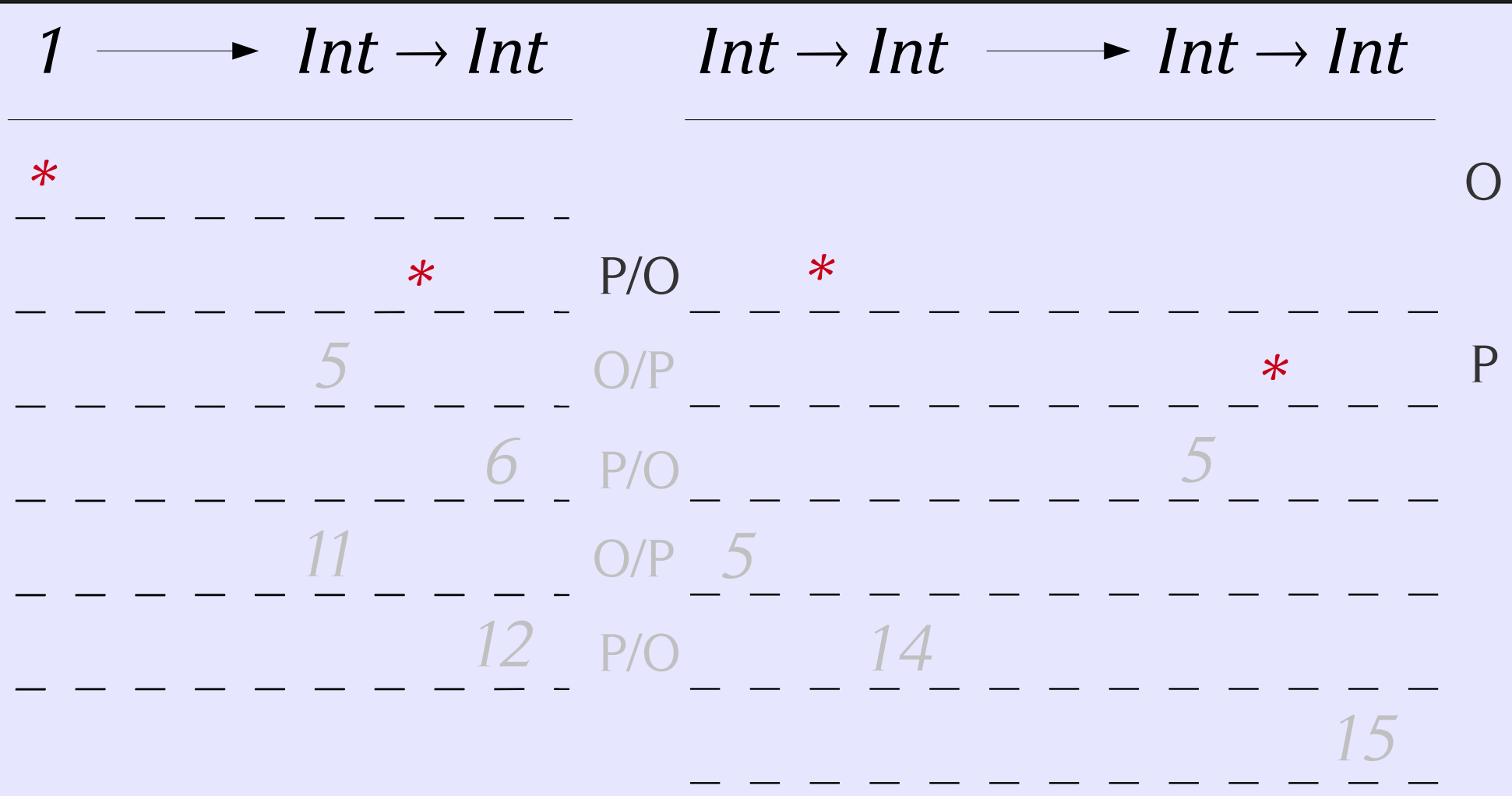

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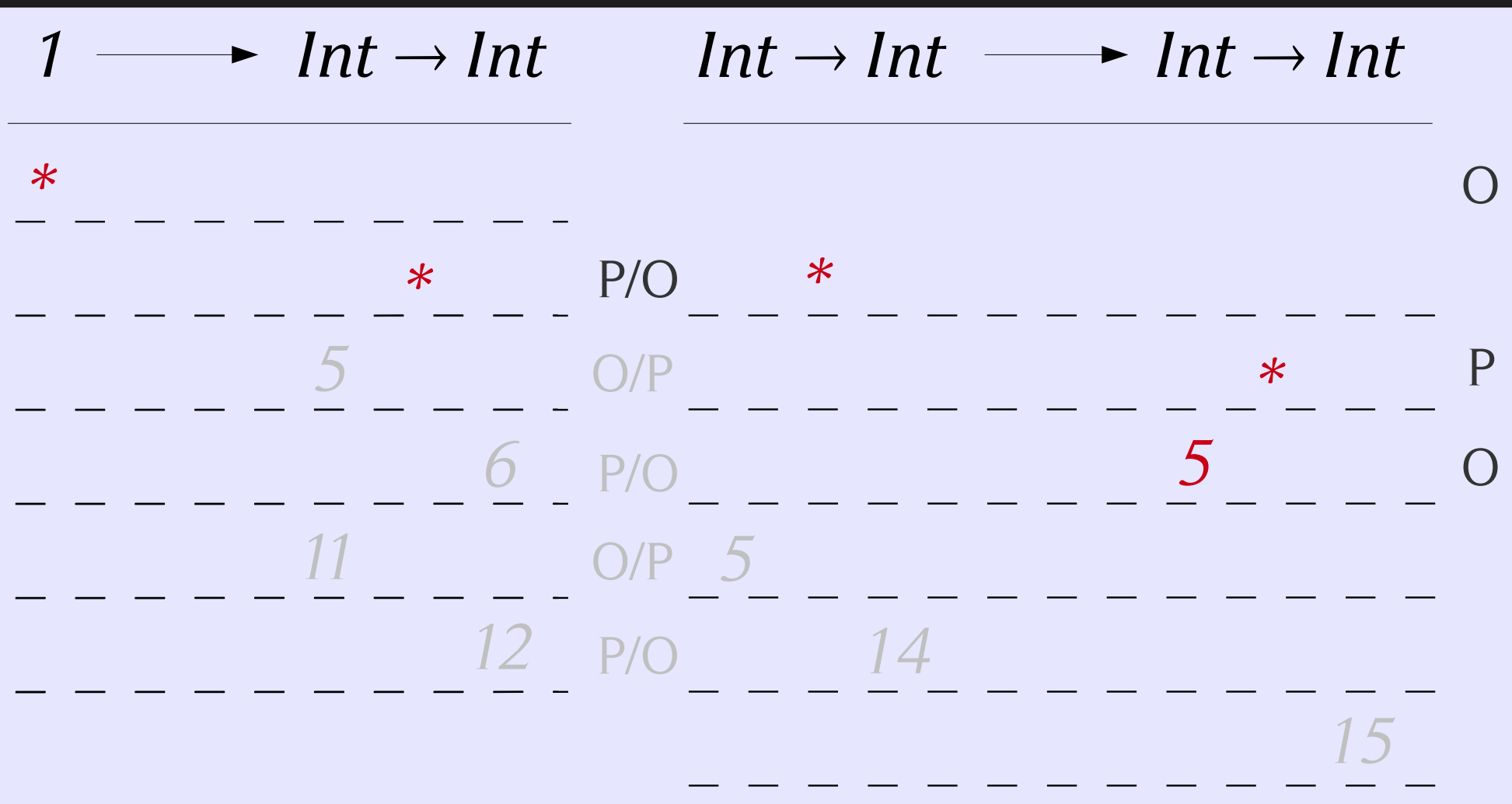

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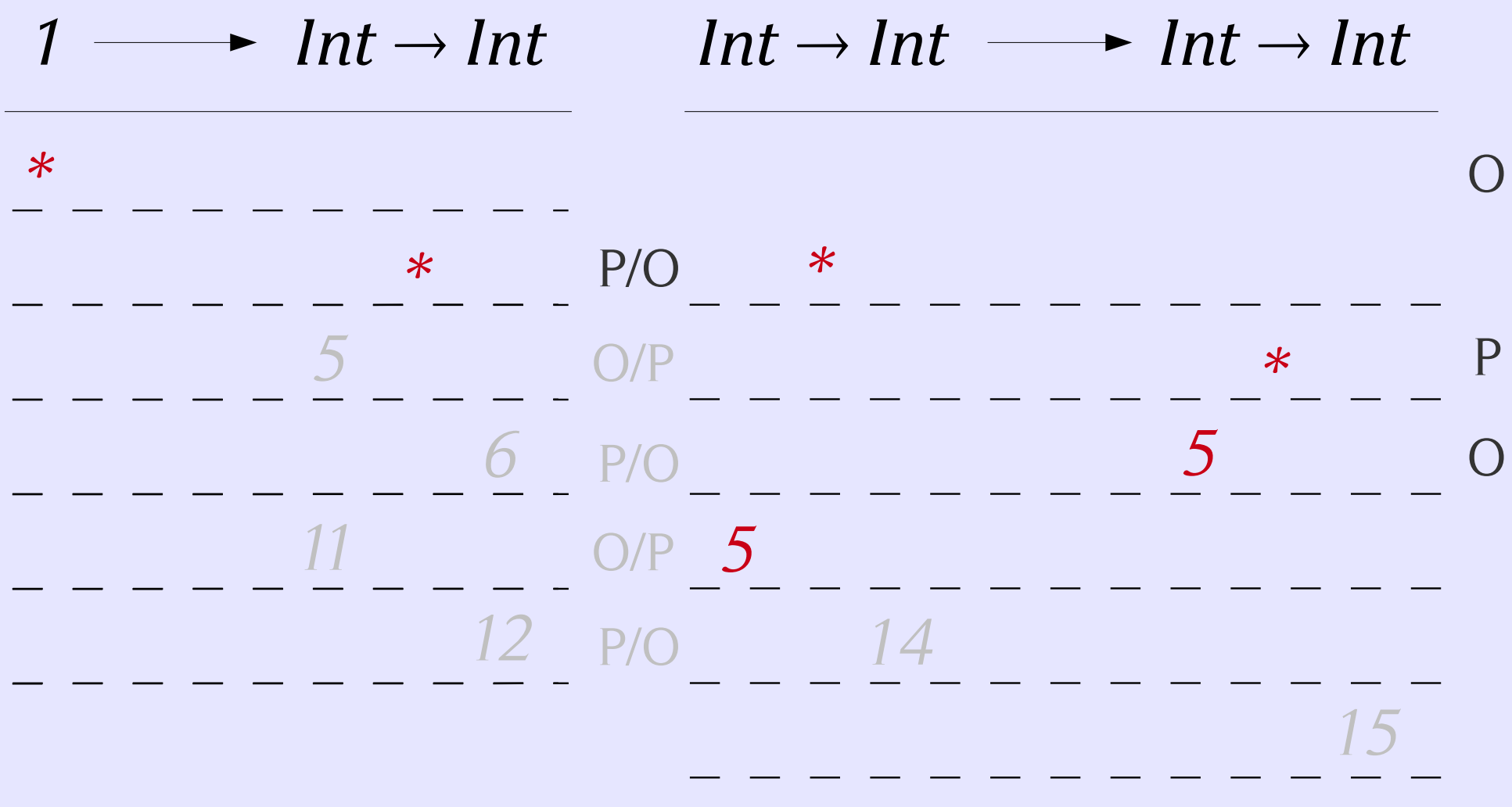

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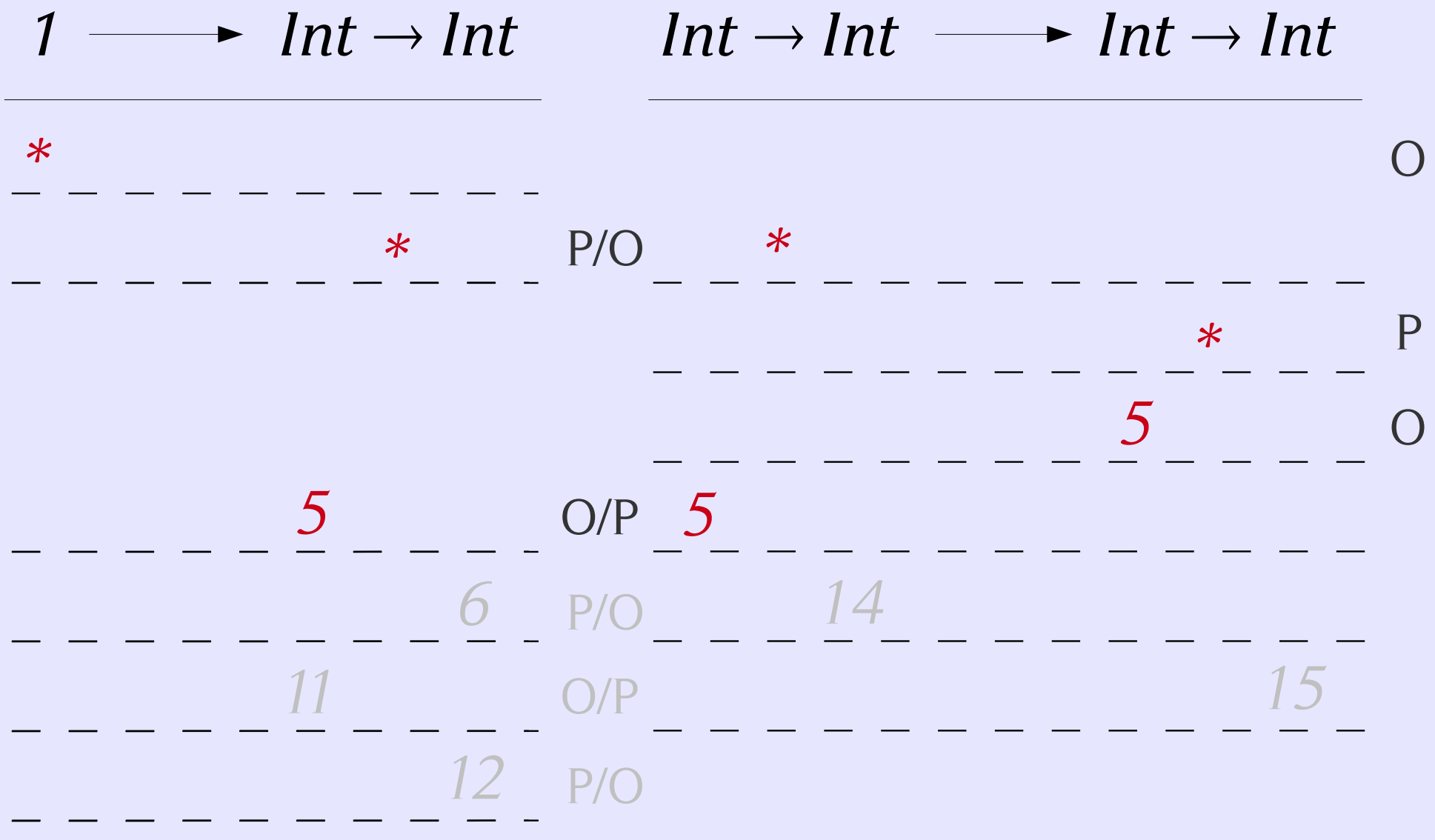

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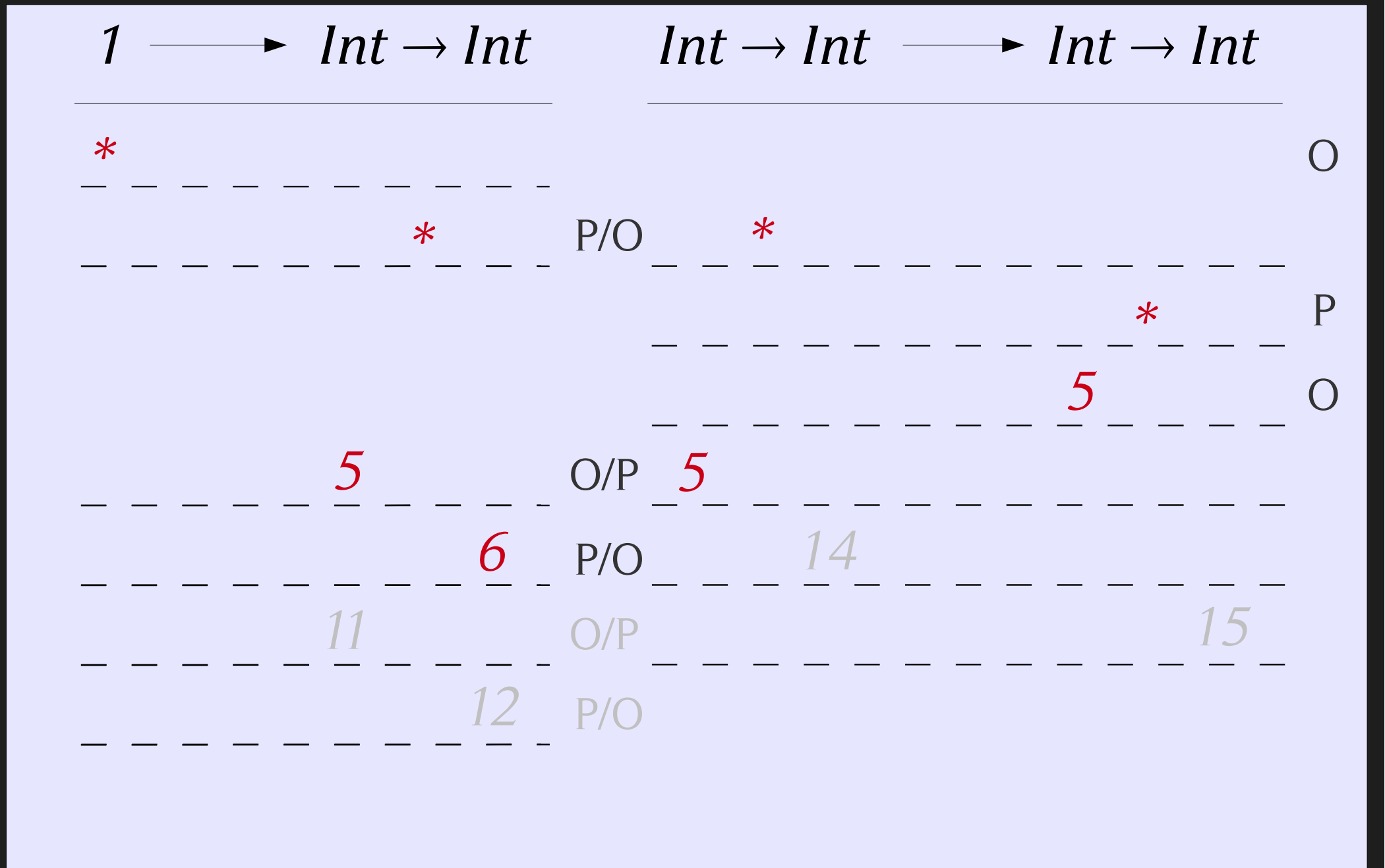

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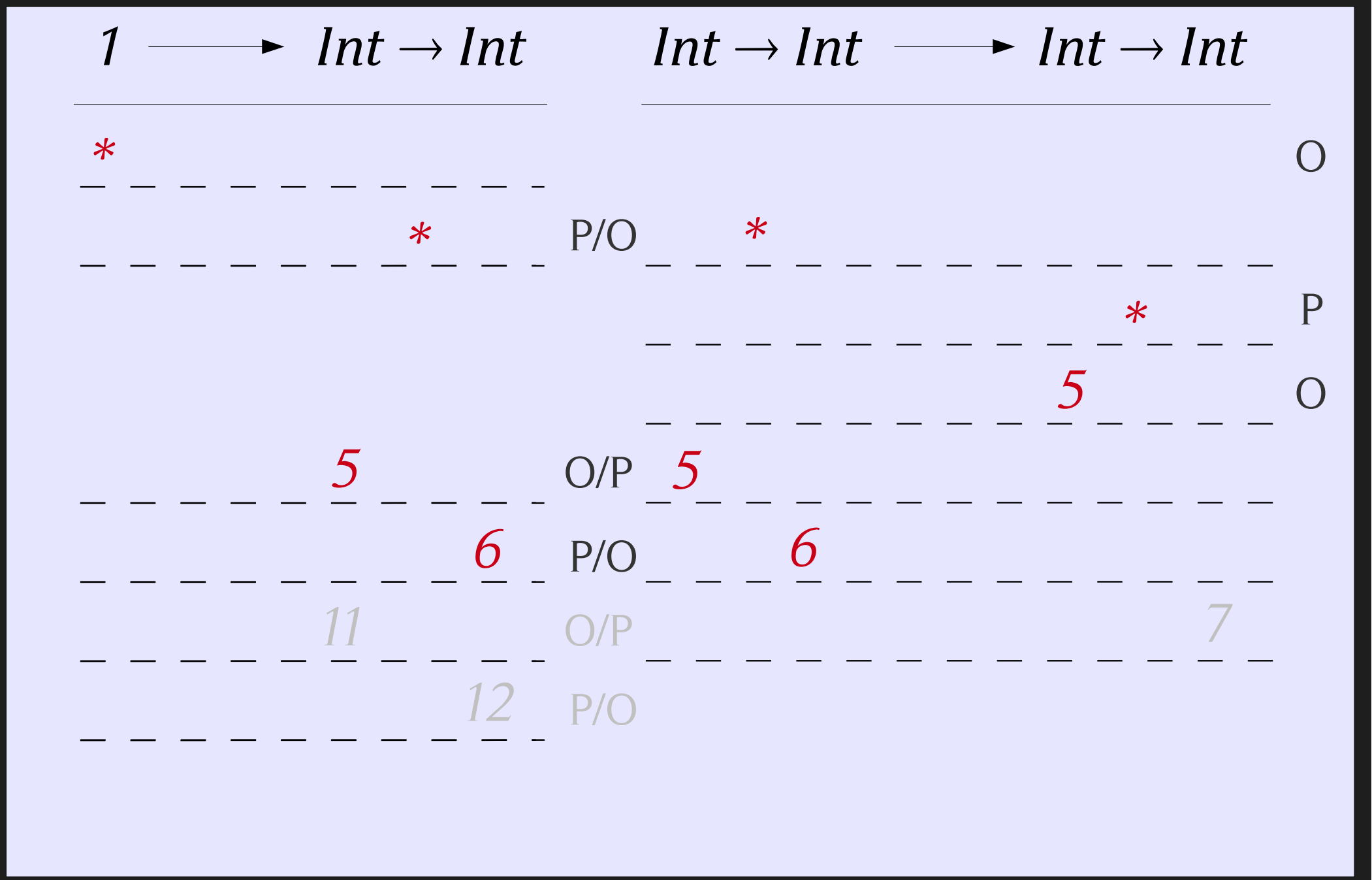

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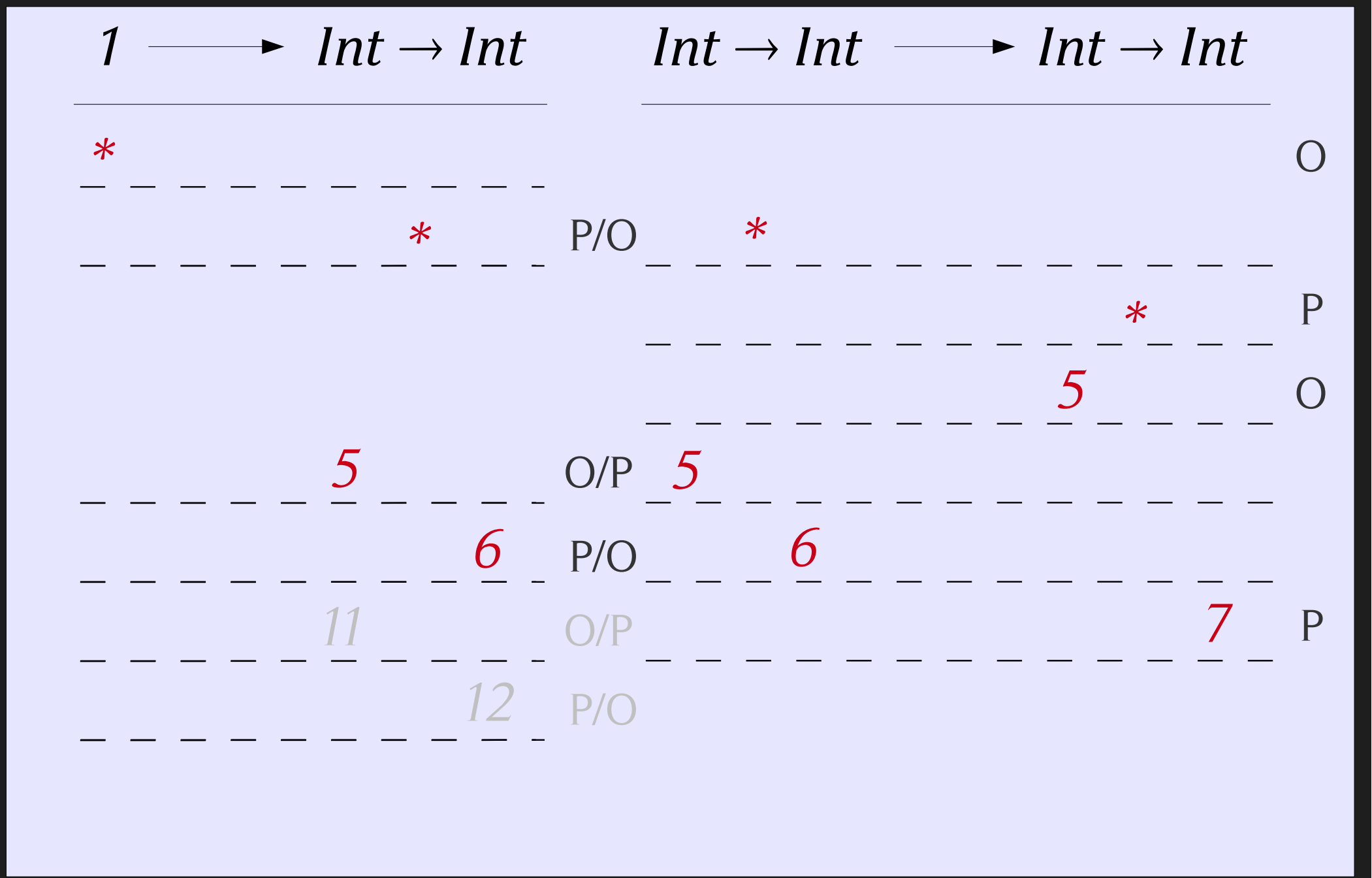

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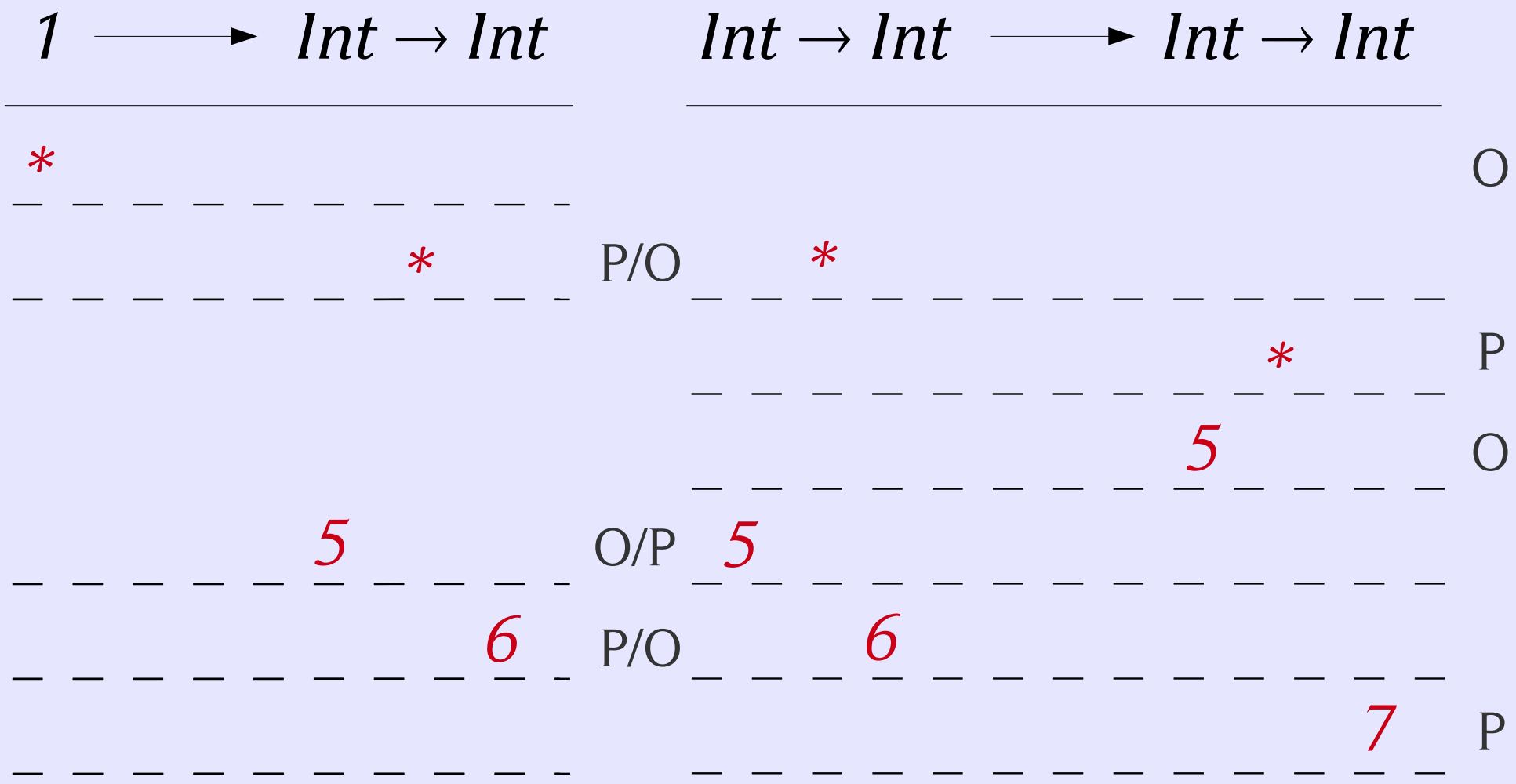

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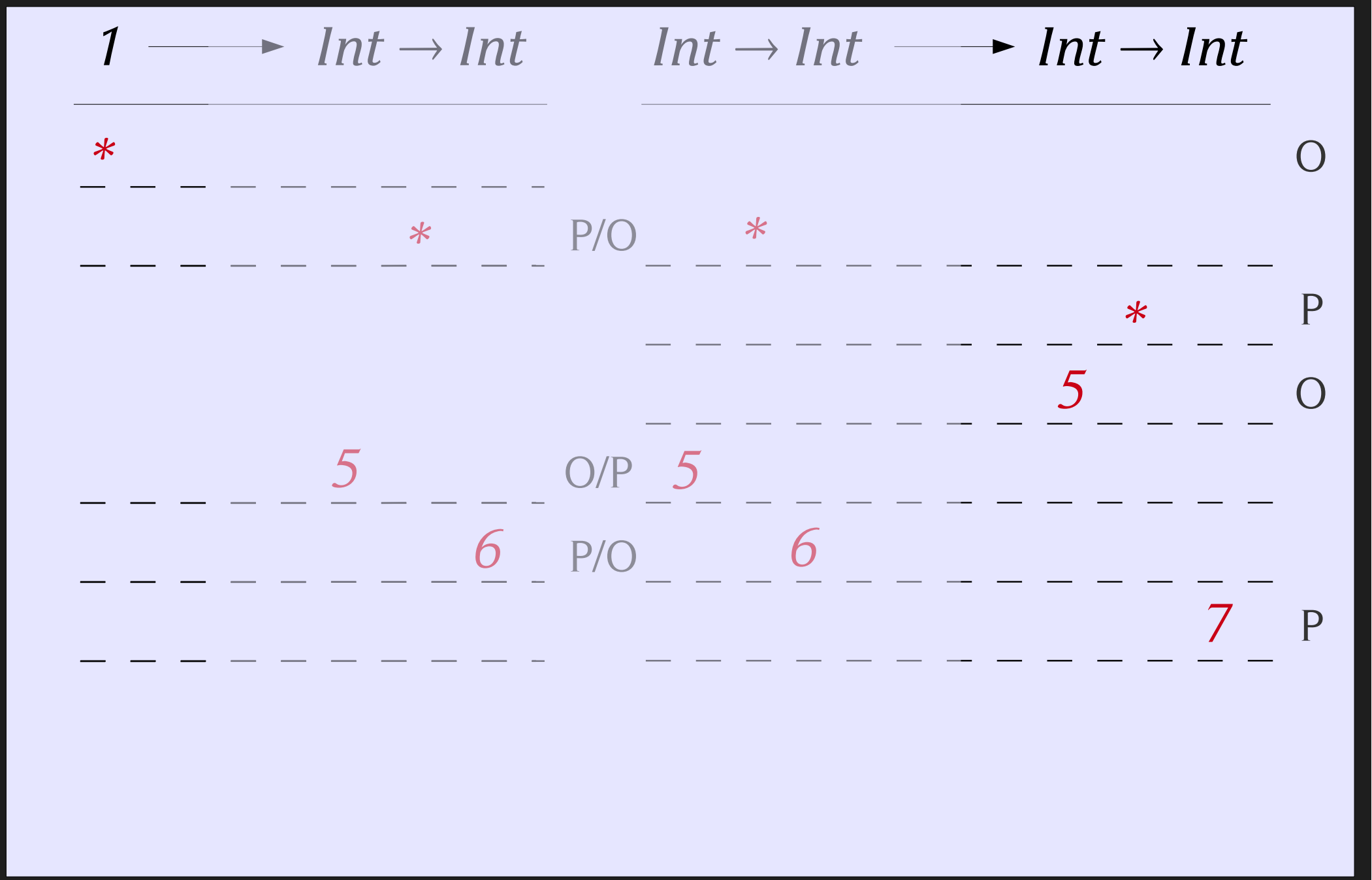

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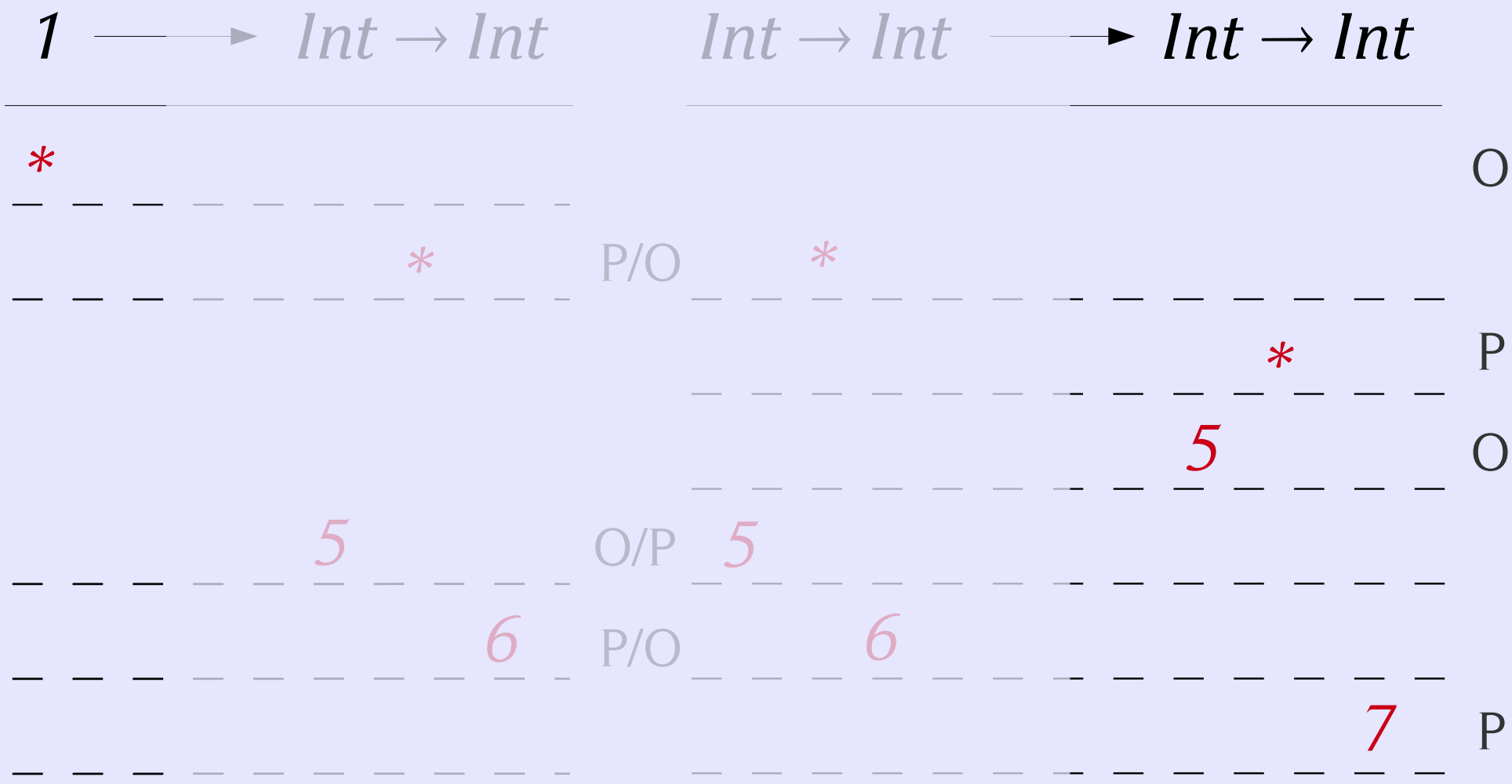

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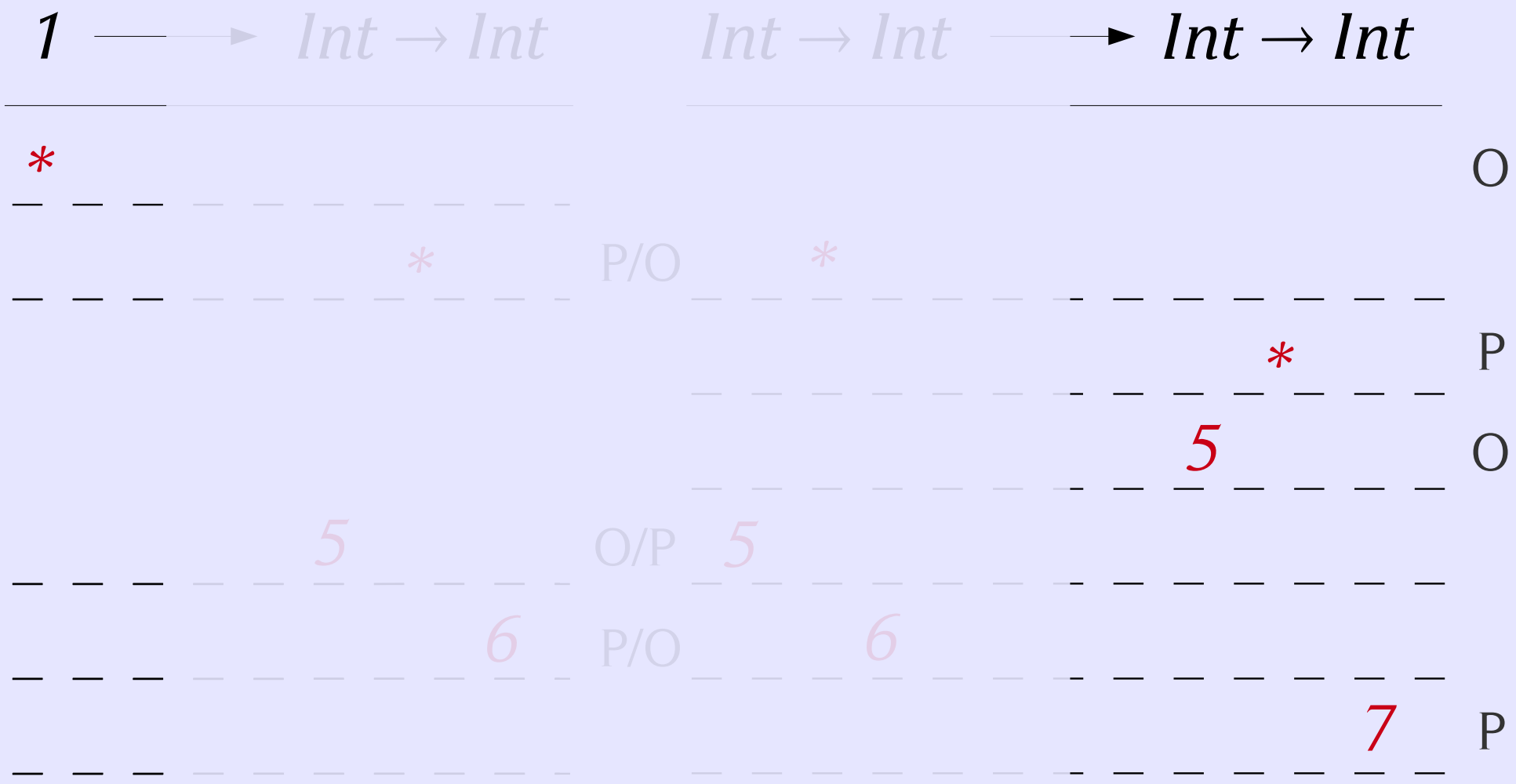

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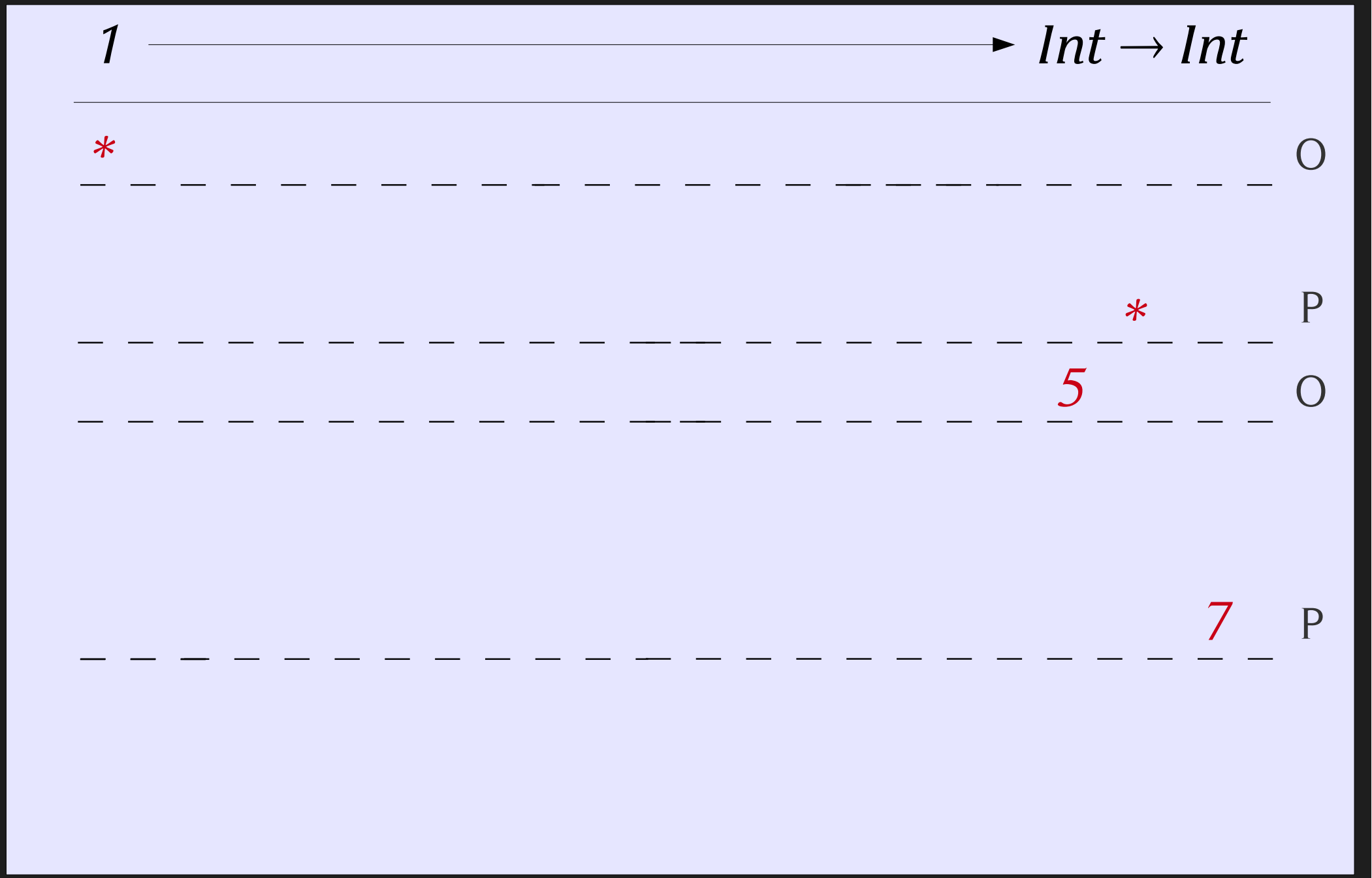

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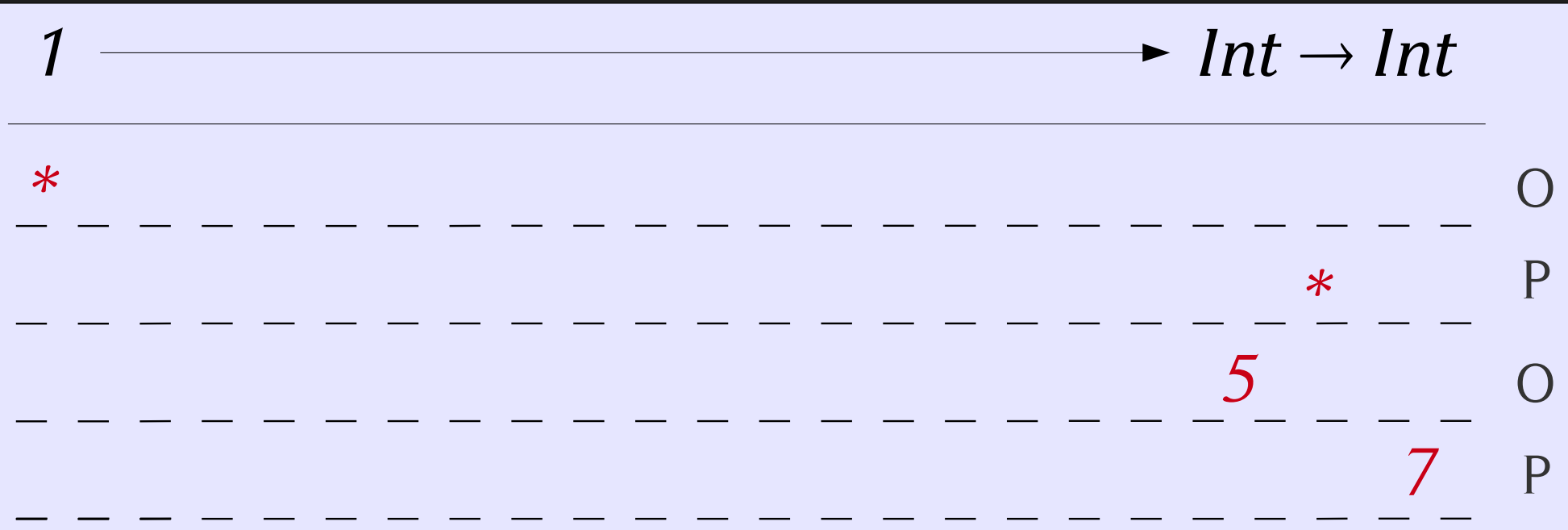

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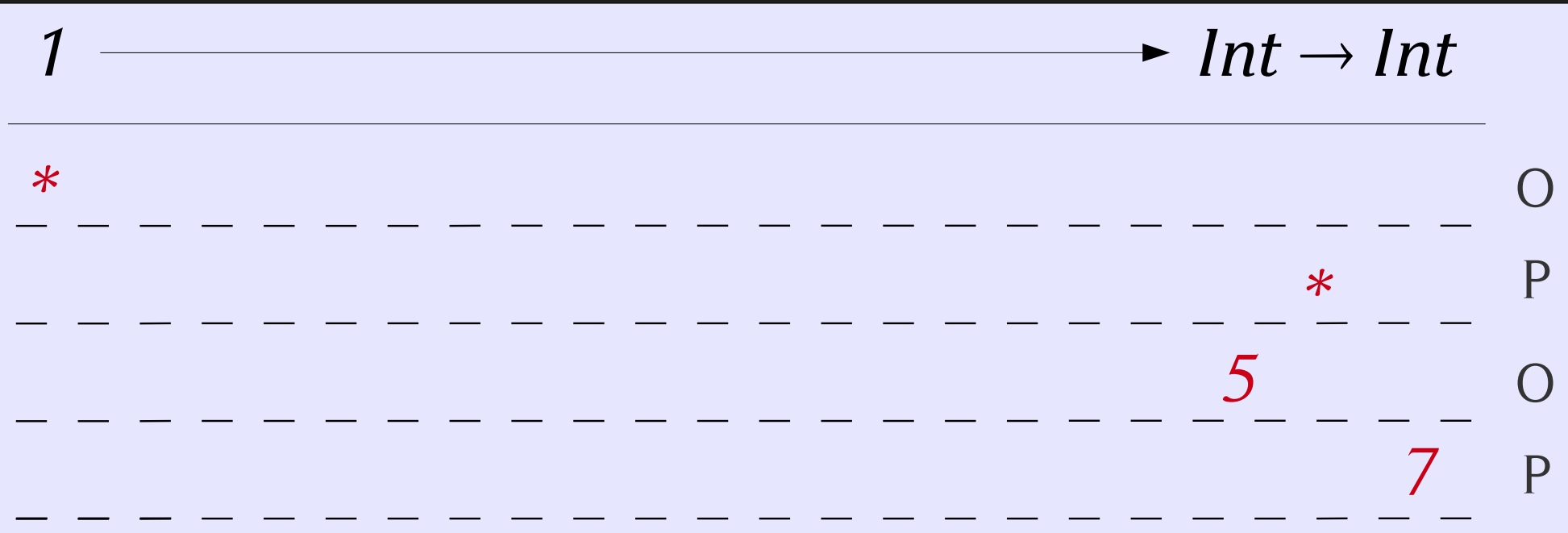

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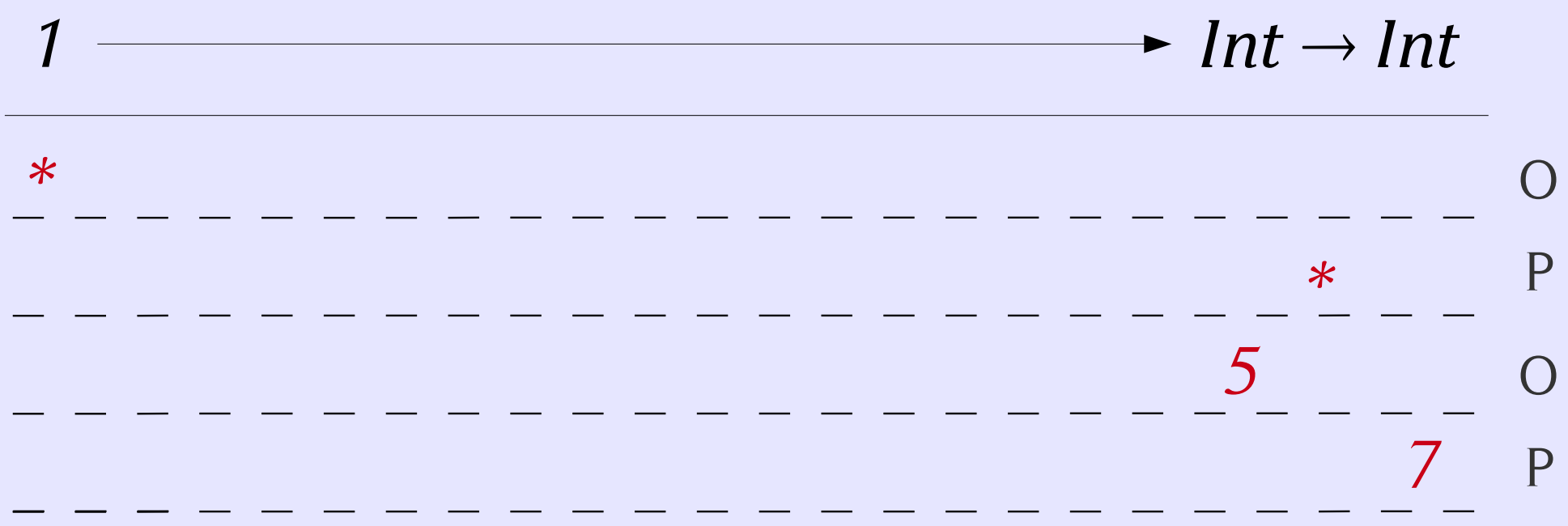

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- Qualitative games
- Programs = *strategies* for Proponent
- Families (i.e. *categories*) of games

Quick storyline

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Introduced in early 90's

Full abstraction for PCF

- Two teams in the UK; one in Siegen
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Introduced in early 90's

- Two teams in the UK; one in Siegen
- Roots in Mathematical Logic

First stage (1993-2004)

- Models for various programming features
- Program analysis

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Nominal game semantics (2004-)

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- Models of realistic programs (2006-)
- Algorithmic games (2011-)

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+ integer variables (*names*)

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Nominal techniques

- A foundational theory for doing maths with names and name-binding

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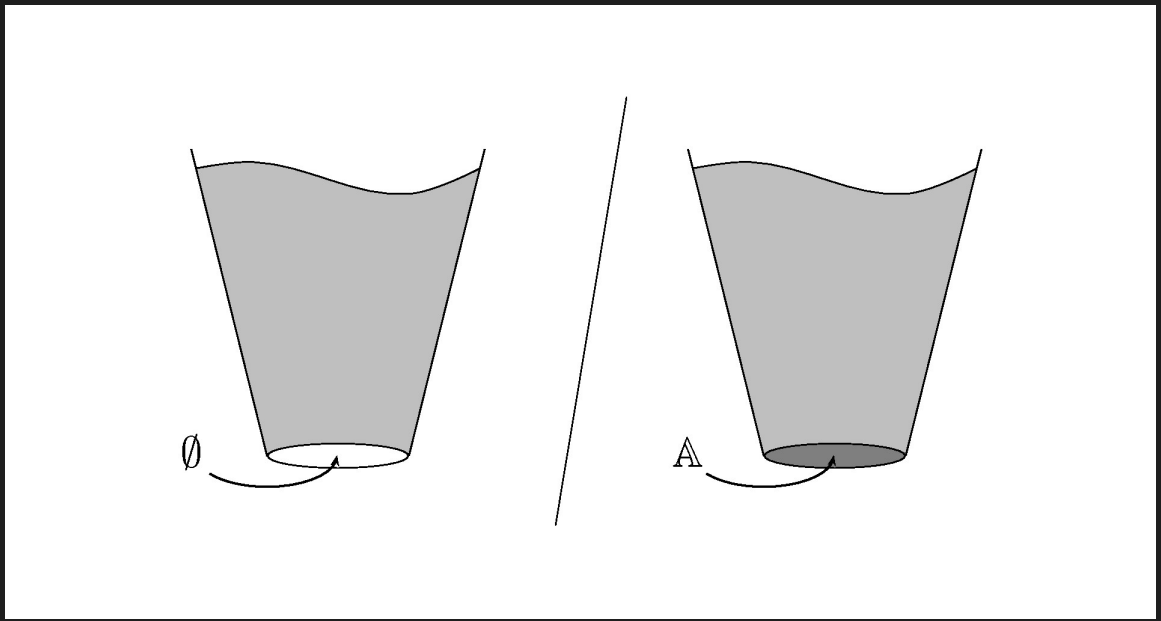
- Two presentations:
  - FM-set theory [Gabbay&Pitts 99, Gabbay 01]
  - Nominal sets [Pitts 01]

FM-sets

- Fraenkel-Mostowski model of ZFA:
  - Basis: empty set + inf. set $\mathcal{A}$ of atoms
  - Constructions have finite support



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- NOM: nominal sets and equivariant functions

Nominal Logic

Example axioms (note sorts should match):

$$\text{И}a.\ \phi(\vec{x}) \iff \exists a.\ a\#\vec{x} \wedge \phi(\vec{x}) \qquad \text{(Q)}$$

$$(a\ a') \cdot (b\ b') \cdot x = ((a\ a') \cdot b\ (a\ a') \cdot b') \cdot (a\ a') \cdot x \qquad \text{(E1)}$$

$$b\#x \implies (a\ a') \cdot b \# (a\ a') \cdot x \qquad \text{(E2)}$$

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NL gives us a strong handle on names. For example:

- $\phi(\vec{x}) \iff \phi((a\ a') \cdot \vec{x})$
- $(\exists a.\ a\#\vec{x} \wedge \phi(\vec{x})) \iff (\forall a.\ a\#\vec{x} \implies \phi(\vec{x}))$
- $b \# a.x \iff b = a \vee b \# x$
- $a.x = a'.x' \iff И b.\ (a\ b) \cdot x = (a'\ b) \cdot x'$

Nominal games

Nominal games

$\vdash \lambda x.\texttt{newvar}(0) : \texttt{com} \to \texttt{intvar}$

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let f = [_] in { f() == f() }
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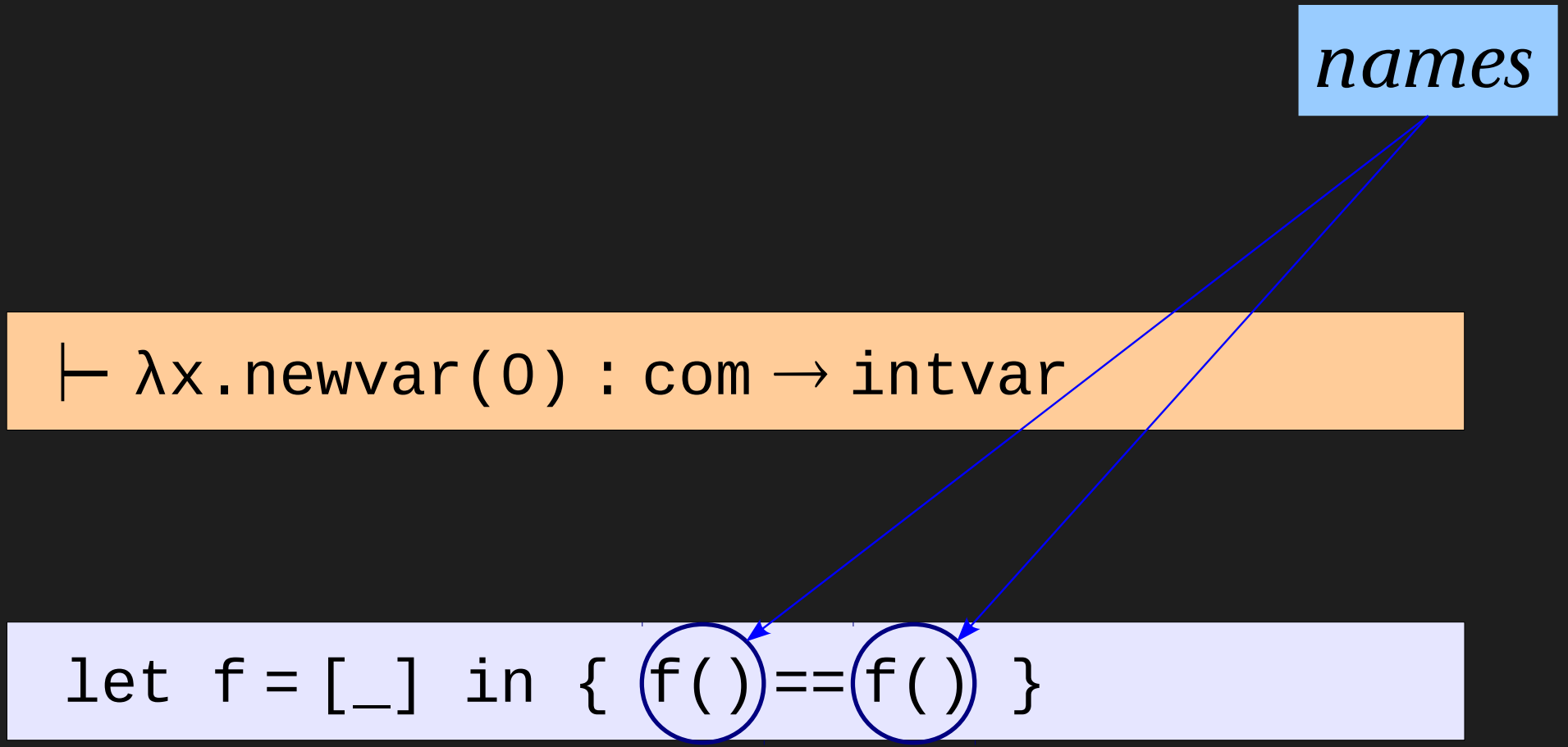

Nominal games

Games in nominal sets

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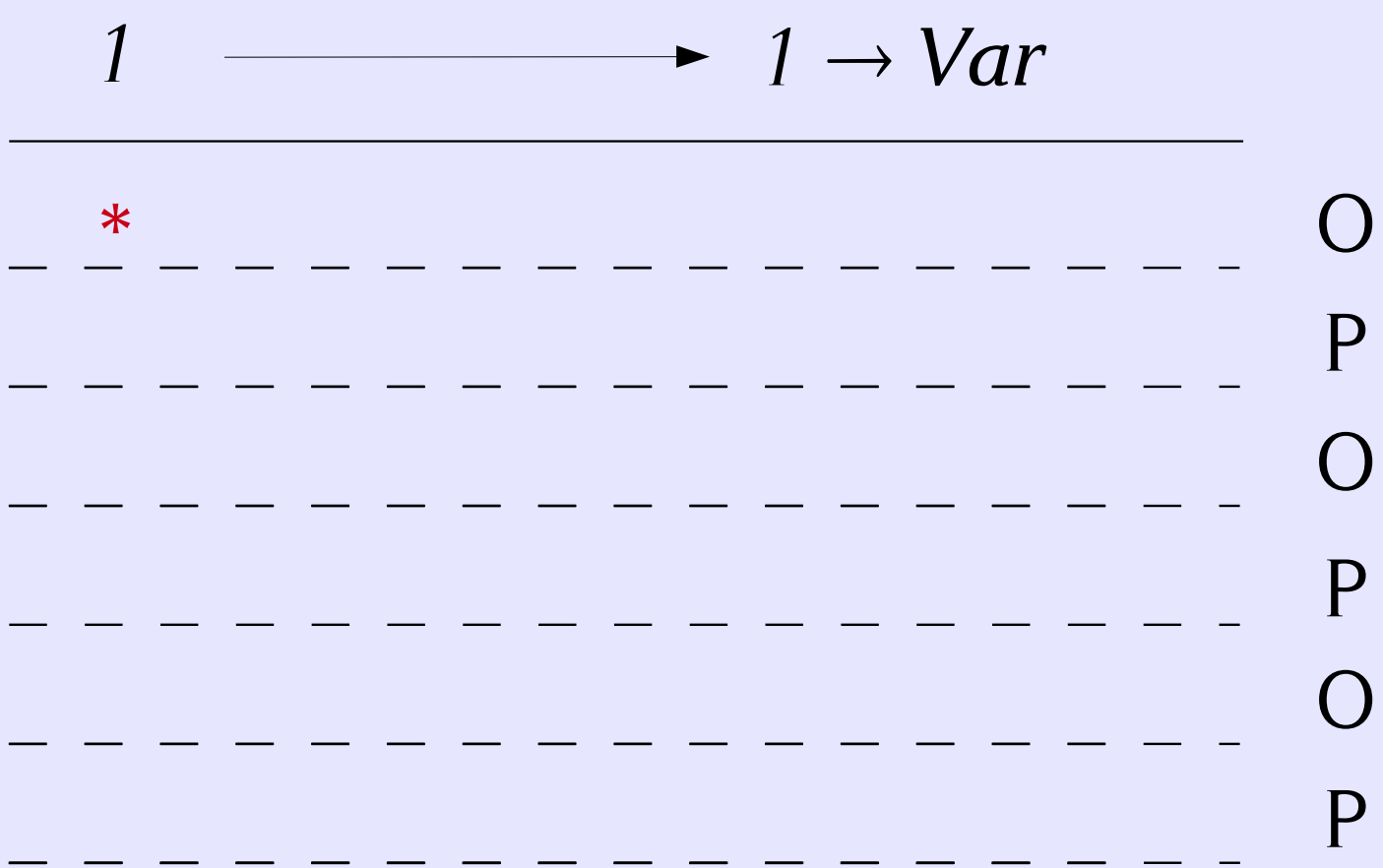

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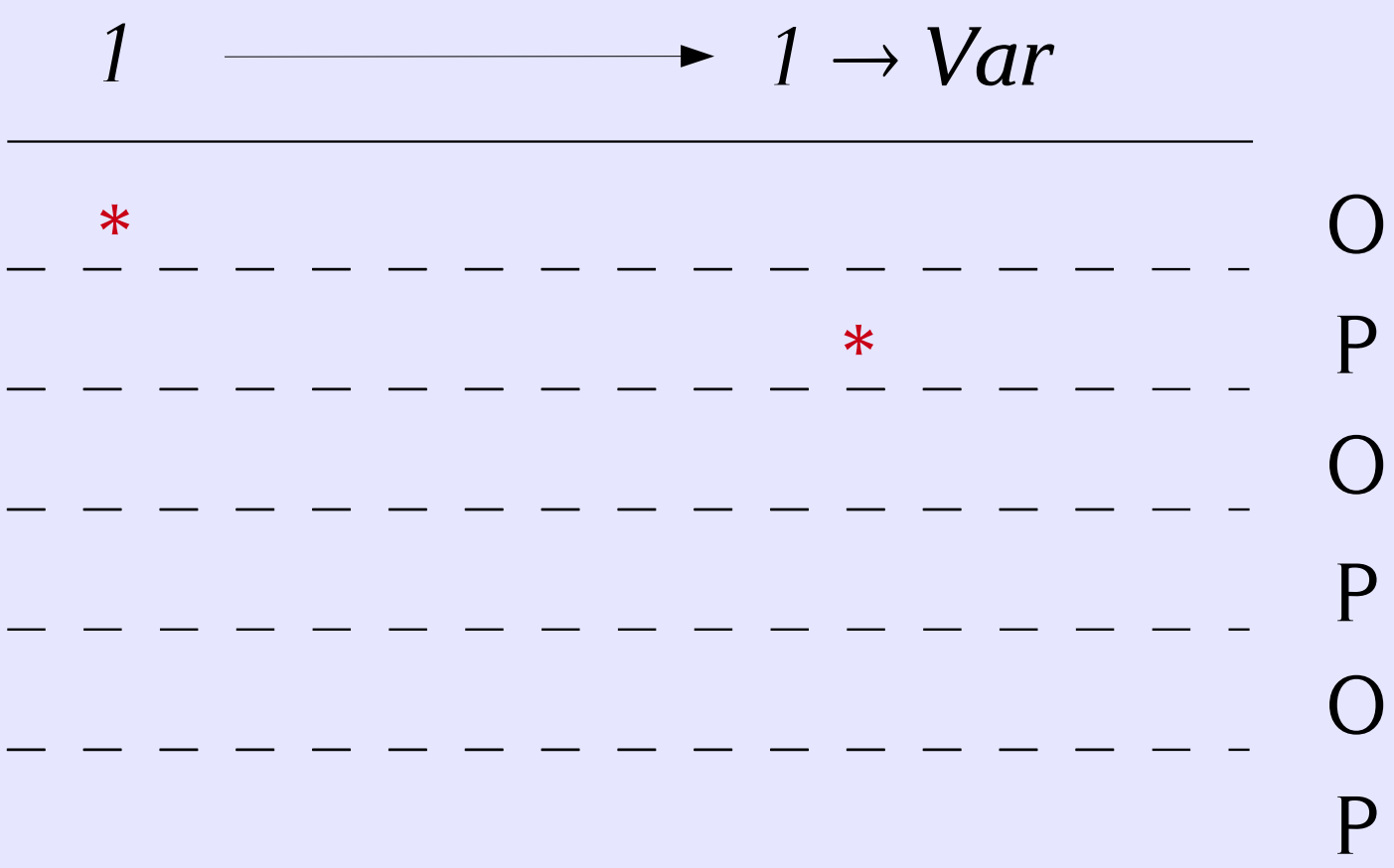

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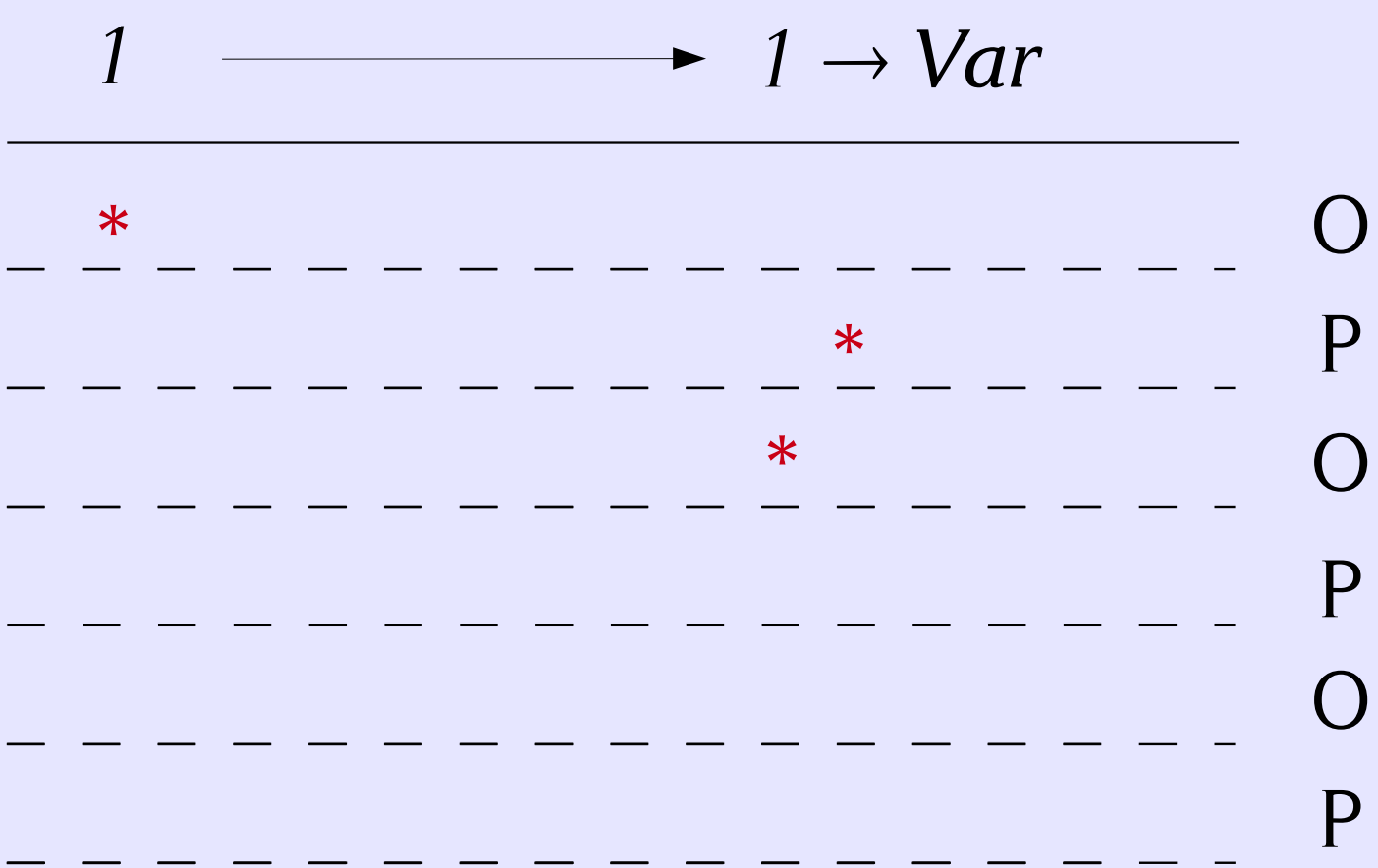

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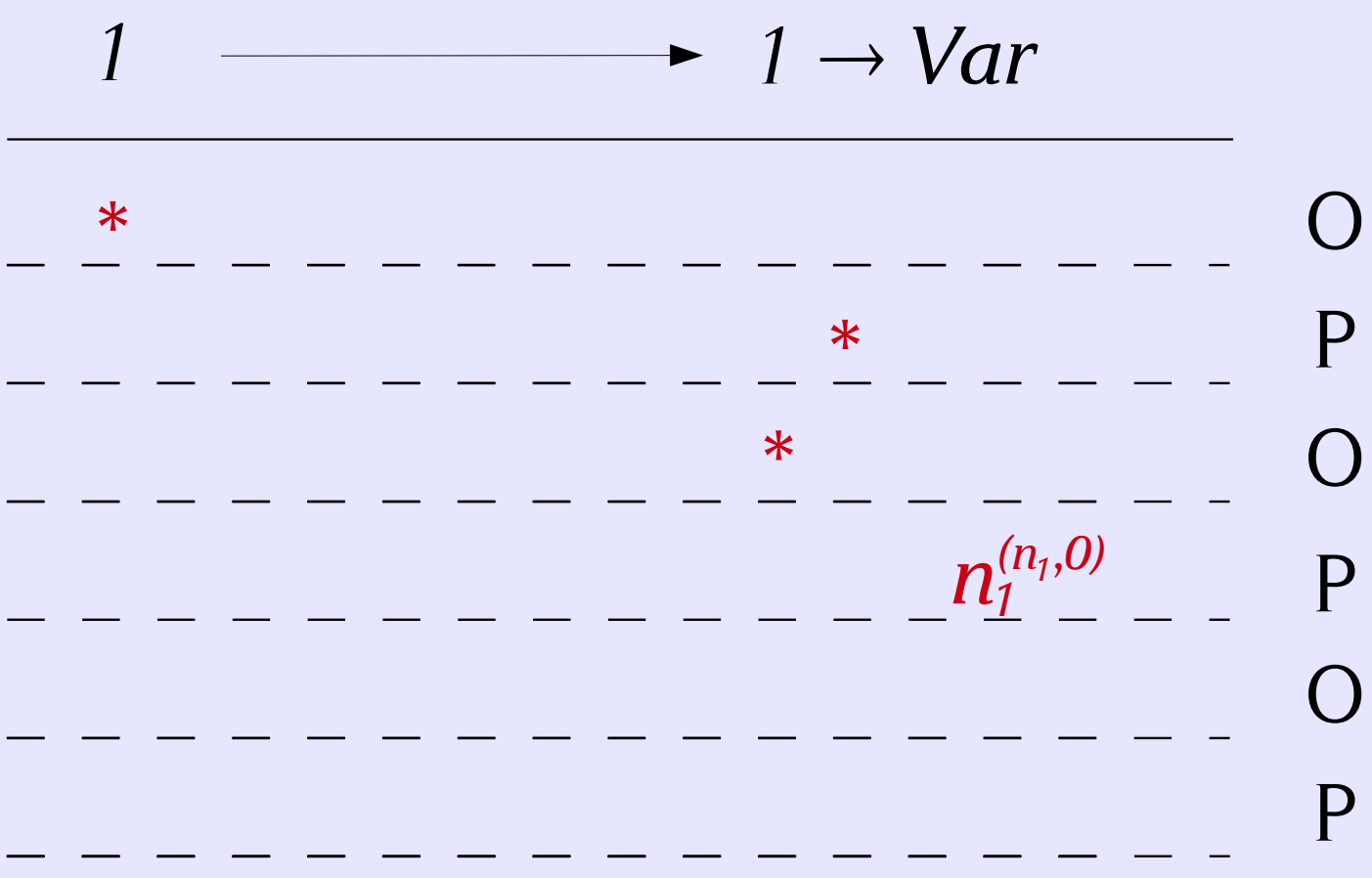

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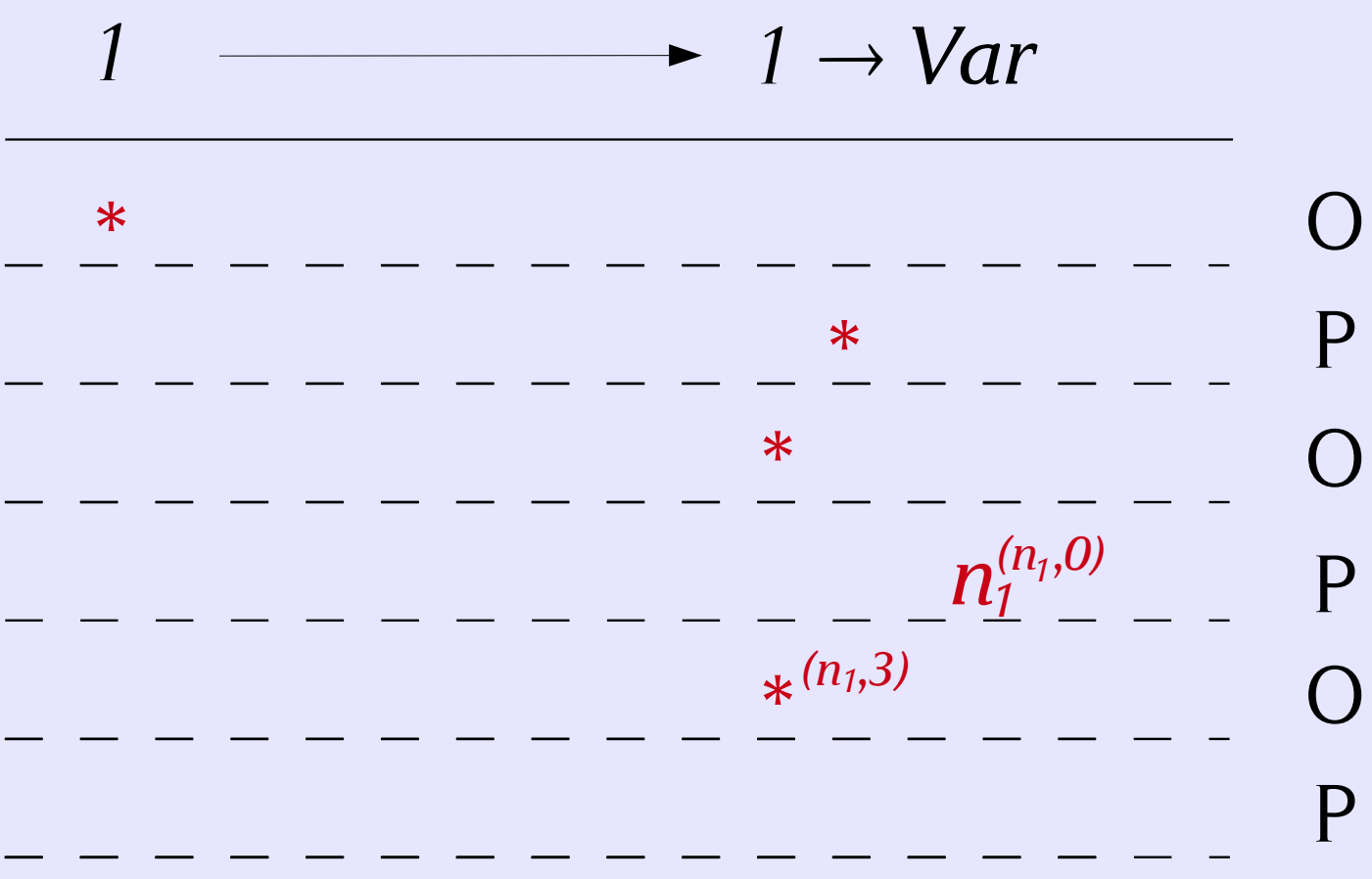

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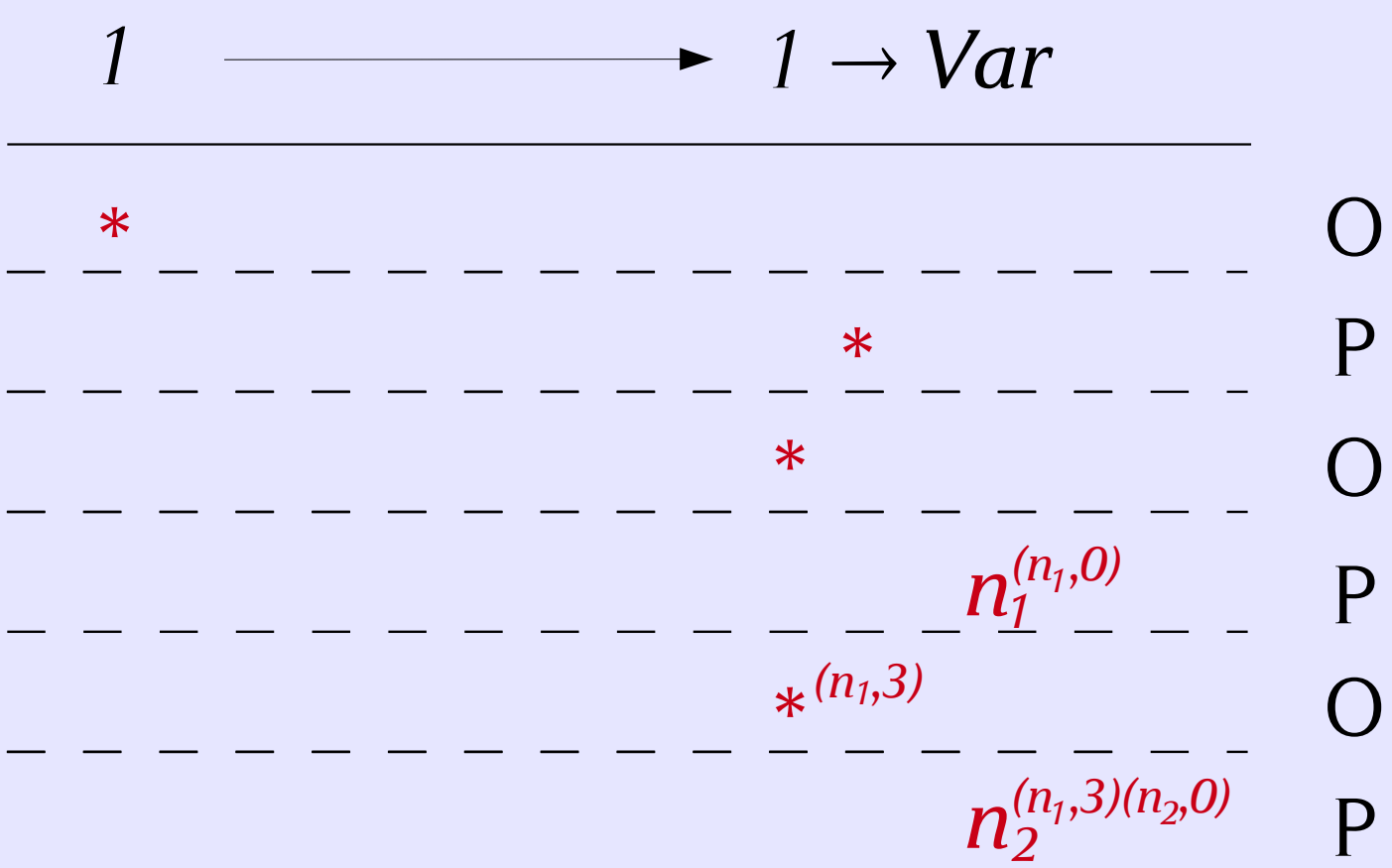

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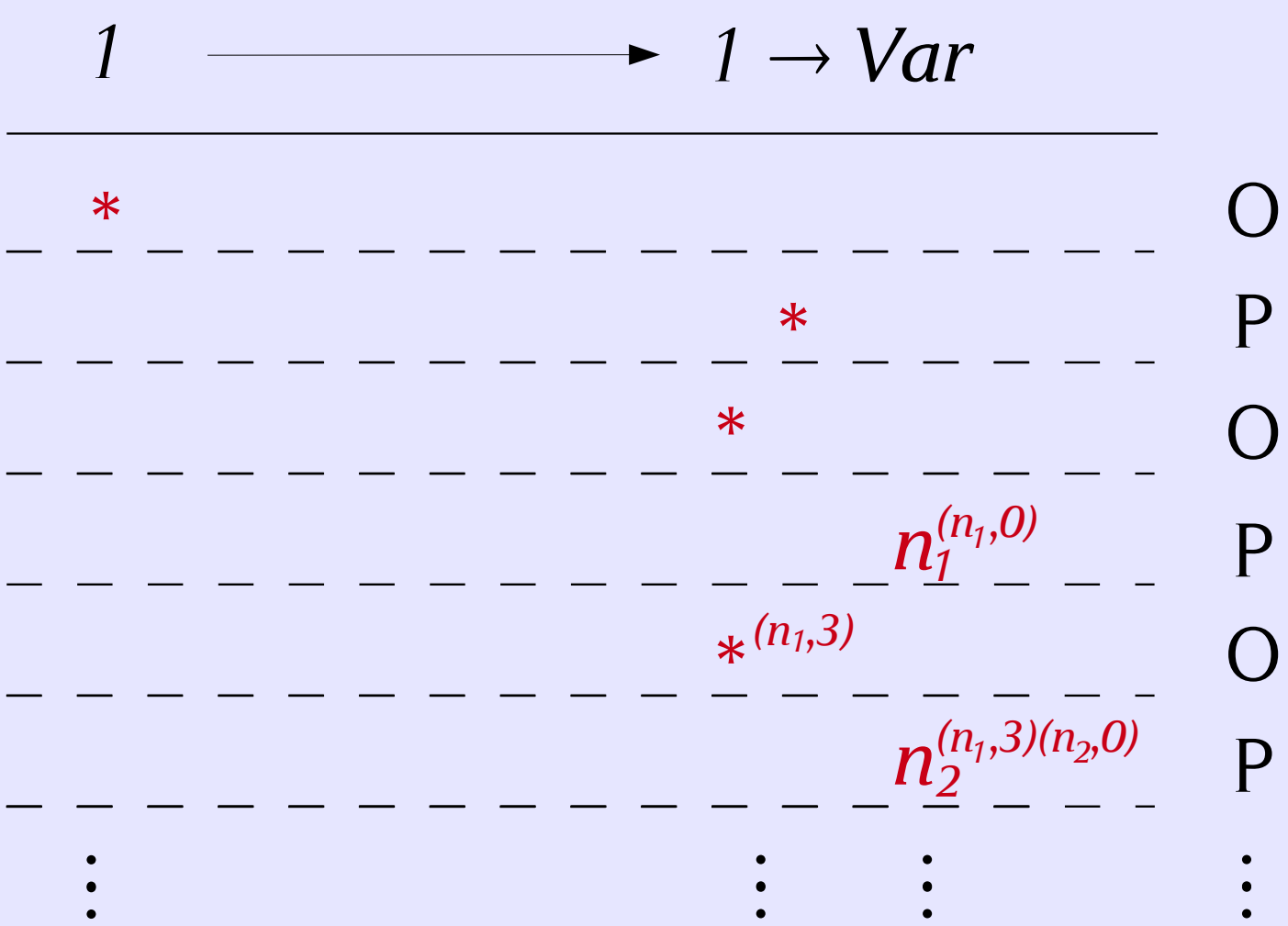

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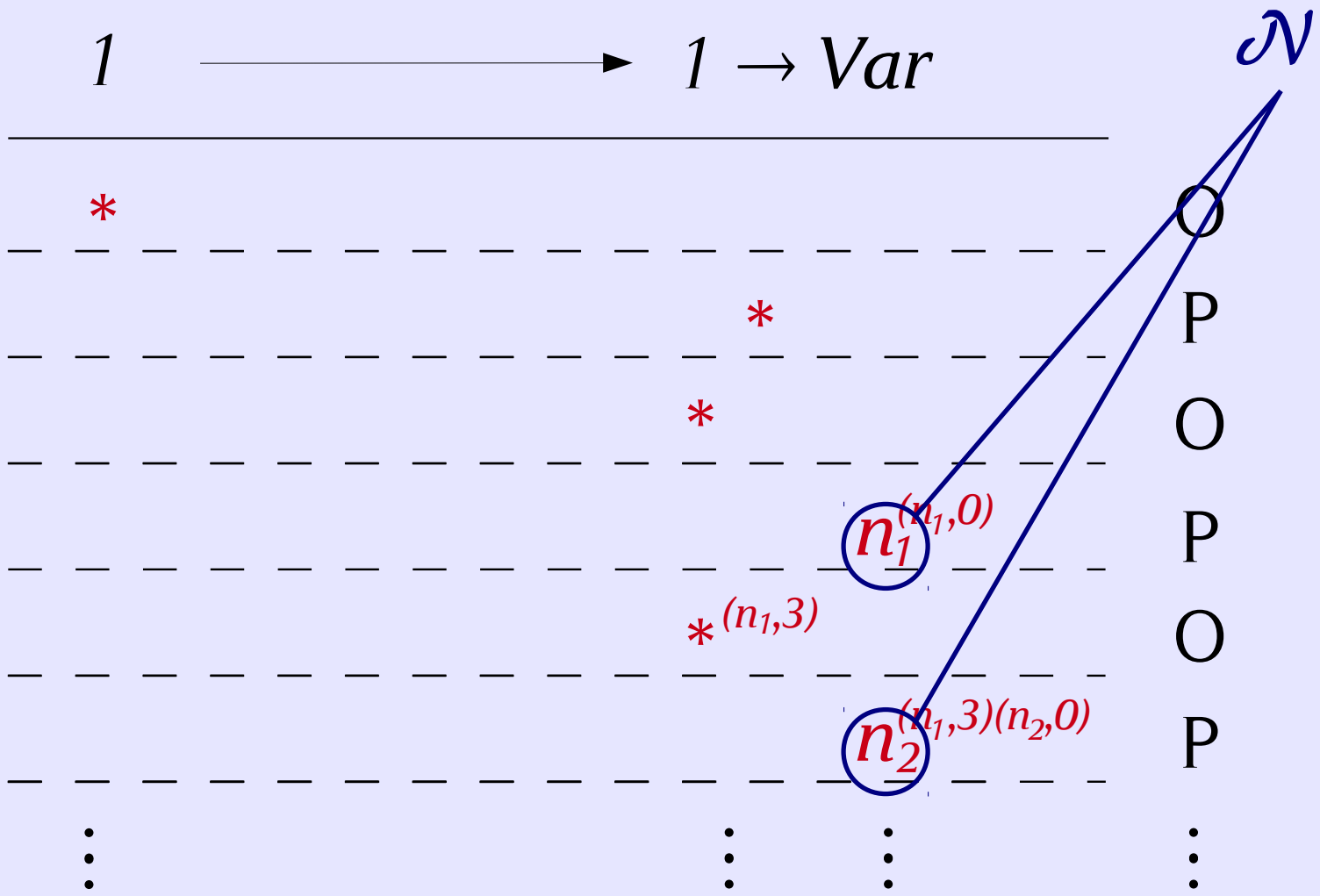

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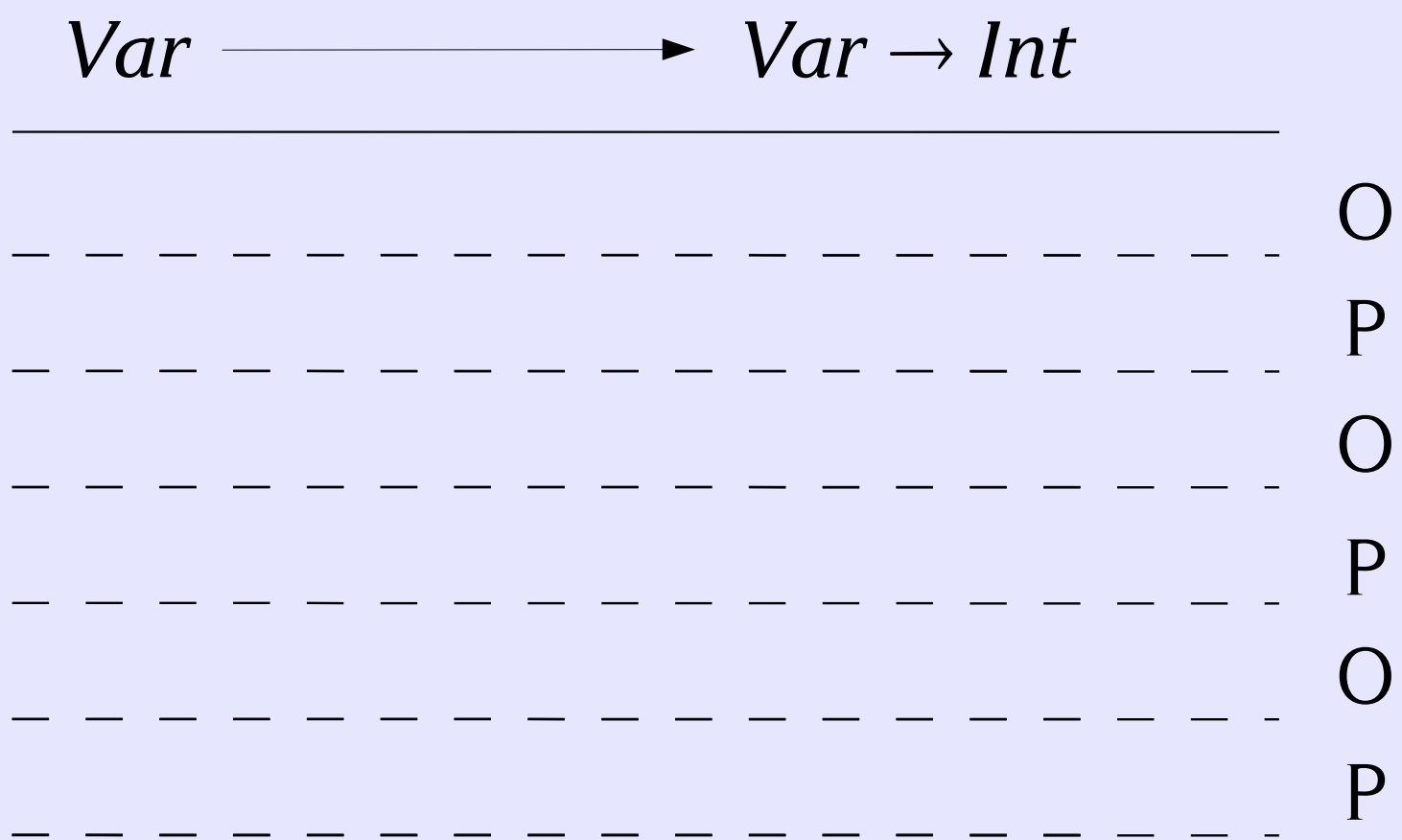

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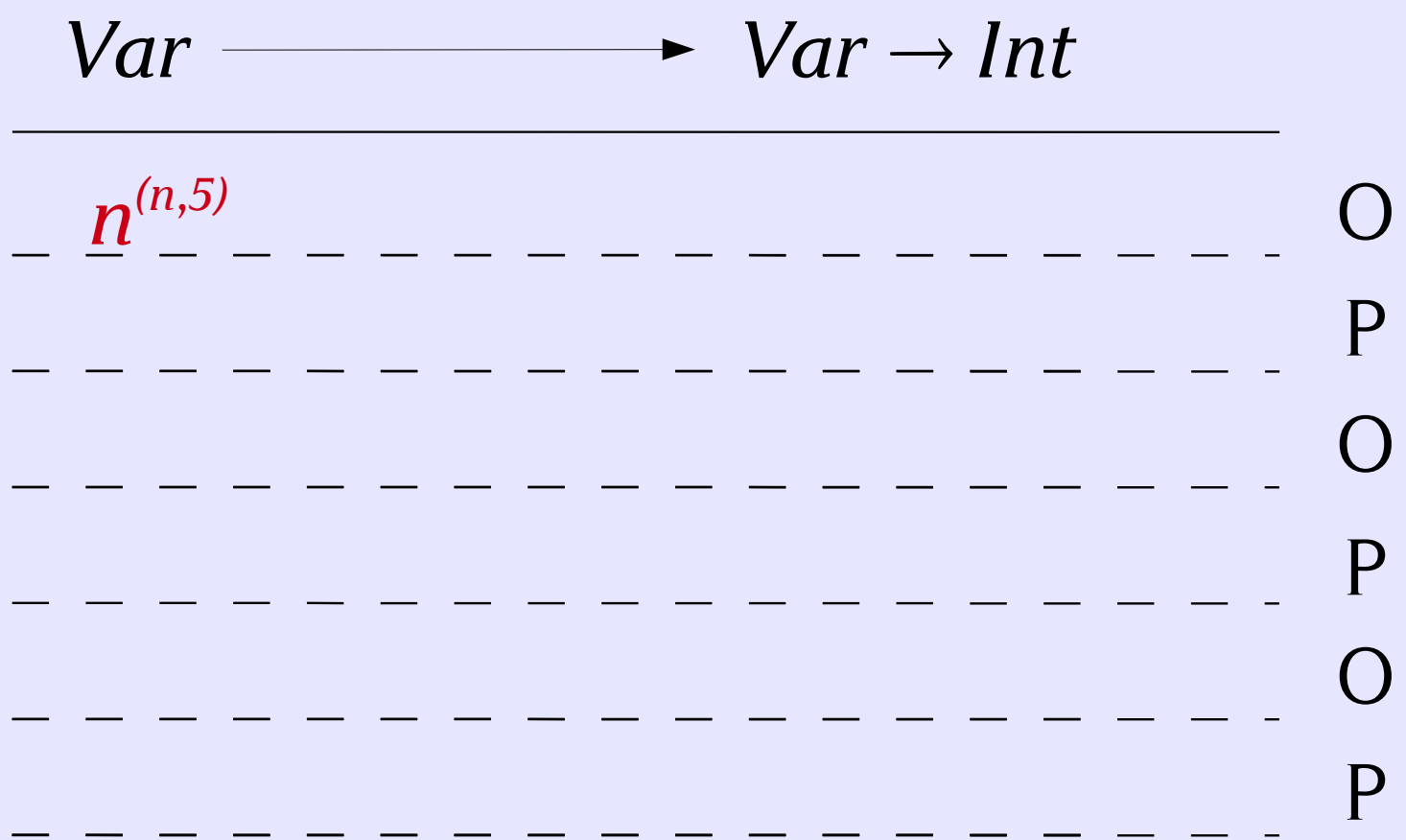

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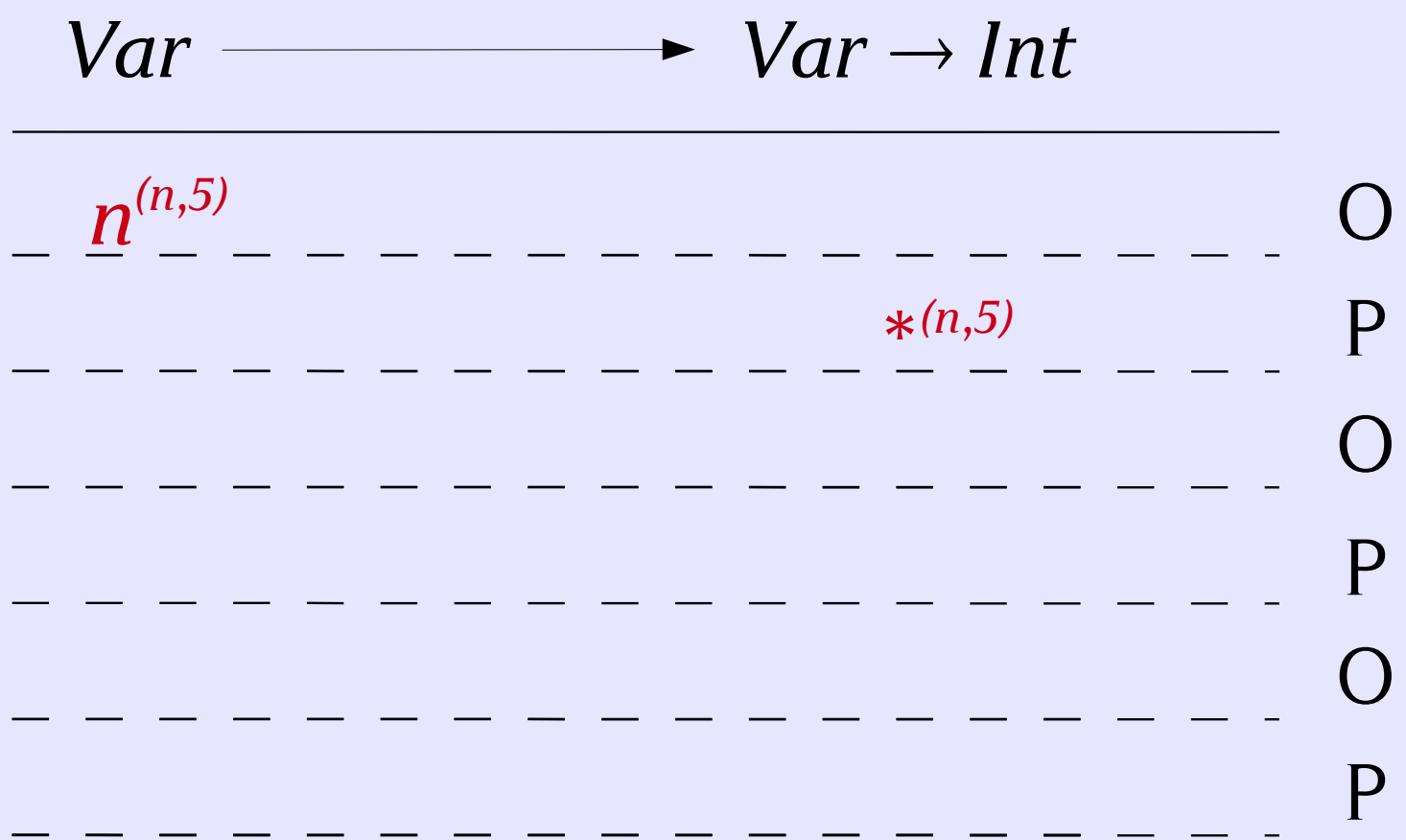

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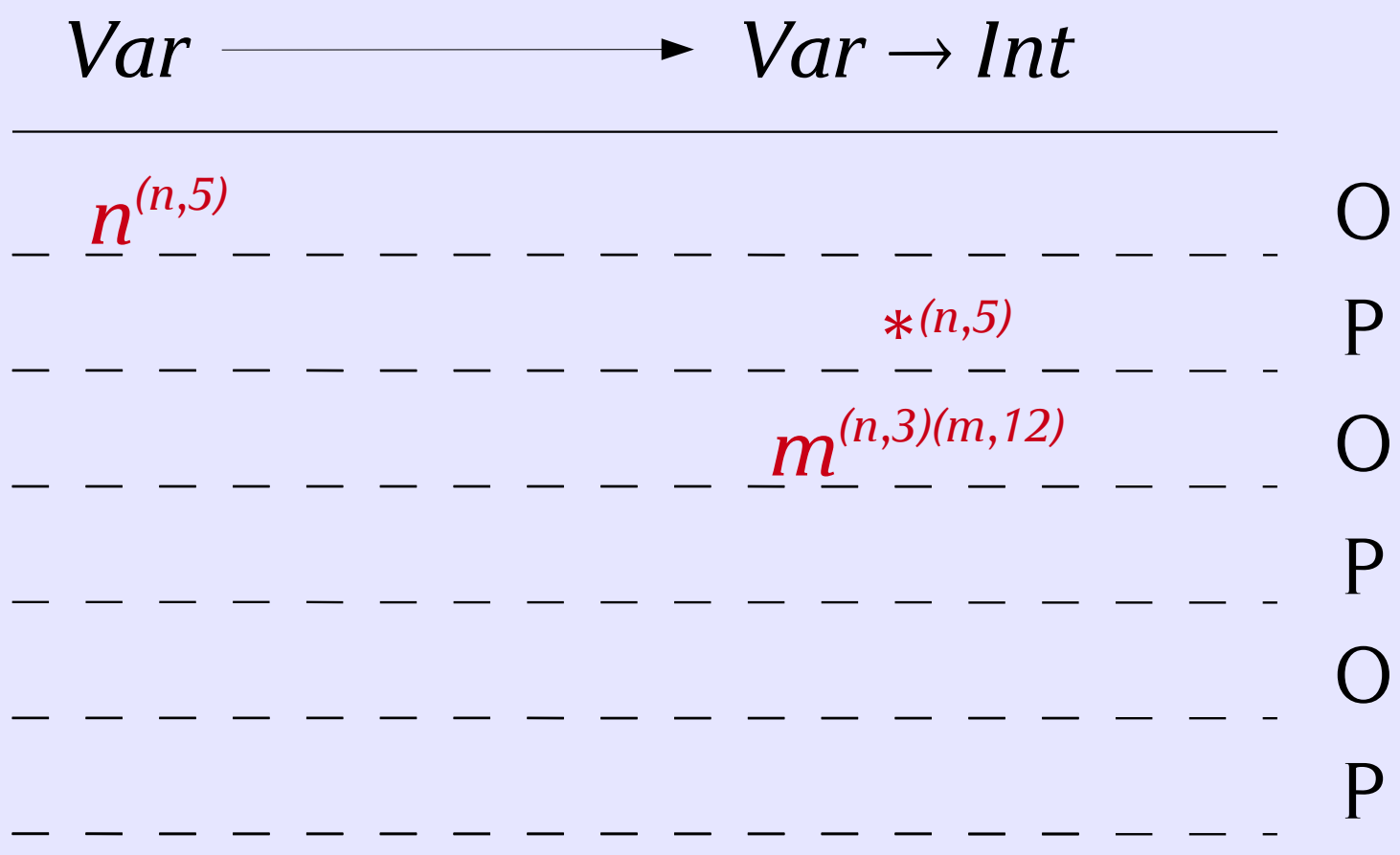

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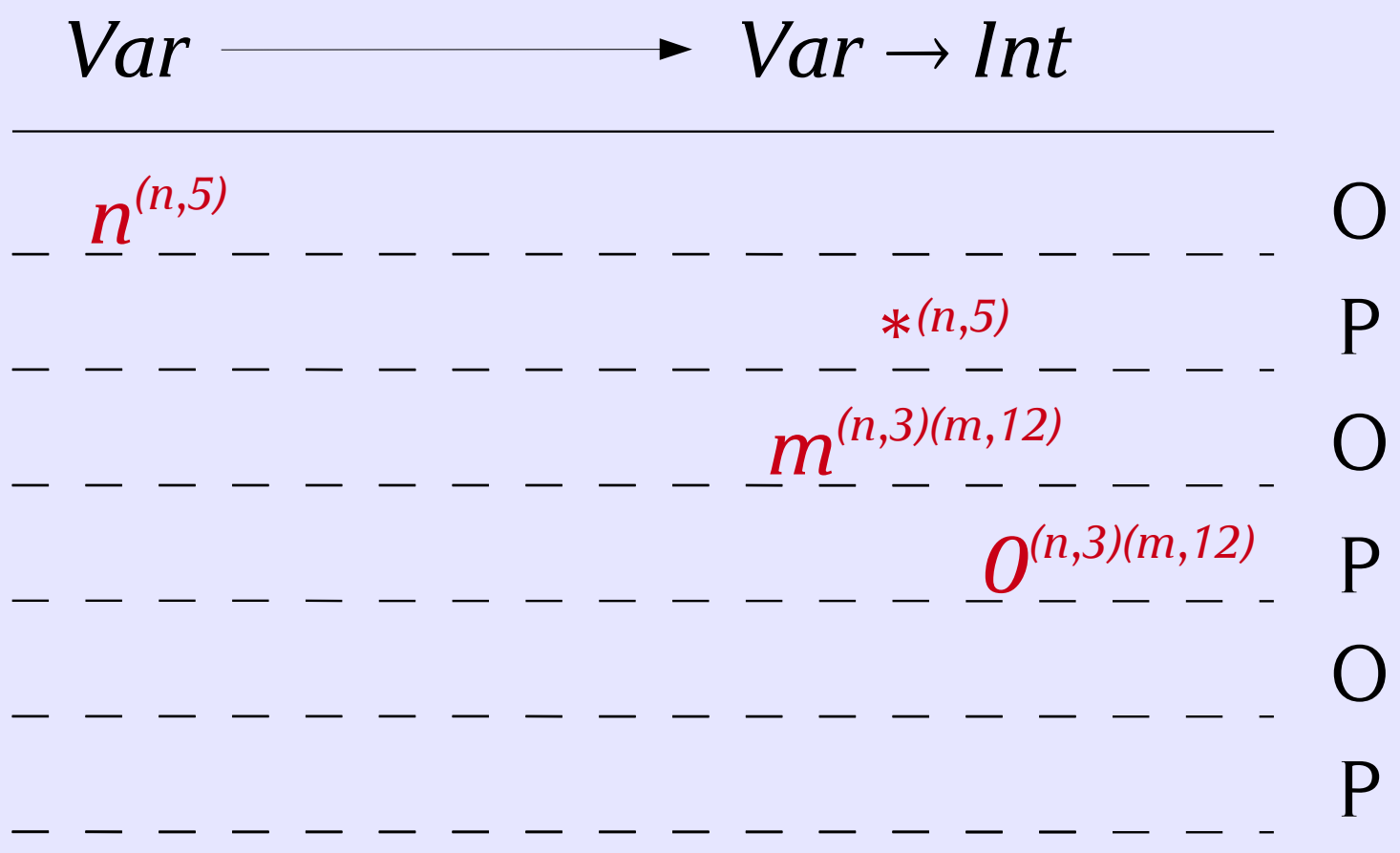

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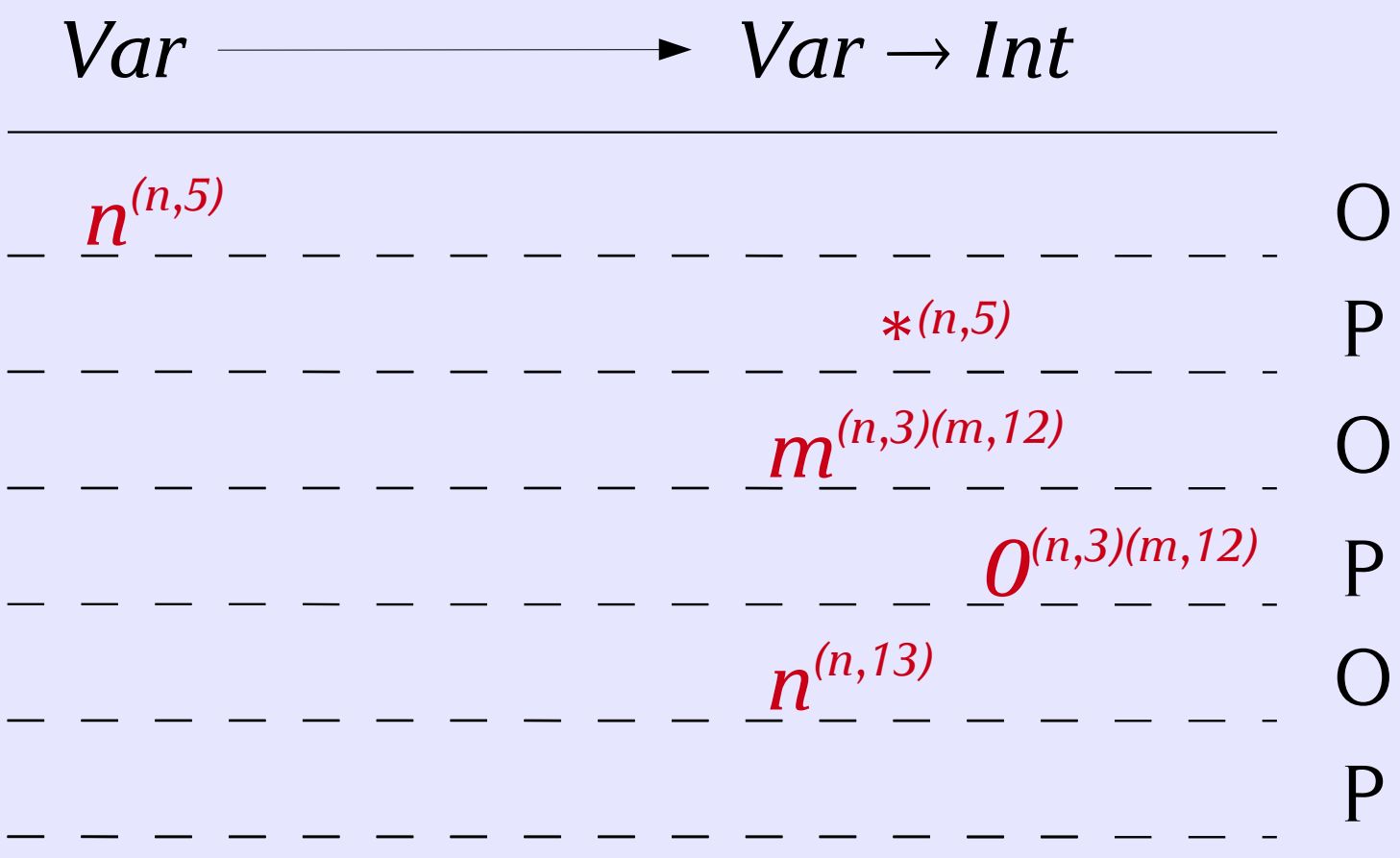

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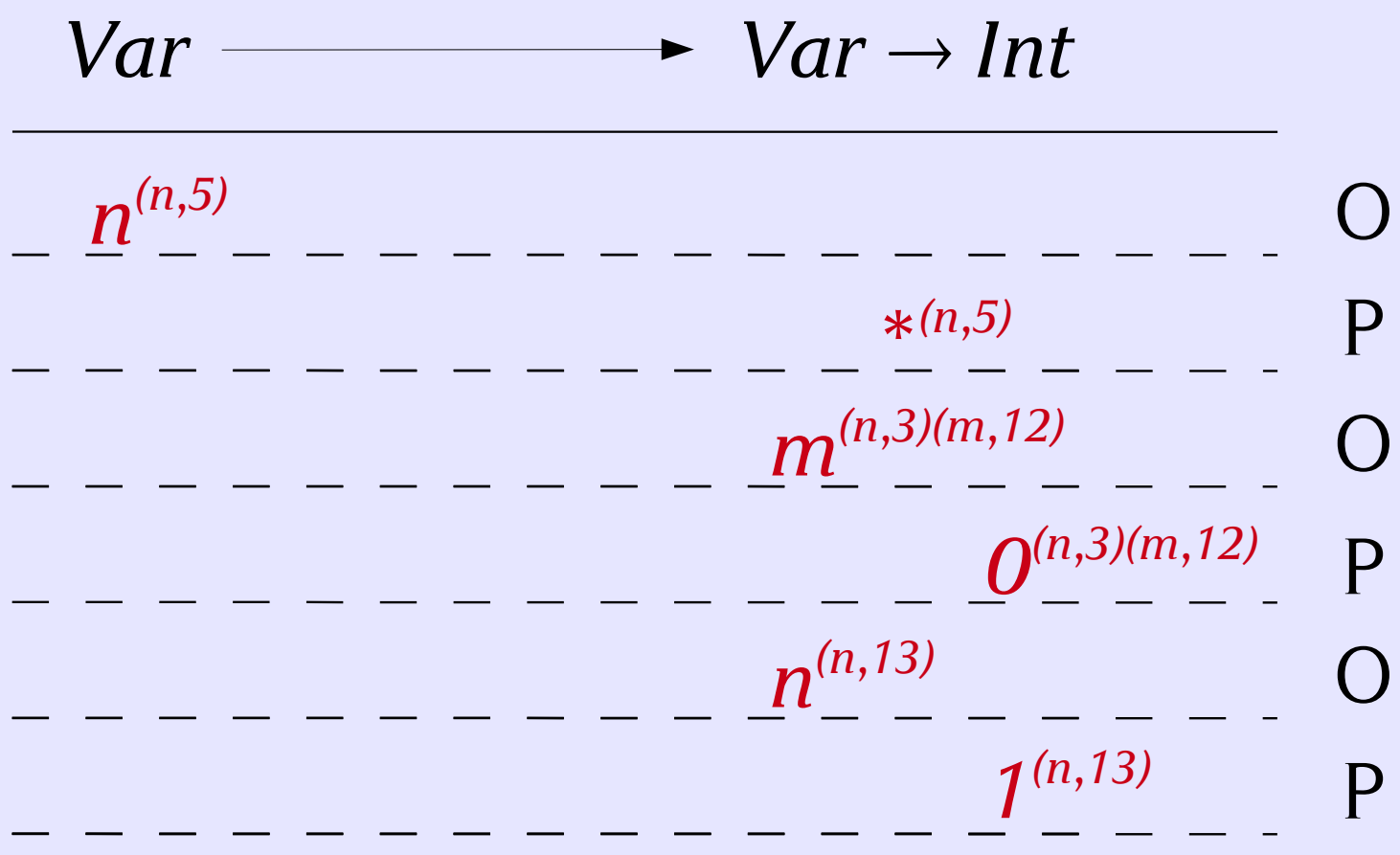

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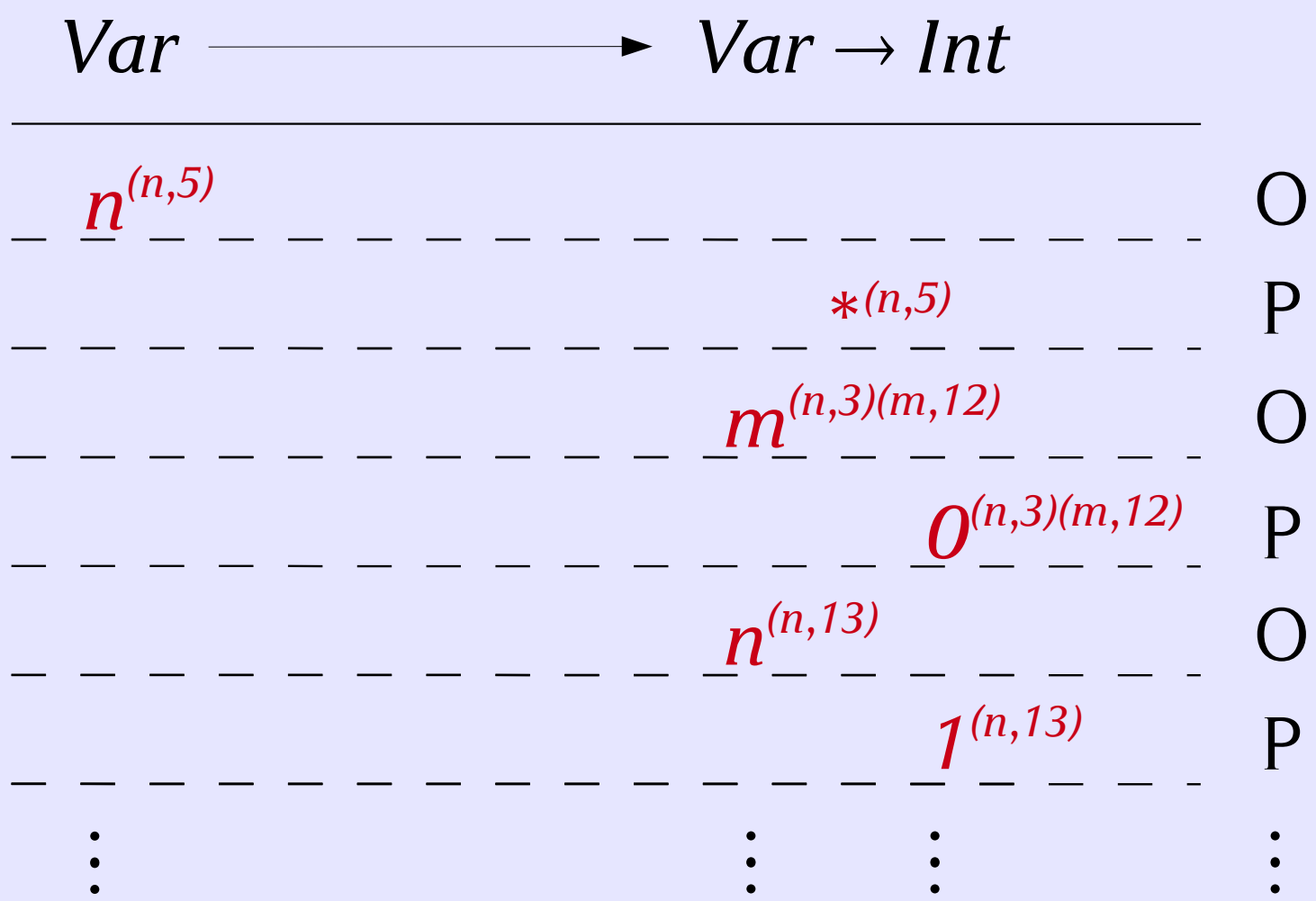

Games with new resources

Models of realistic programs

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Fresh-register automata [*POPL'11*]

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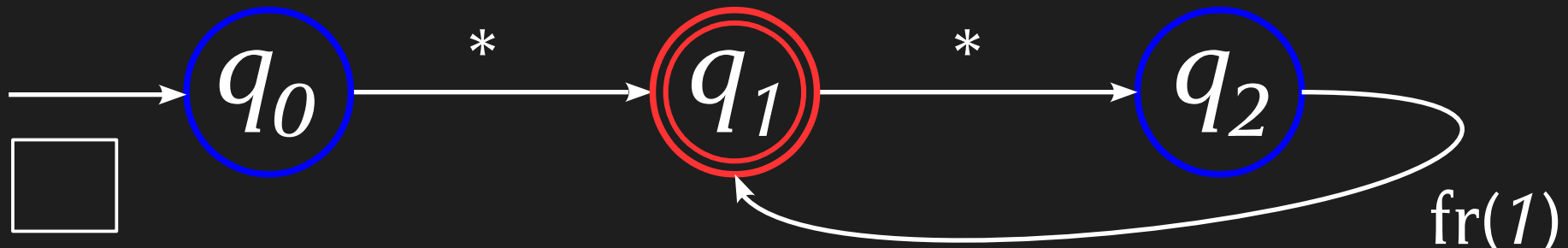

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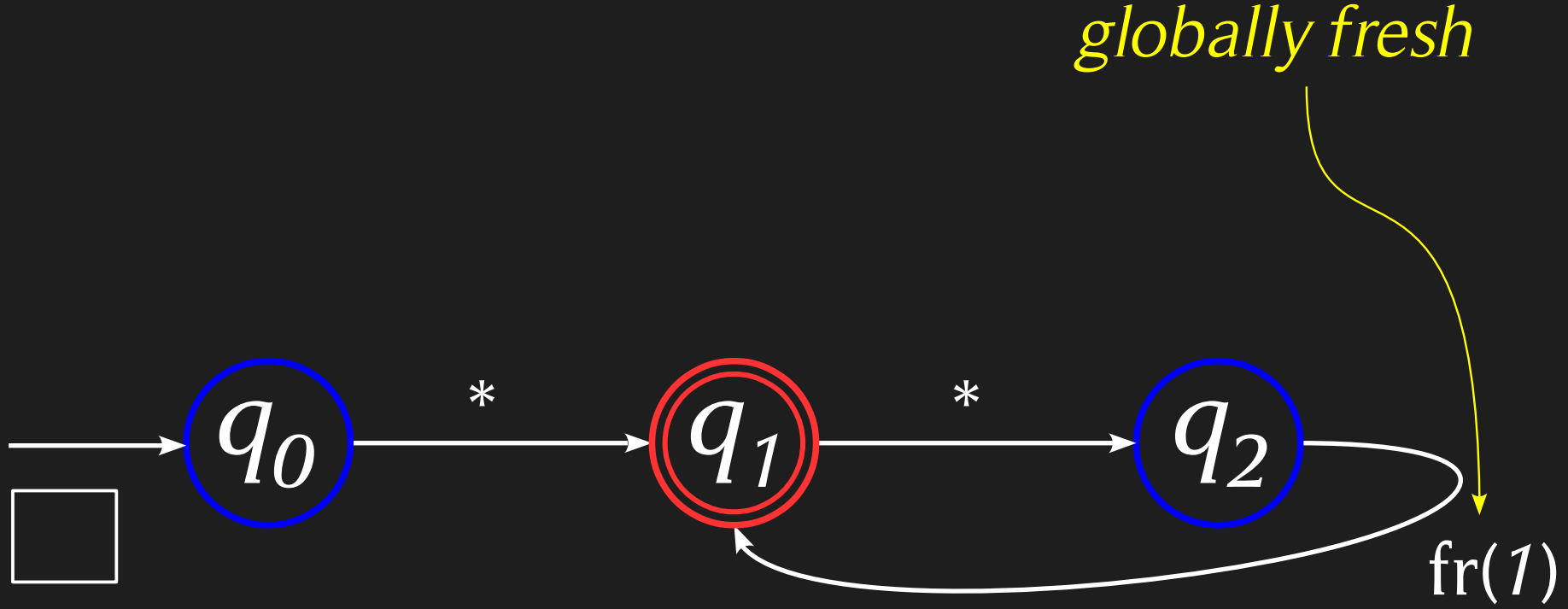

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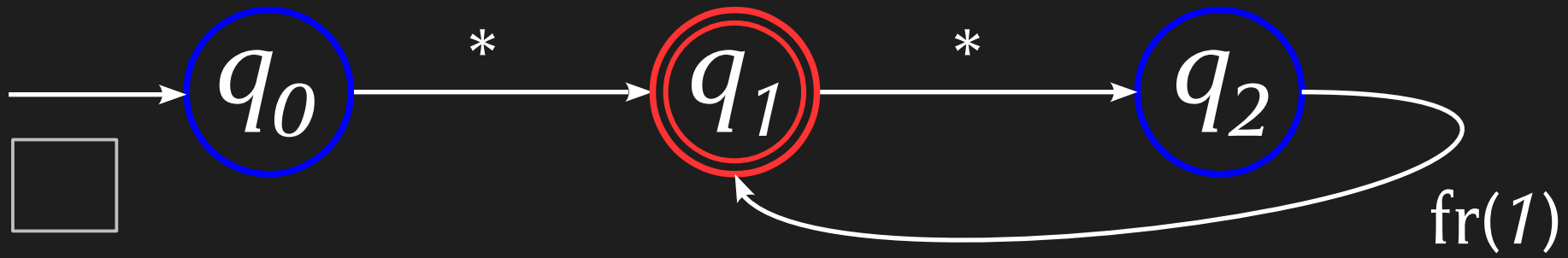

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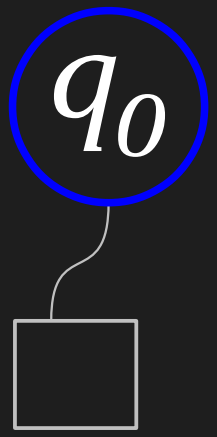

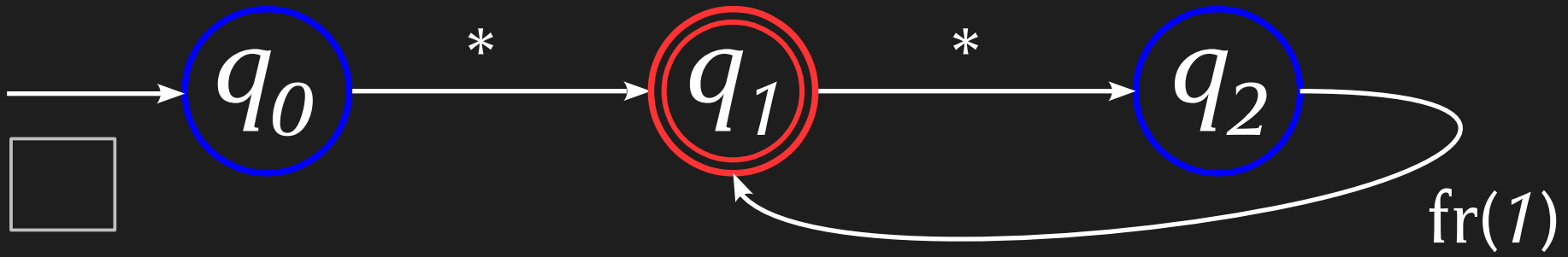

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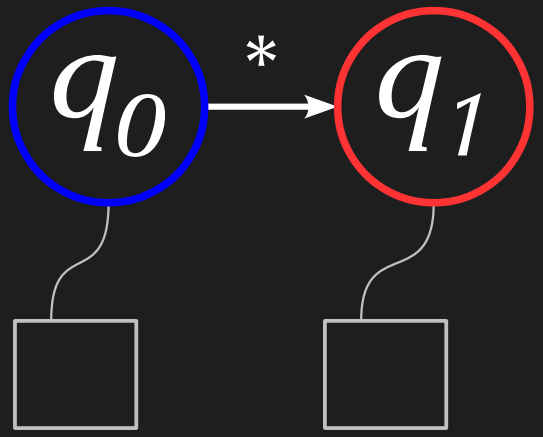

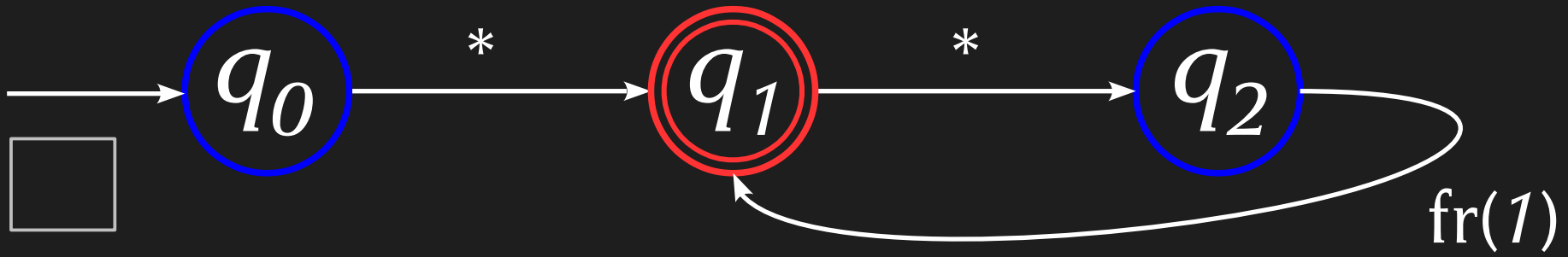

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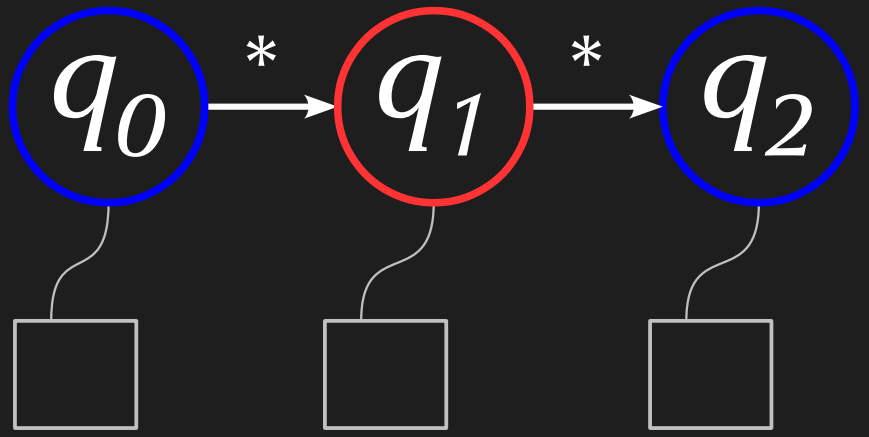

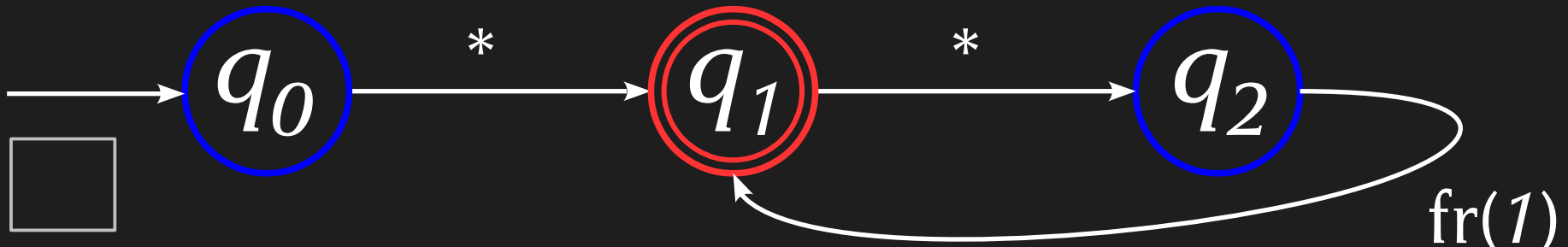

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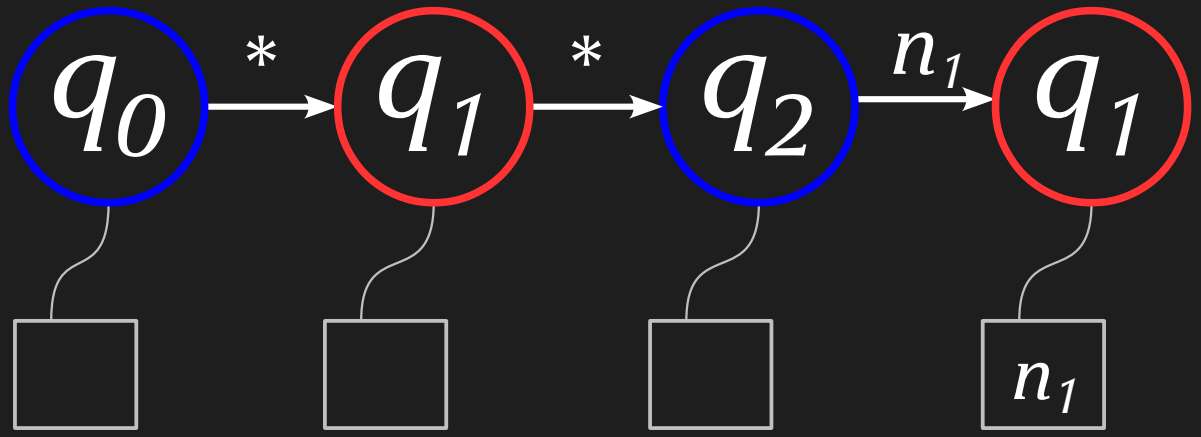

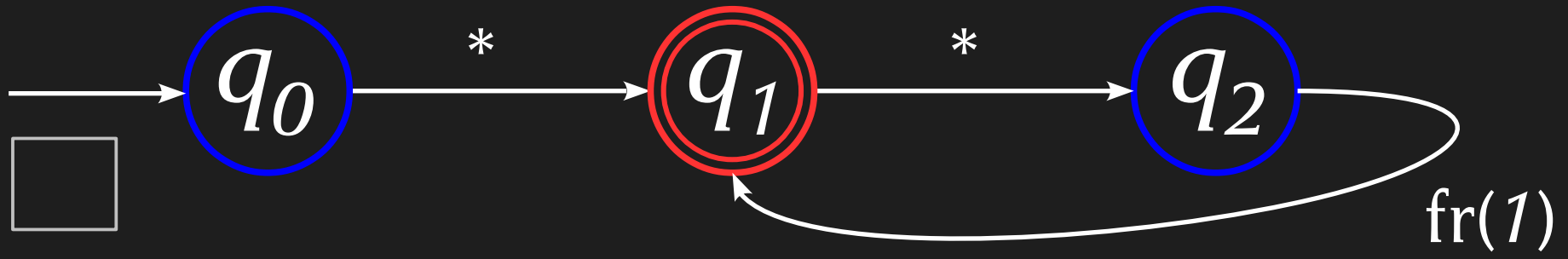

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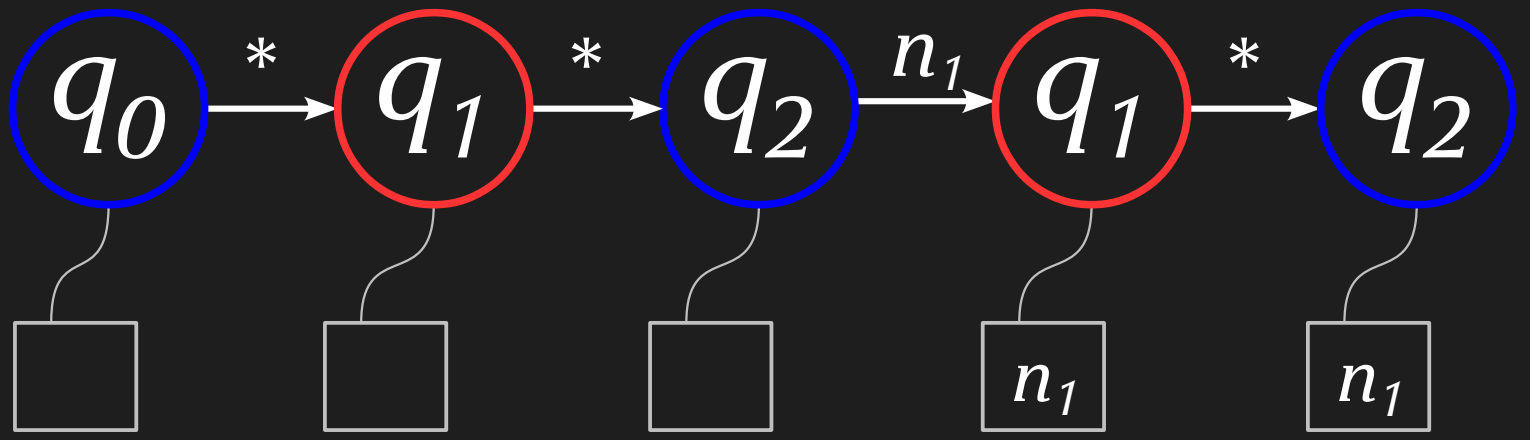

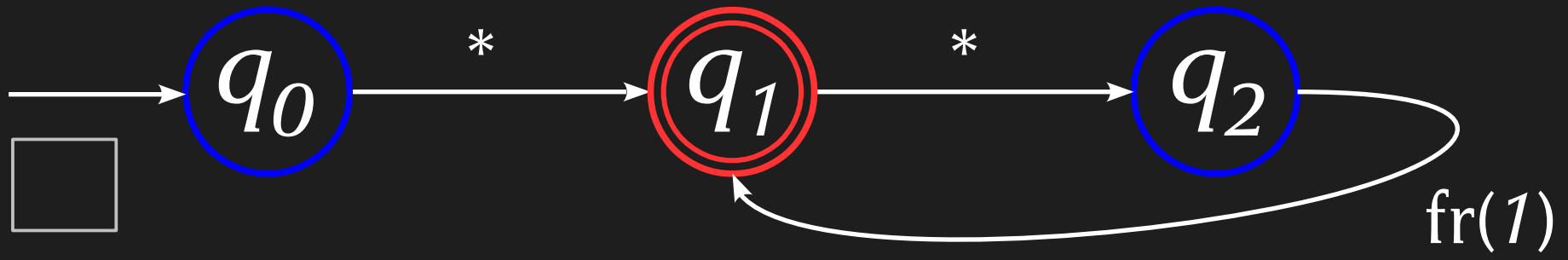

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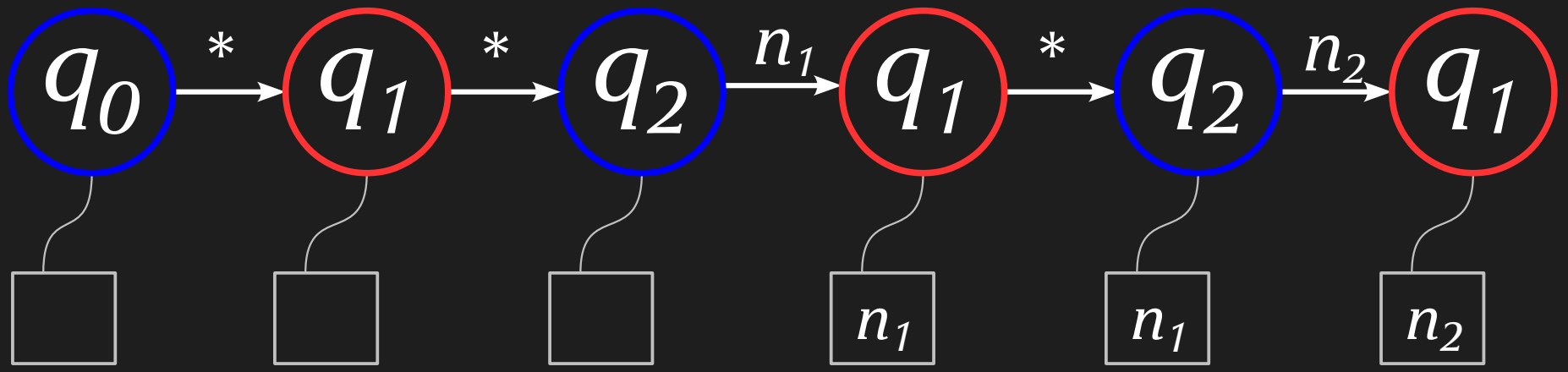

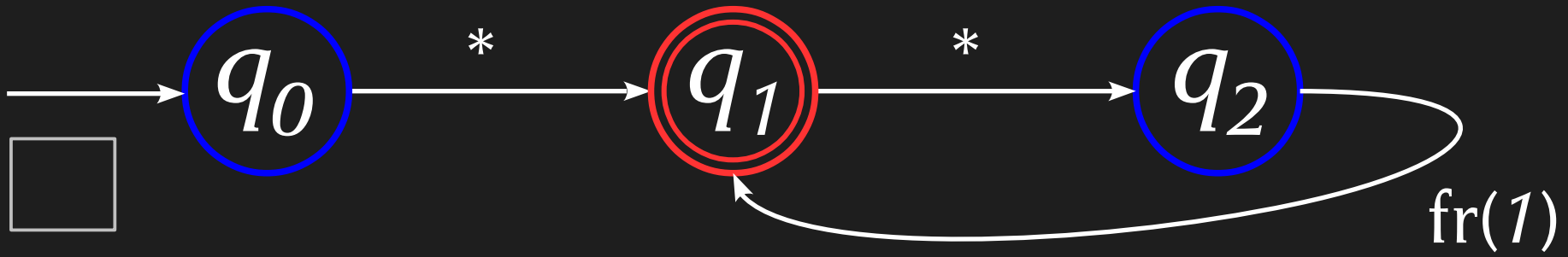

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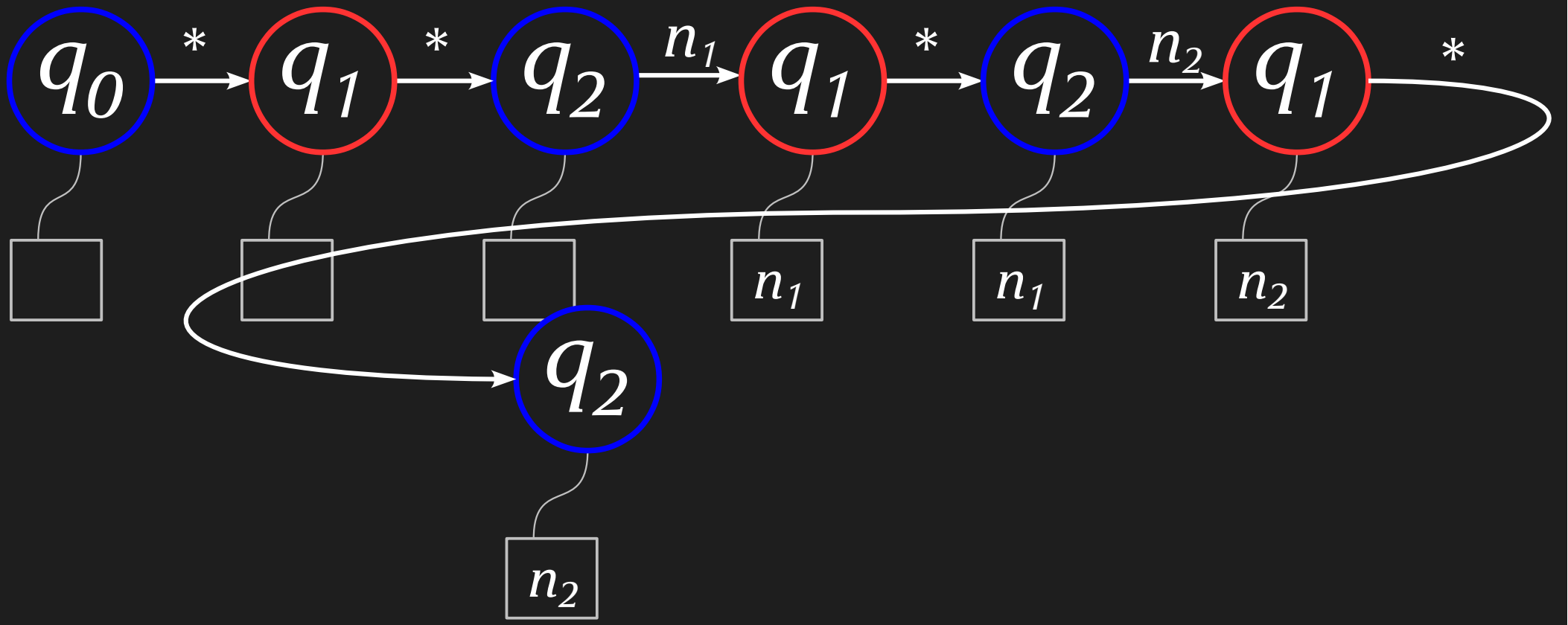

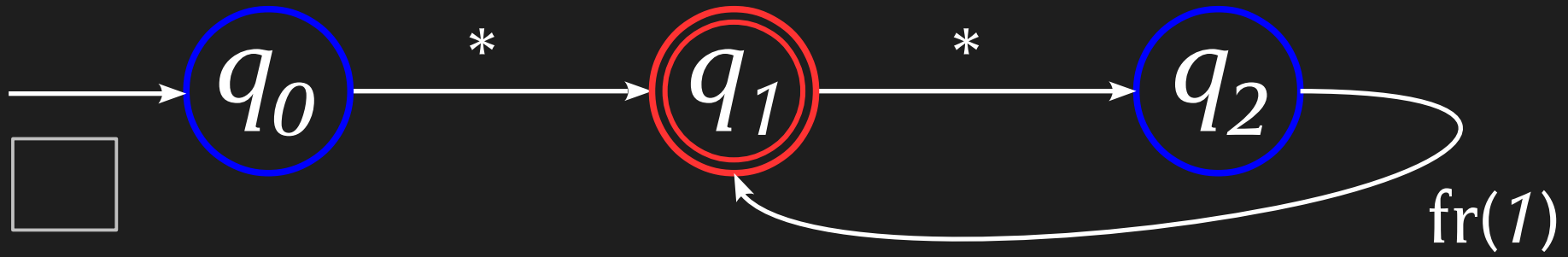

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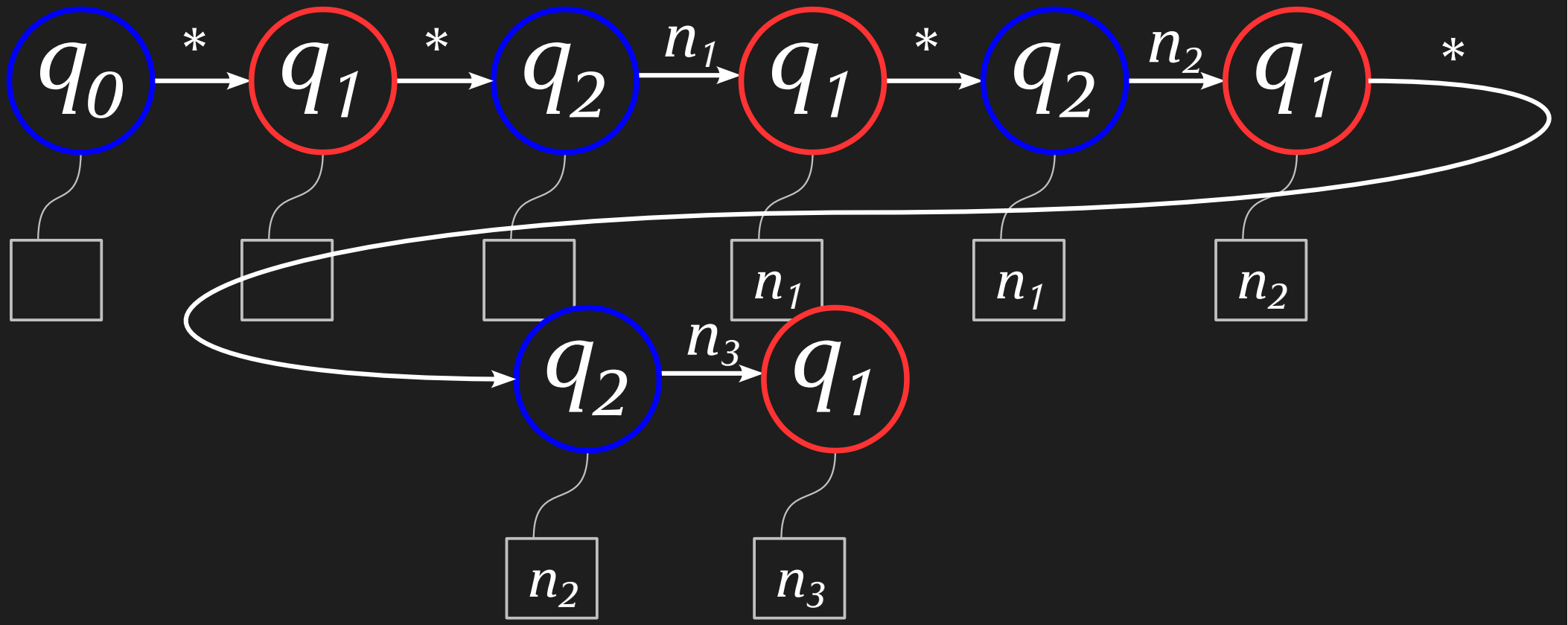

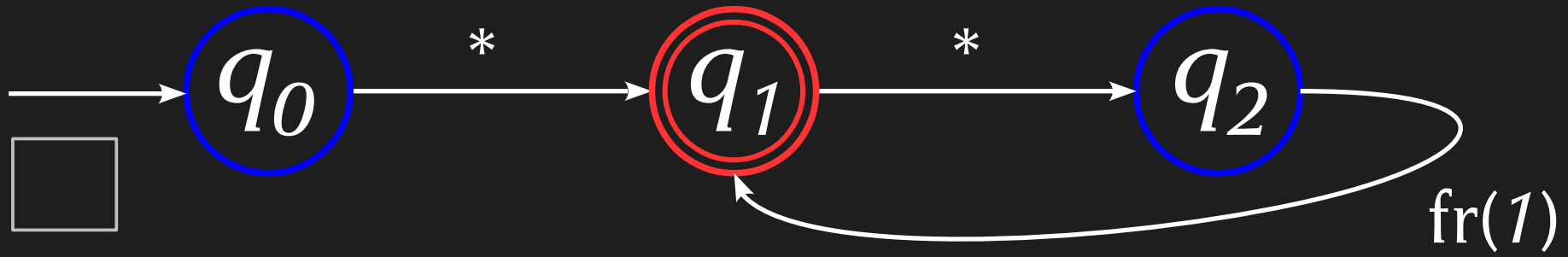

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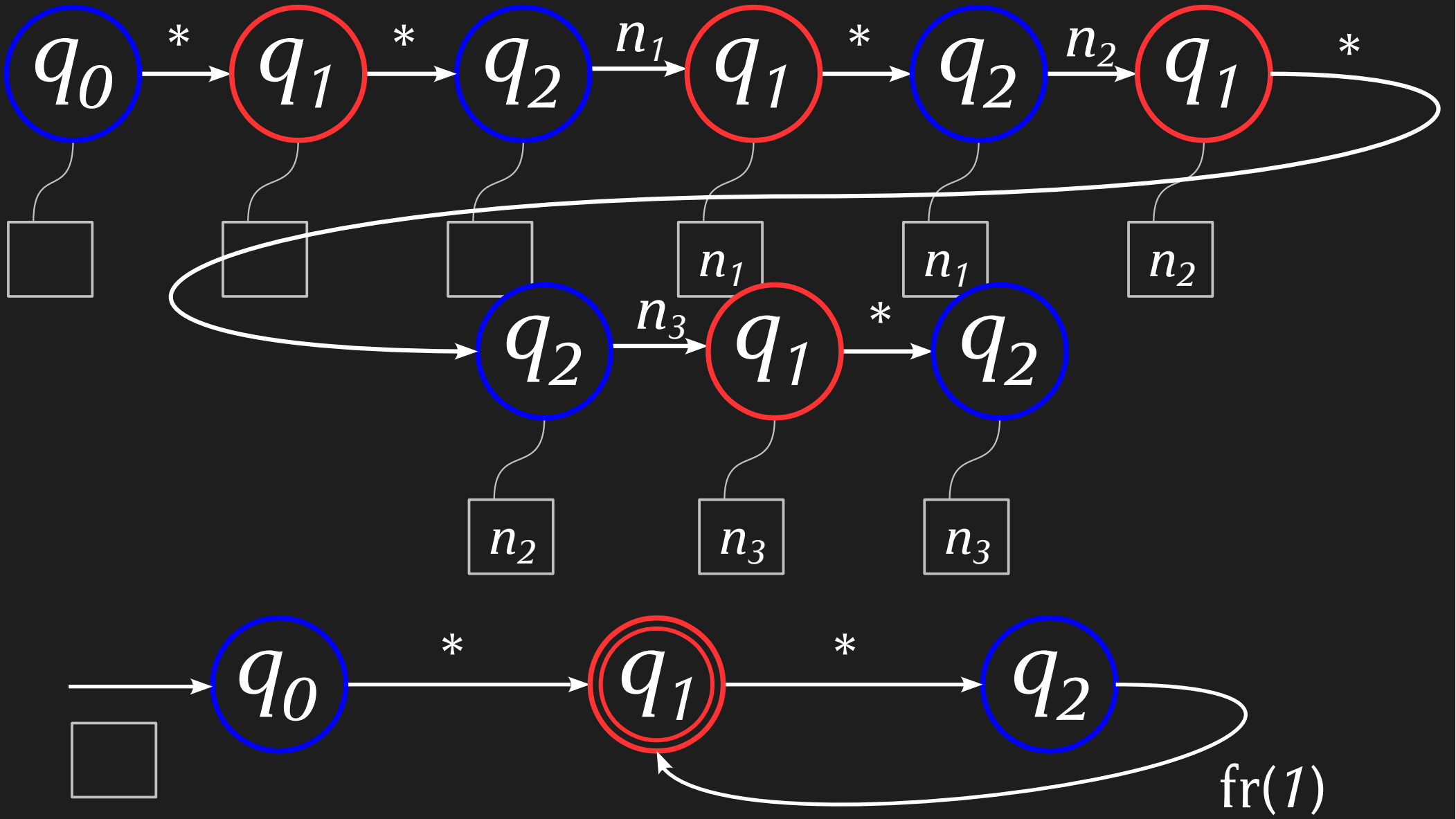

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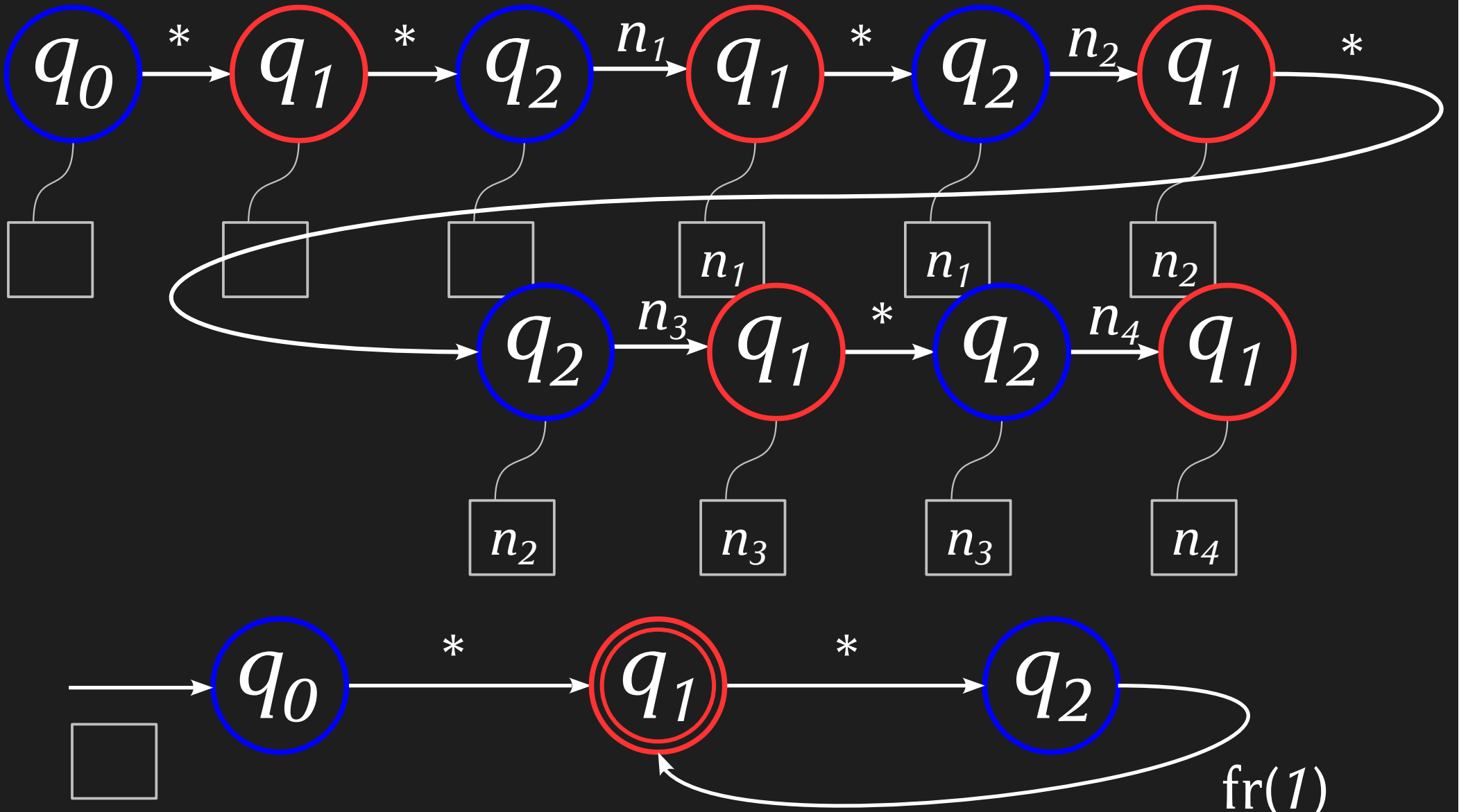

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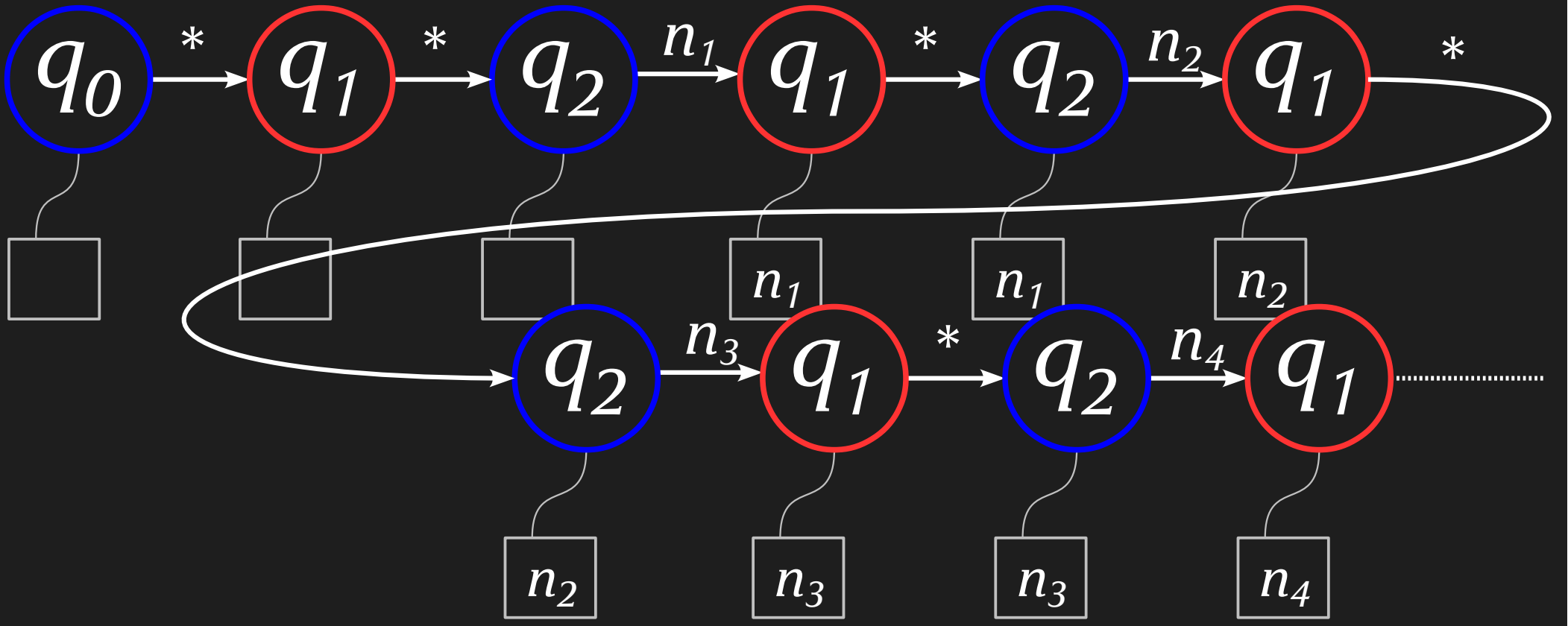

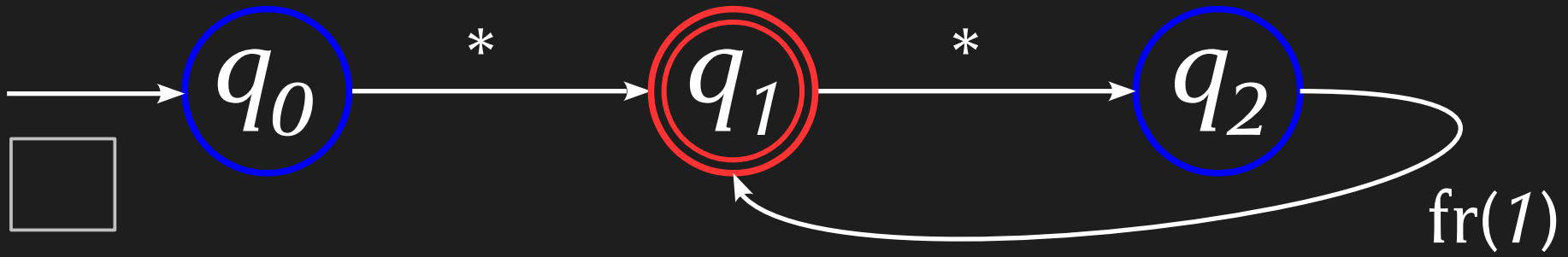

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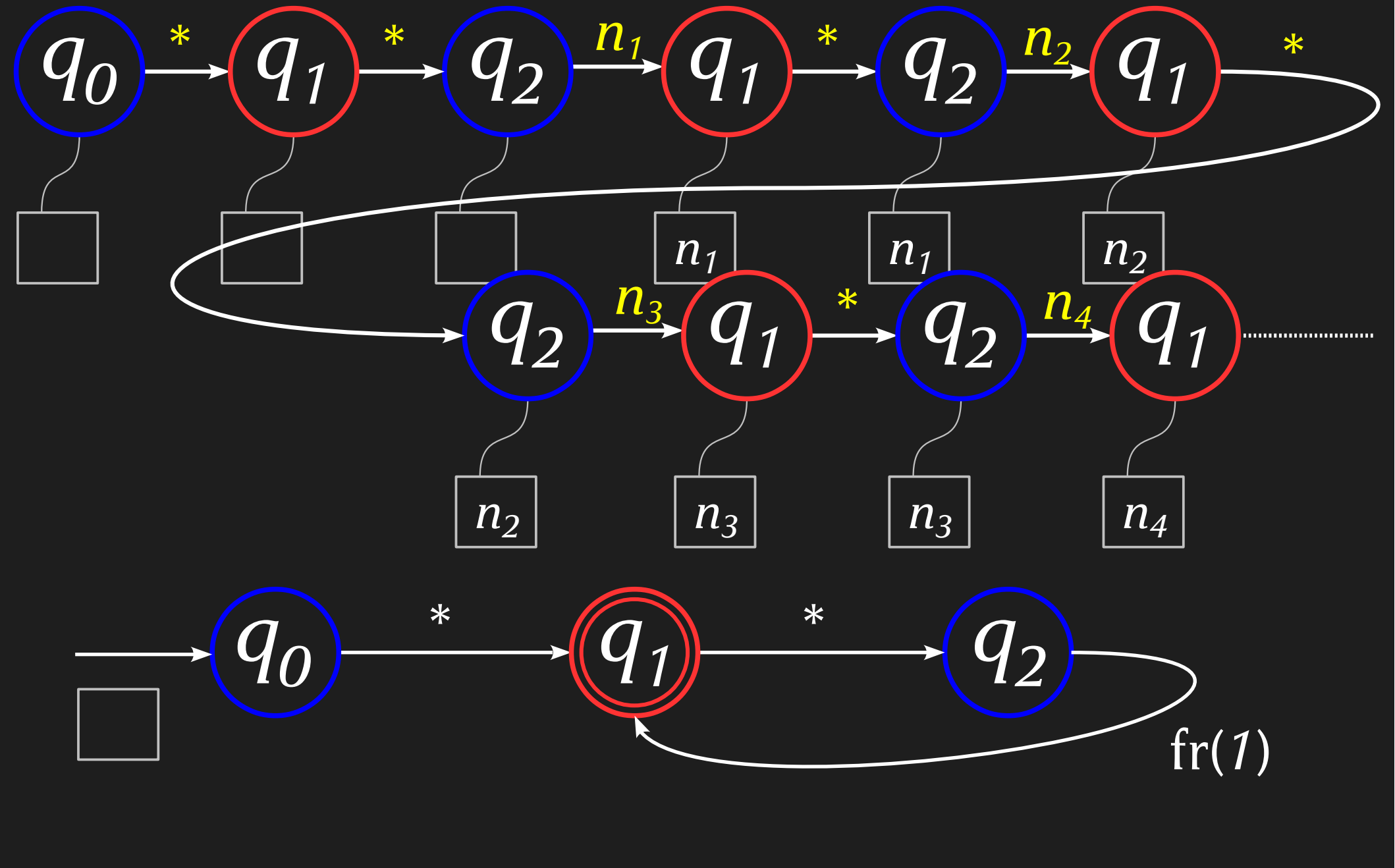

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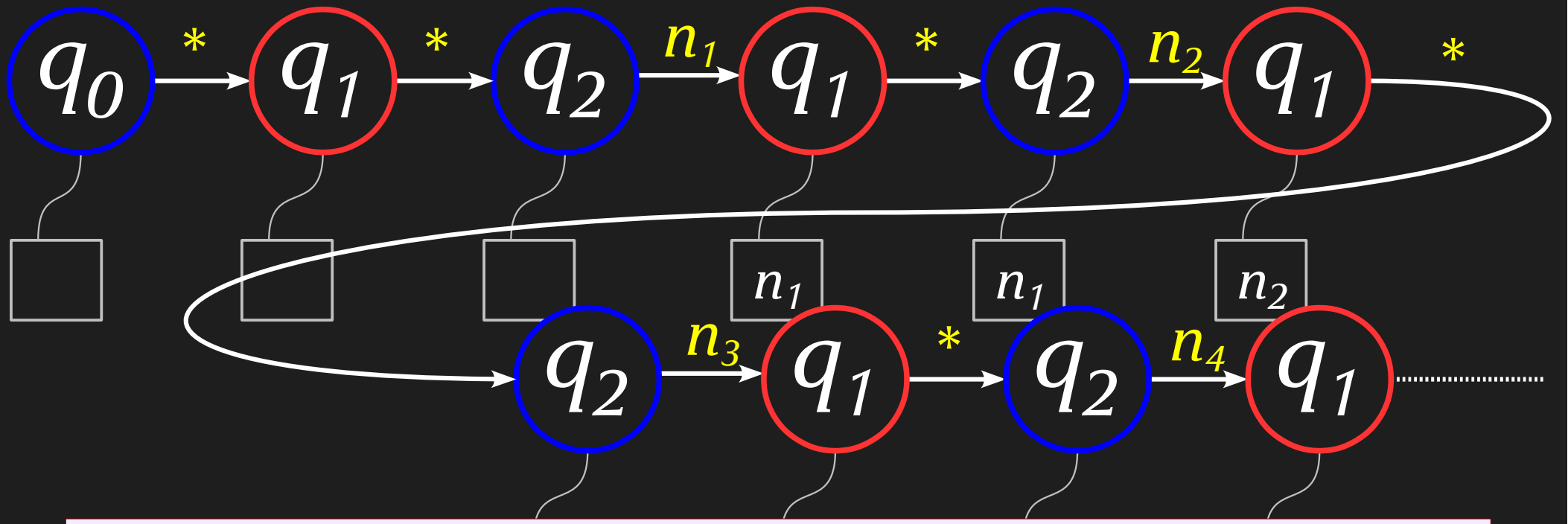
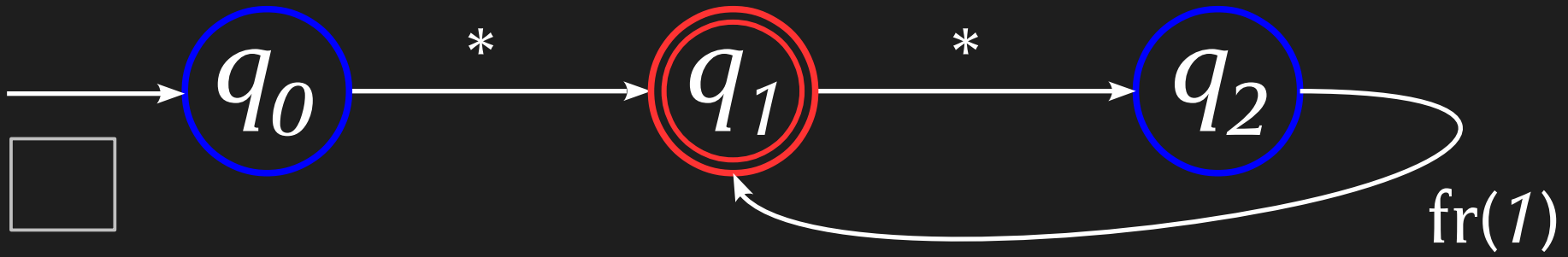


$$\vdash \lambda z.\mathtt{newvar}(0) : \mathtt{com} \to \mathtt{intvar}$$



Proponent / *Opponent*

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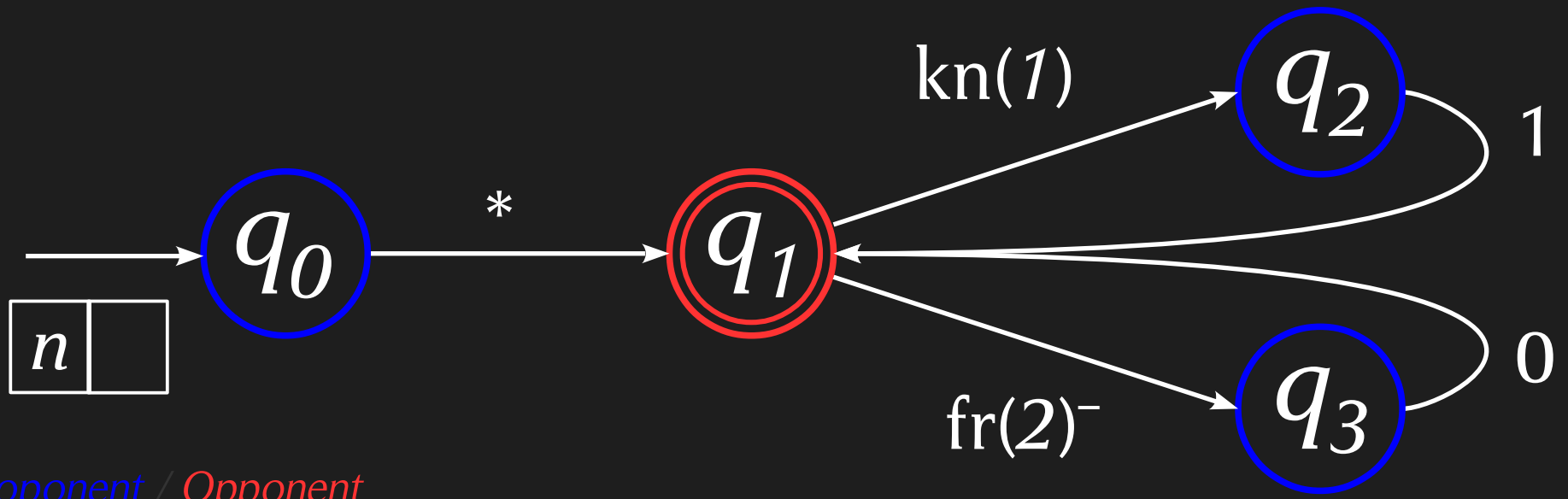

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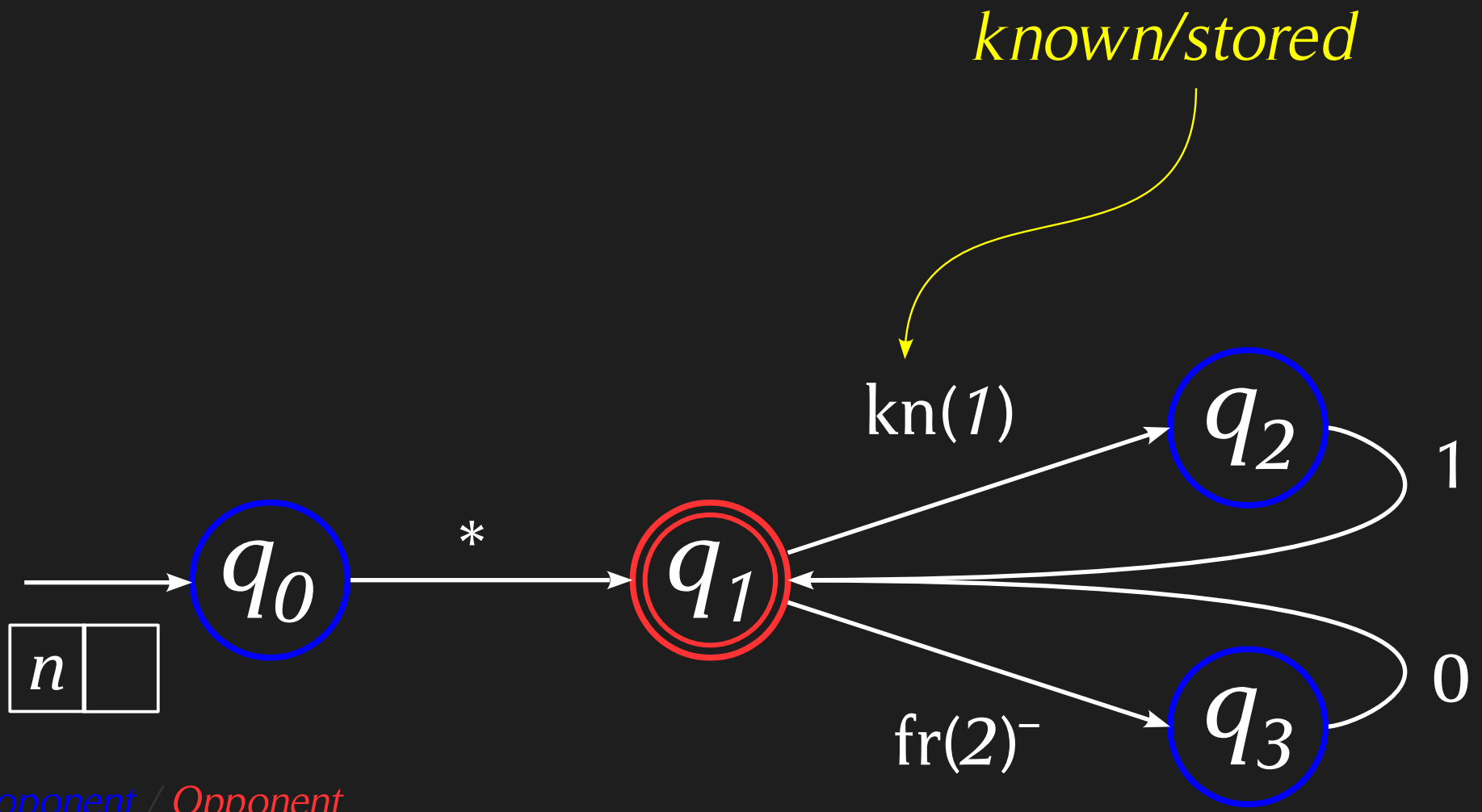

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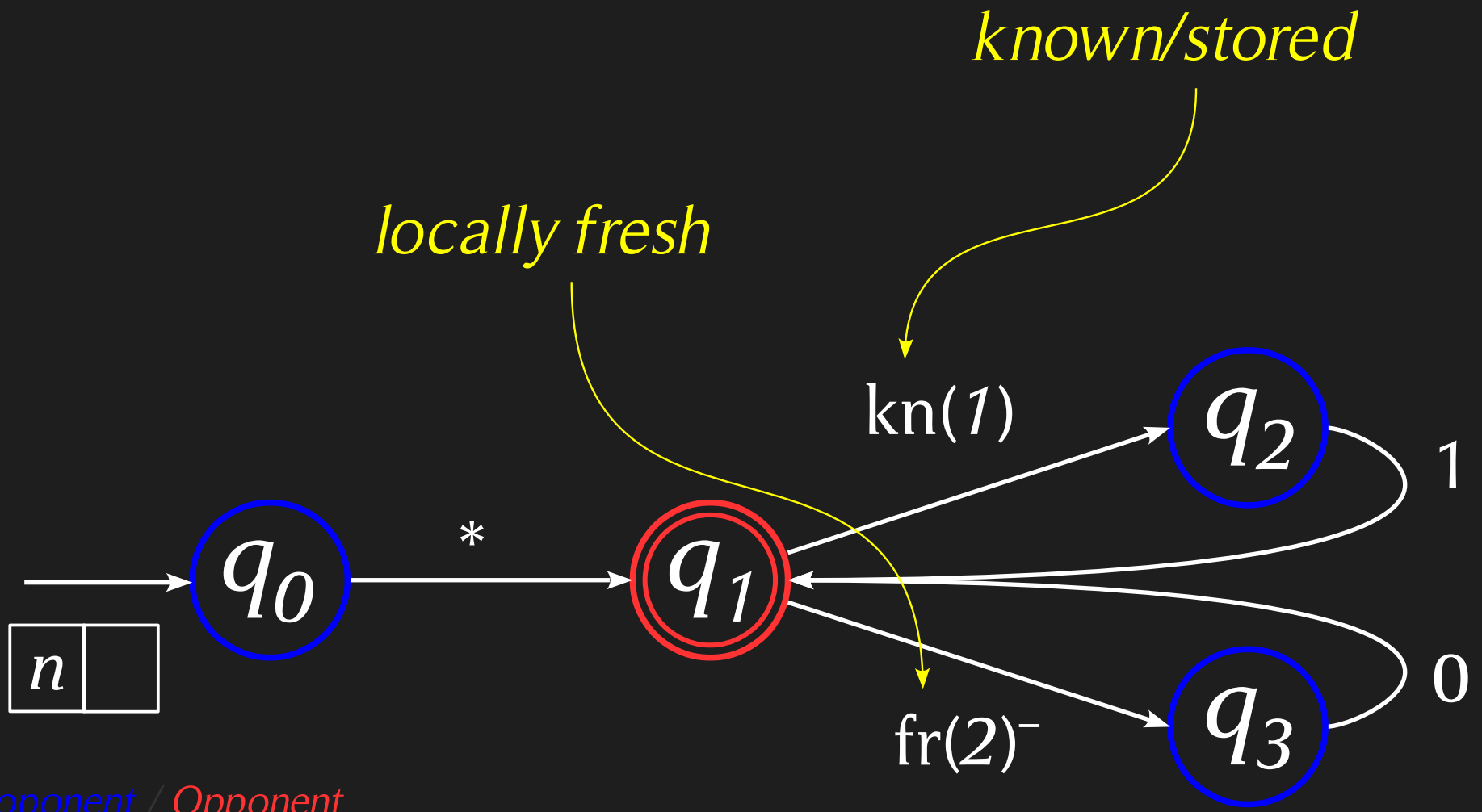

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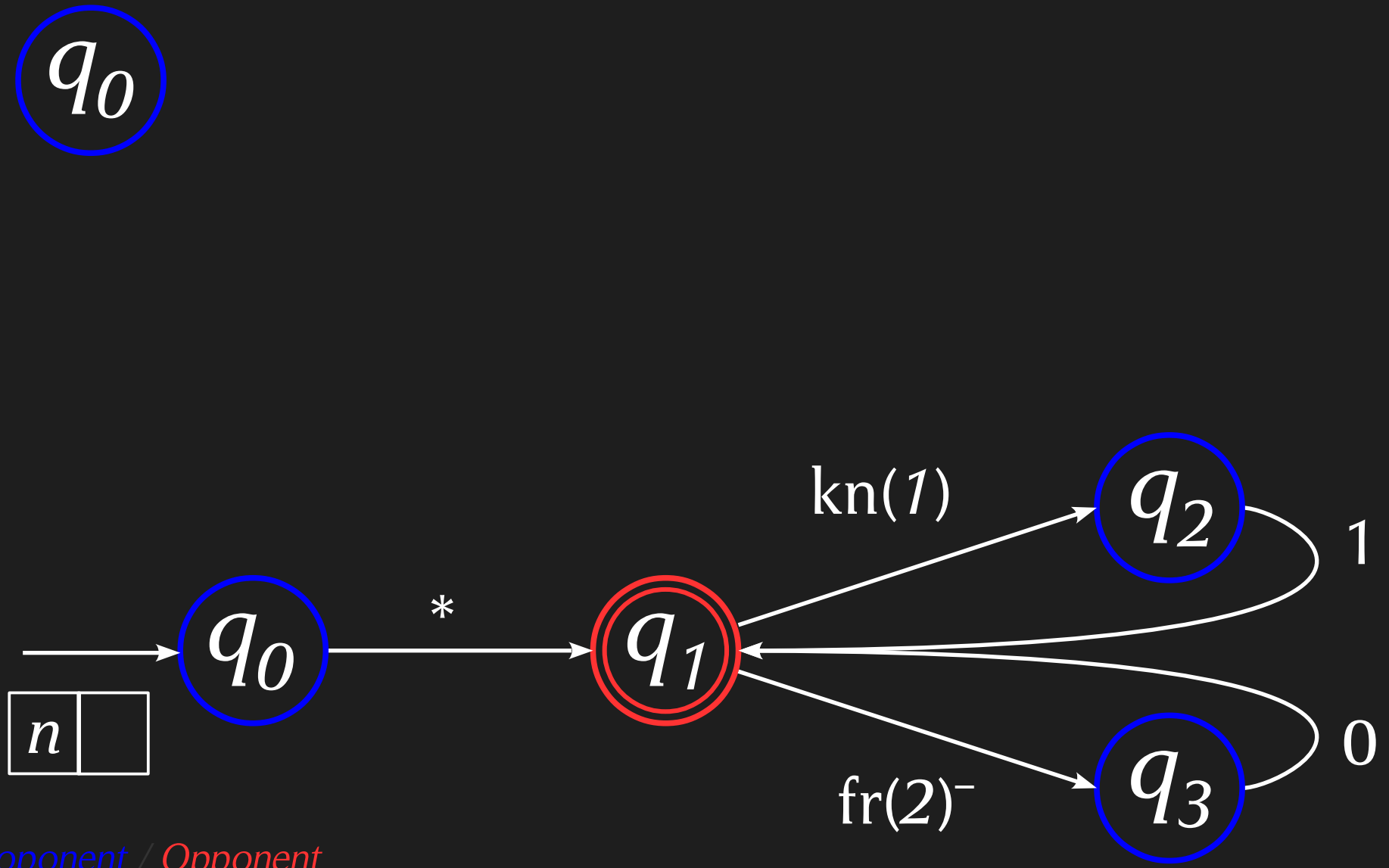

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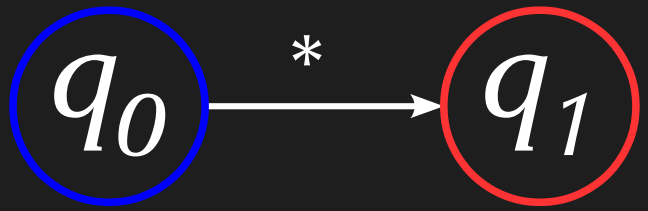

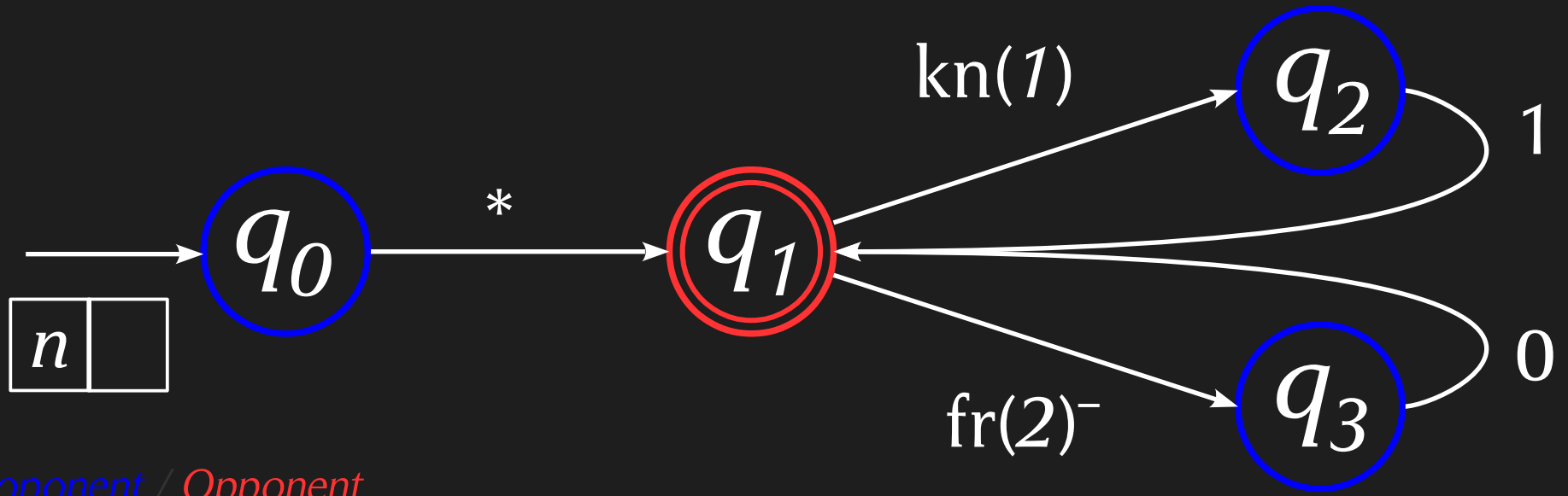

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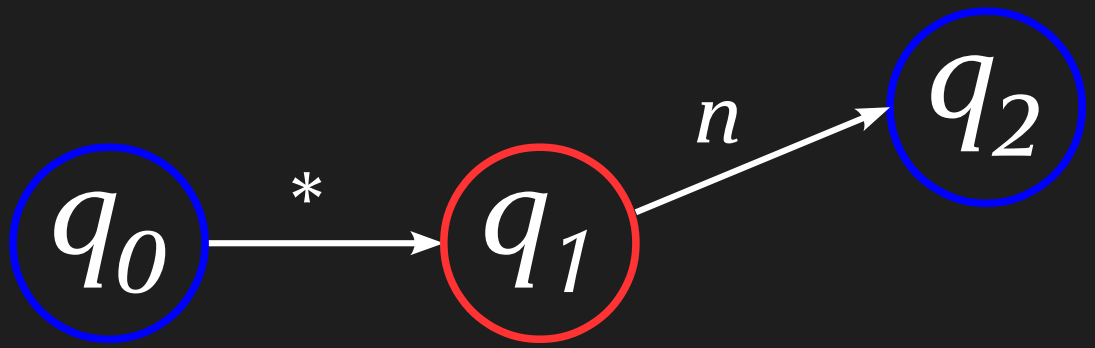

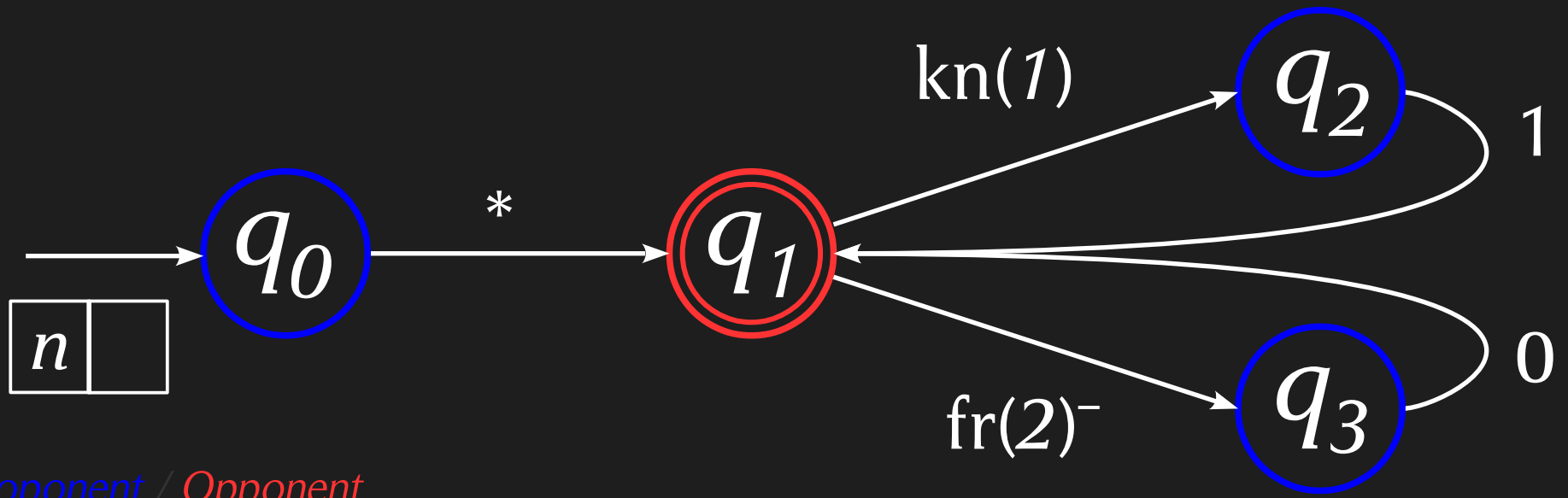

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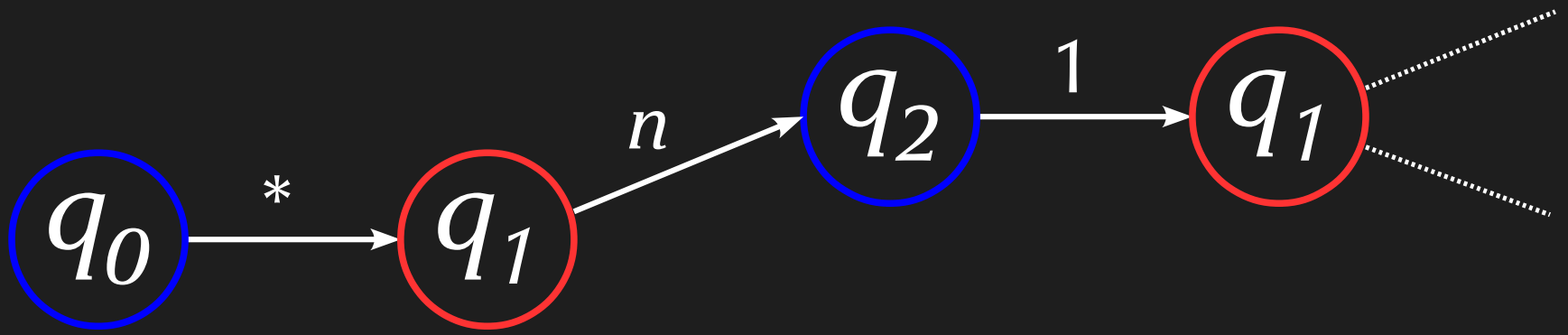

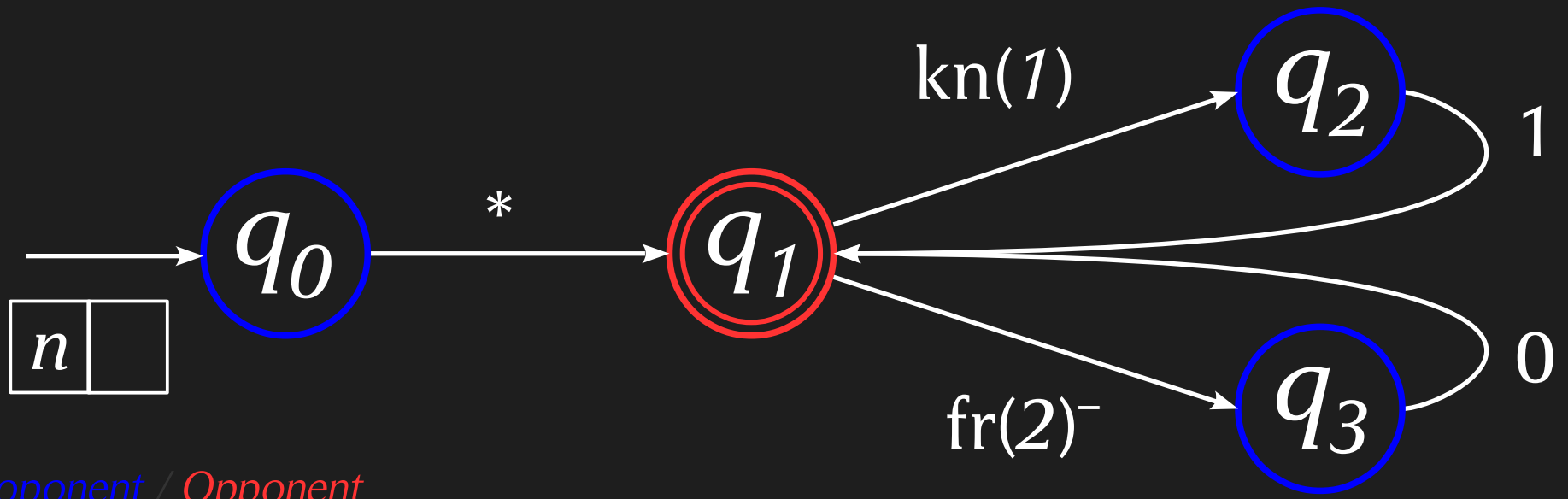

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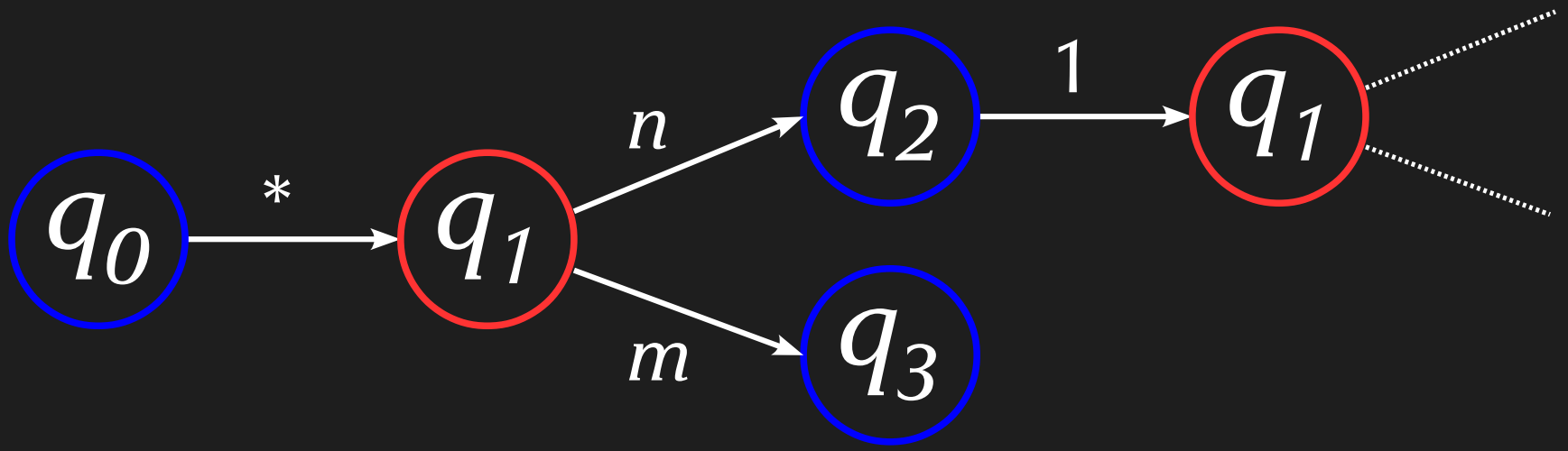

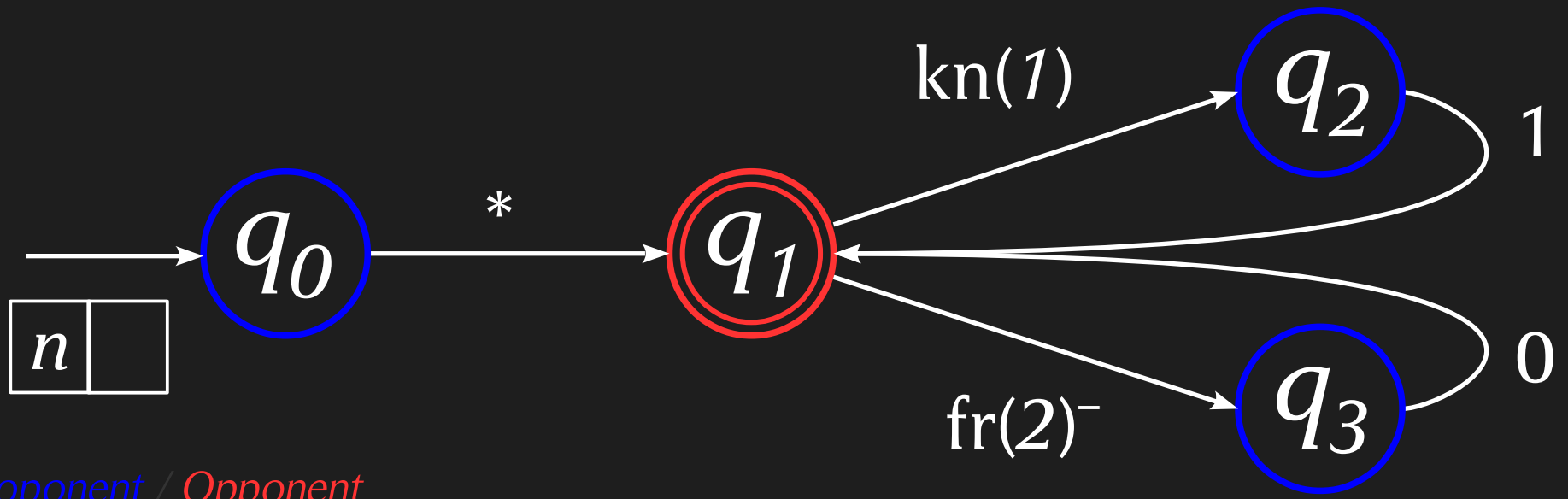

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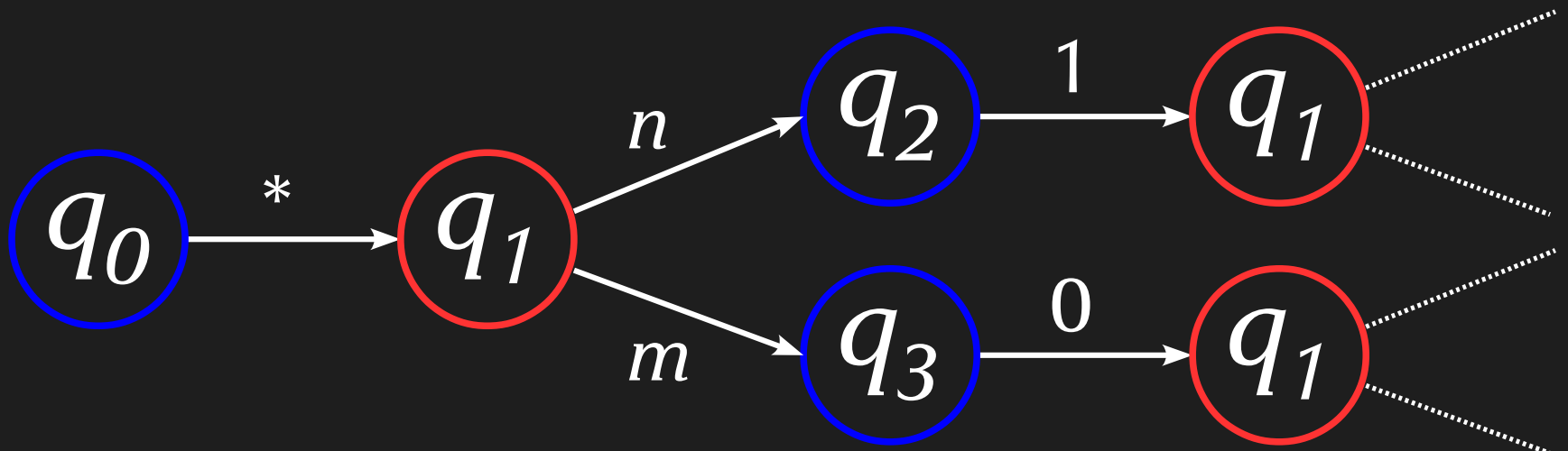

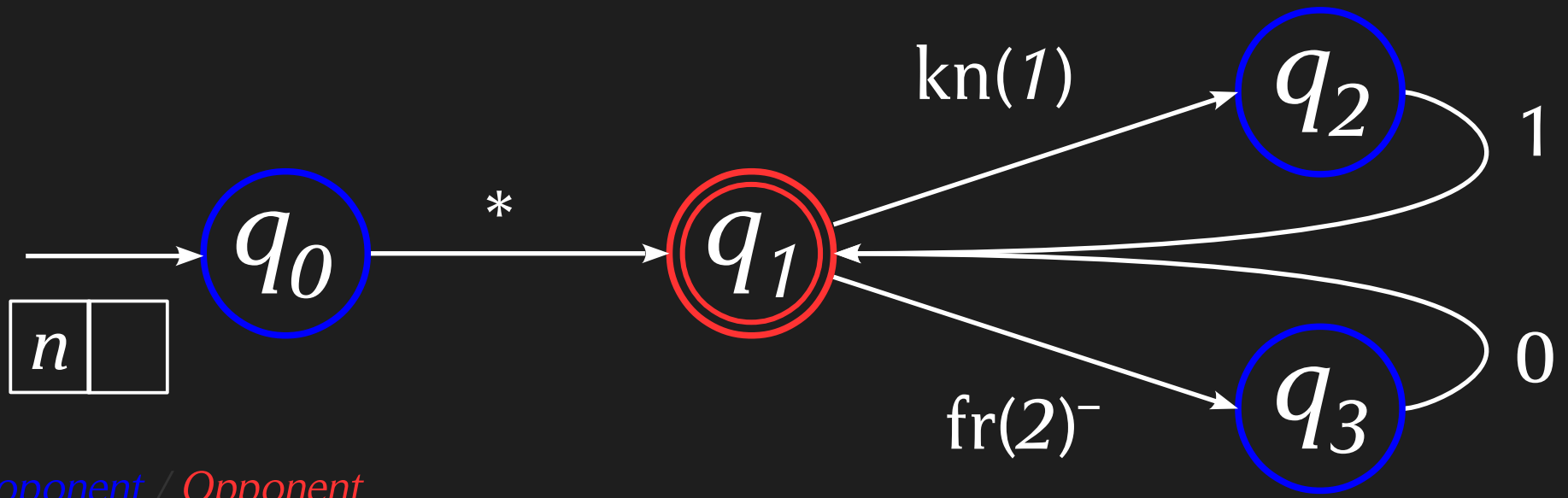

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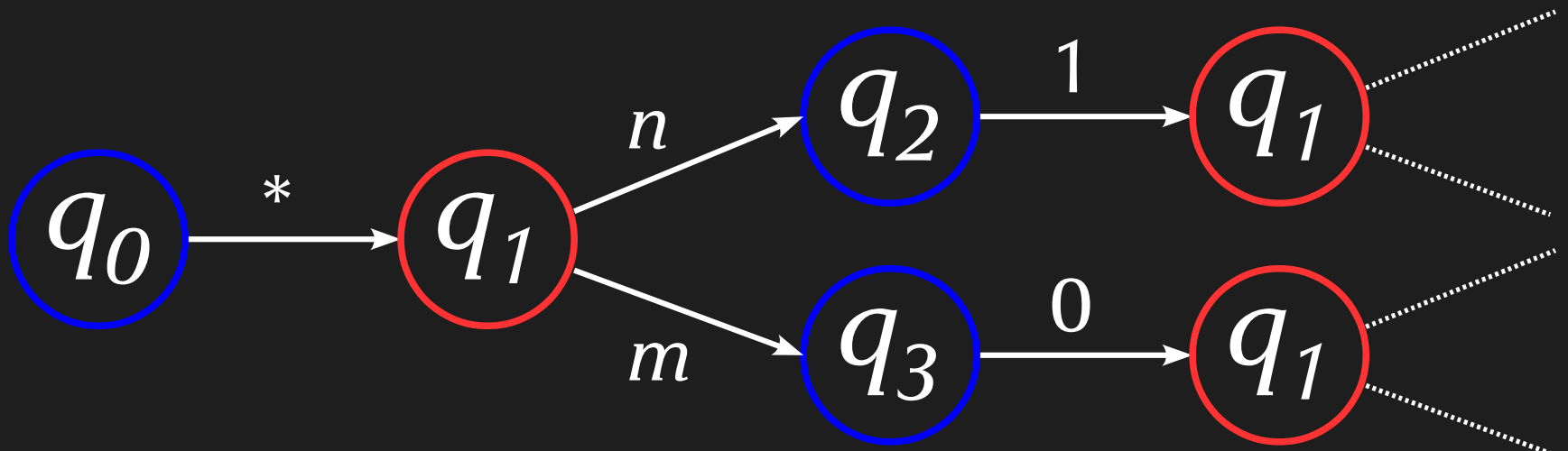
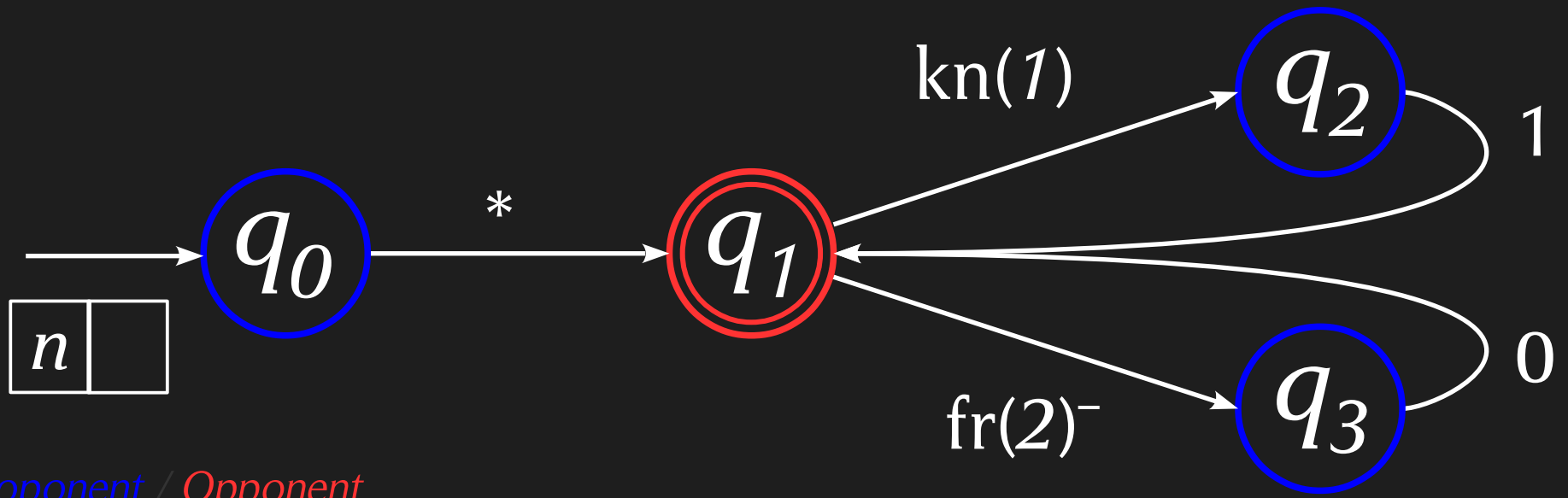



```
x : intvar ⊢ λy.(x == y) : intvar → int
```



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Games with new resources

Models of realistic programs (2007-)

- General name-features [Tz.07, Tz.08]
- Direct models [Laird 06, Laird 08, Murawski&Tz.09, Mur.&Tz.11]

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$$M \cong N \iff \mathcal{A}_M \sim \mathcal{A}_N$$

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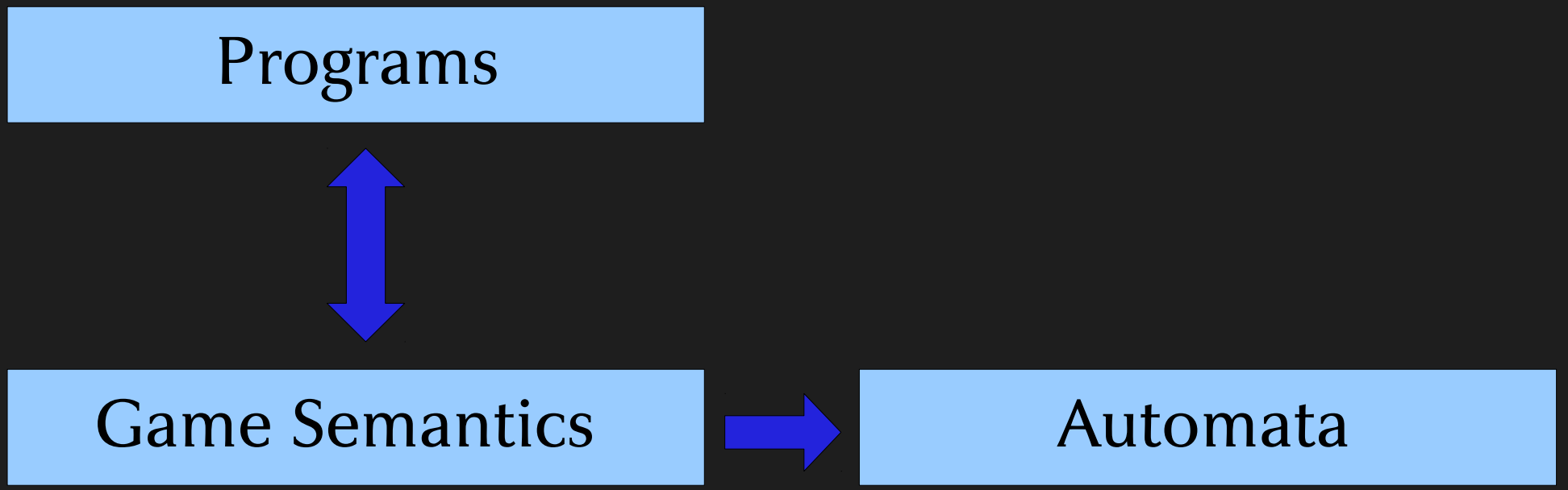
Reasoning about new resources



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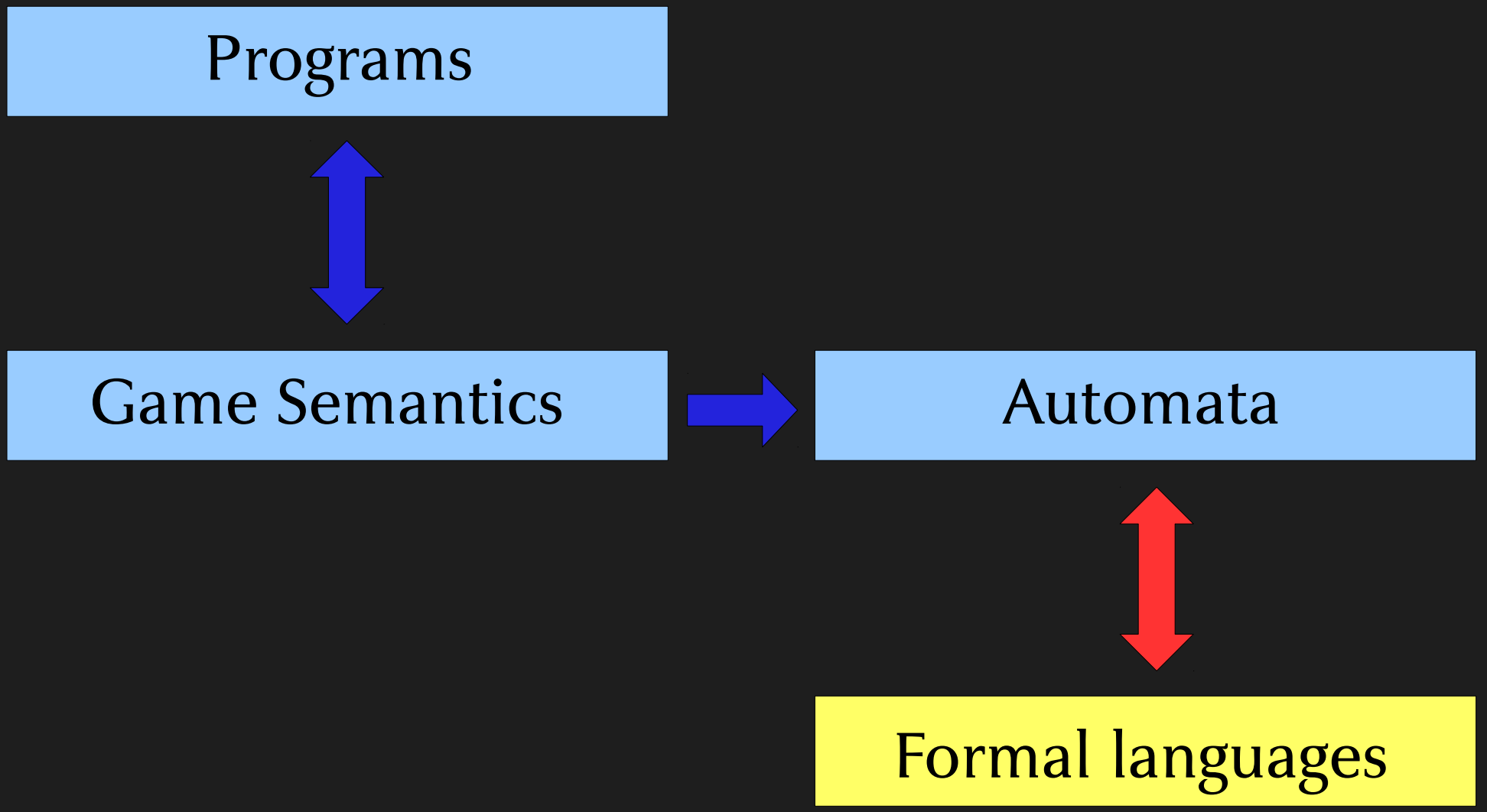

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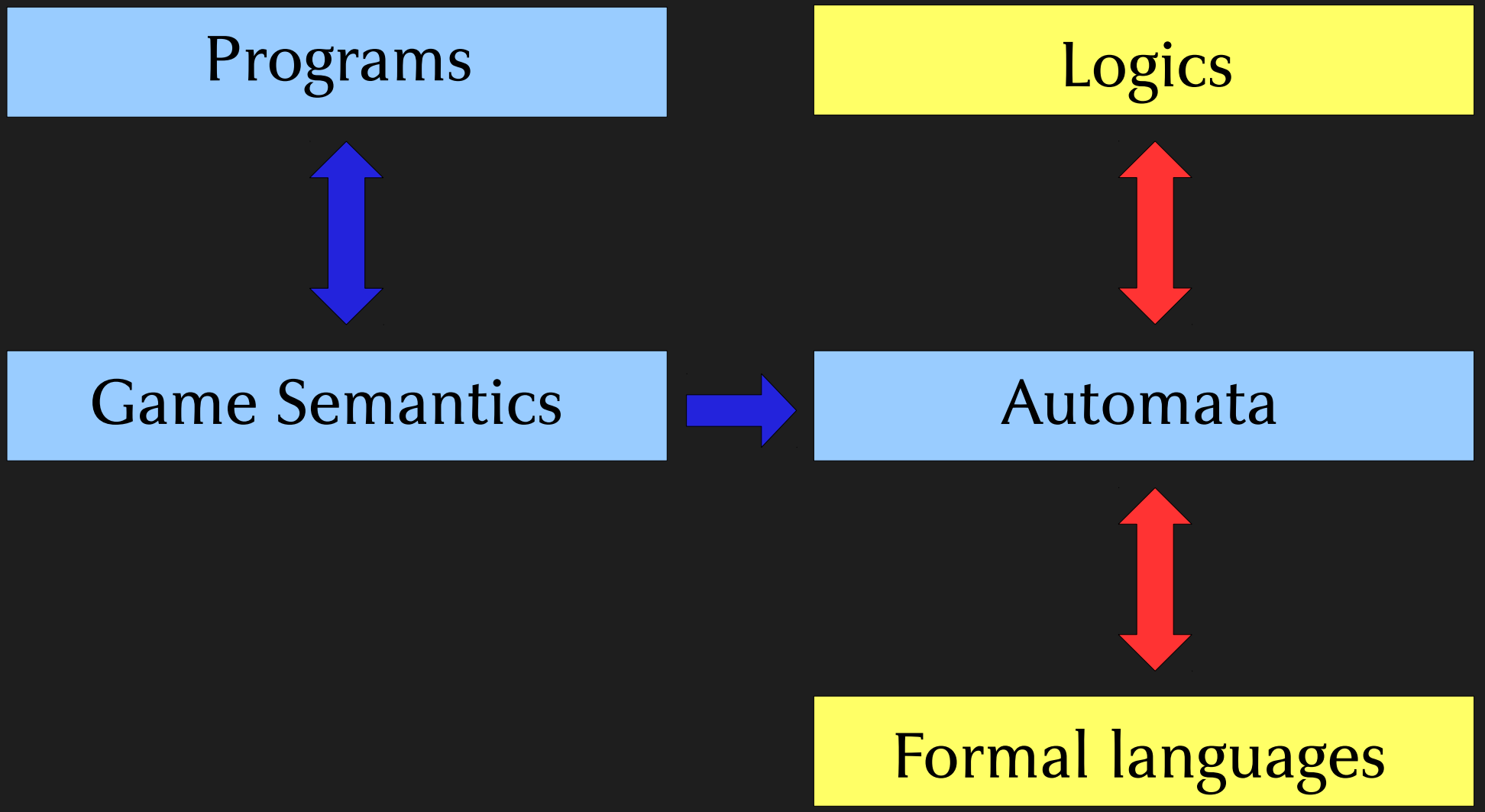

thank you!