# Games with names

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# What this talk is about

Generation of new resources is a pervasive feature in computation (references, objects, channels, etc.)

We see resources as names

We show how to use game semantics to capture computation with names

# Computation with names

$\lambda x.\mathtt{ref}(0) : \mathtt{com} \to \mathtt{intref}$

```
let f = [_] in { f() == f() }
```

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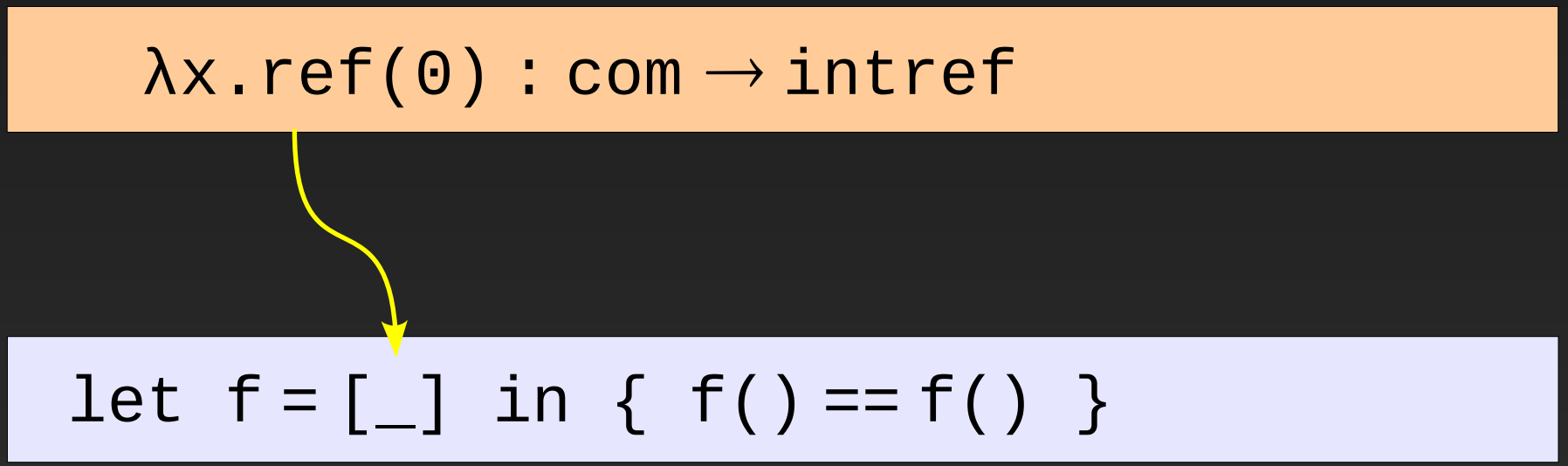

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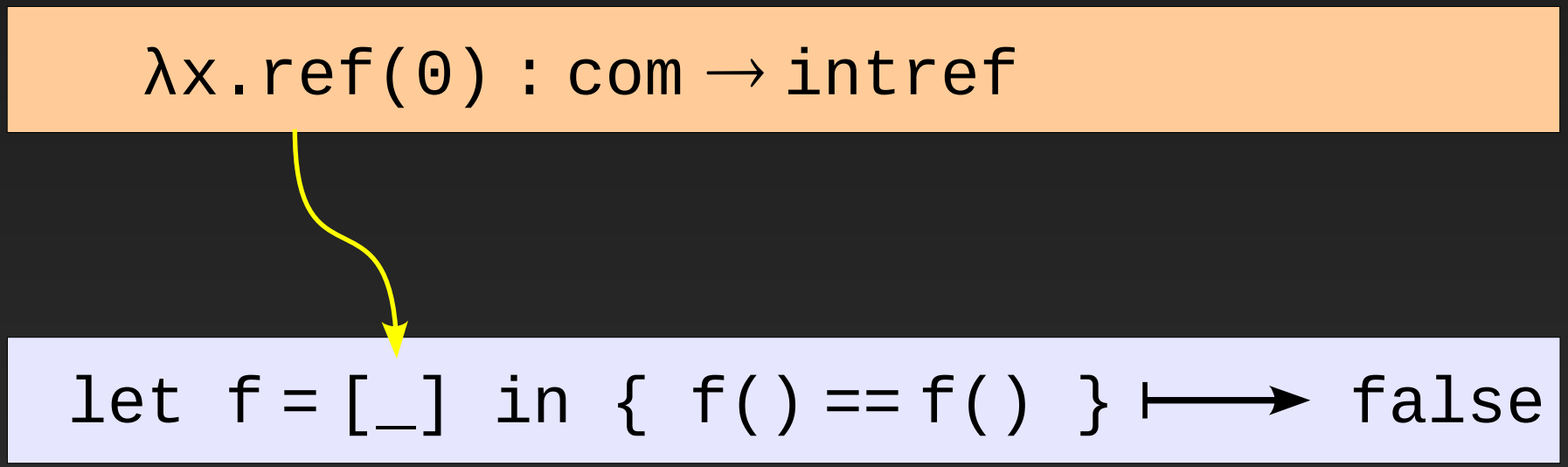

# Example: Reduced ML

basic programming language with functions + integer references

$$P \cong P'$$

*equivalent behaviour in every context*

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$$\mathsf{let}\, x = \mathsf{ref}(0)\, \mathsf{in}\ \lambda y.\, (x == y) \quad \cong \quad \lambda y.\, \mathsf{false}\ : \mathsf{intref} \to \mathsf{int}$$

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$$f : \mathsf{intref} \to \mathsf{int} \ \vdash\ \lambda y.\, f(\mathsf{ref}(0))$$

$$\cong\ \mathsf{let}\, x = \mathsf{ref}(0)\, \mathsf{in}\, \lambda y.\, x := 0; f(x)\ : \mathsf{com} \to \mathsf{int}$$

# Two ways to model references

Reynolds

- *Idealized Algol (1978)*

References are *pairs:*

$$\mathtt{intref} = (\mathtt{com} \to \mathtt{int}) \times (\mathtt{int} \to \mathtt{com})$$

$$\longmapsto (\mathbf{1} \to \mathbf{Z}) \times (\mathbf{Z} \to \mathbf{1})$$

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## Pitts & Stark
- *nu-calculus (1993)*

References are *names:*

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- Notion of *resource (name):*
  - atomic values
  - infinitely many
  - comparable for equality

# Bad variables

- Many pairs of ref type are *not* references
  - e.g. `mkvar(λx.3,λx.())`
- no notion of *reference equality test*
- spurious non-equivalences:

```
x:=0; !x      vs.   x:=0; 0
x:=0; x:=1    vs.   x:=1
```

# Game Semantics

How can we reason about:

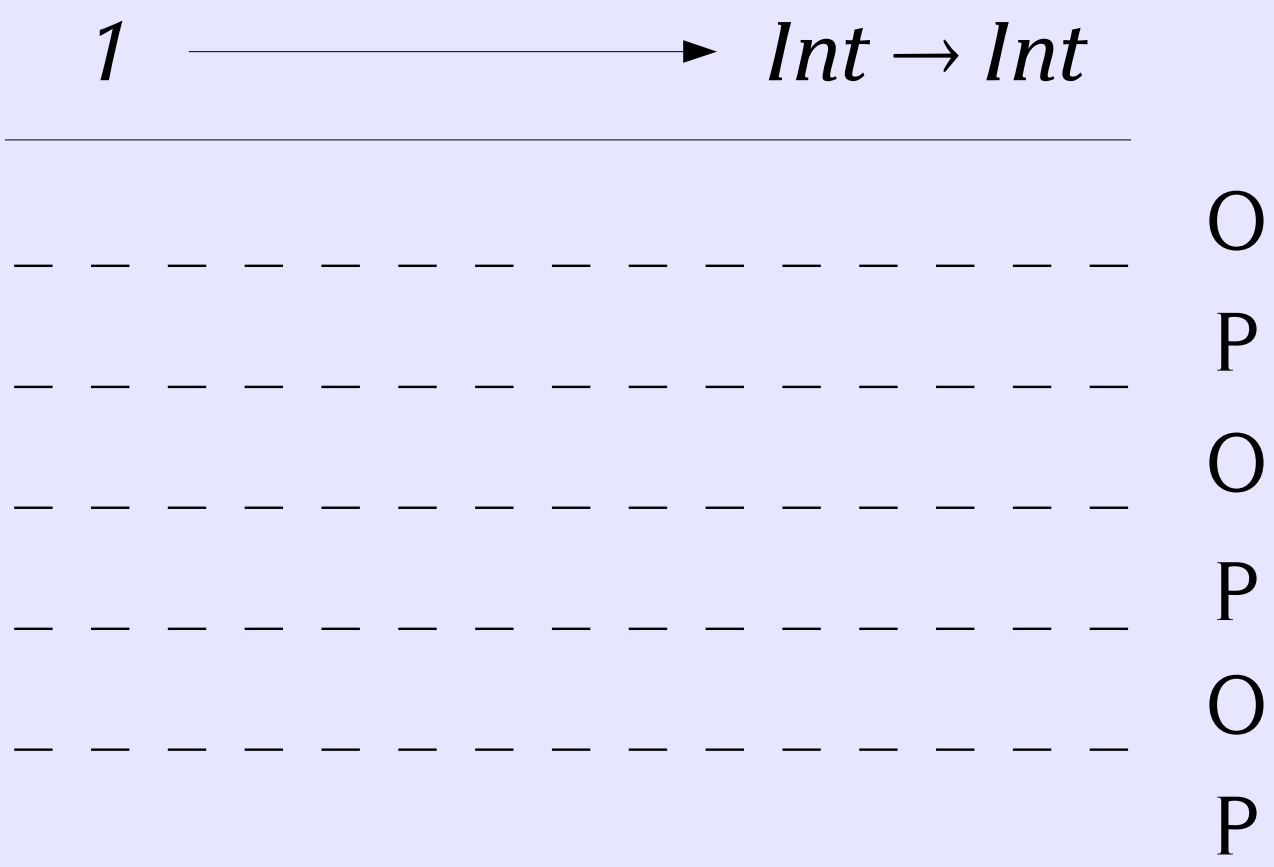
$$P \cong P'$$

# Game Semantics

- Computation is modelled as a 2-player game between:
  - *Proponent (the program)*
  - *Opponent* (the environment)

# Example

$\lambda x.x+1 : \texttt{int} \to \texttt{int}$



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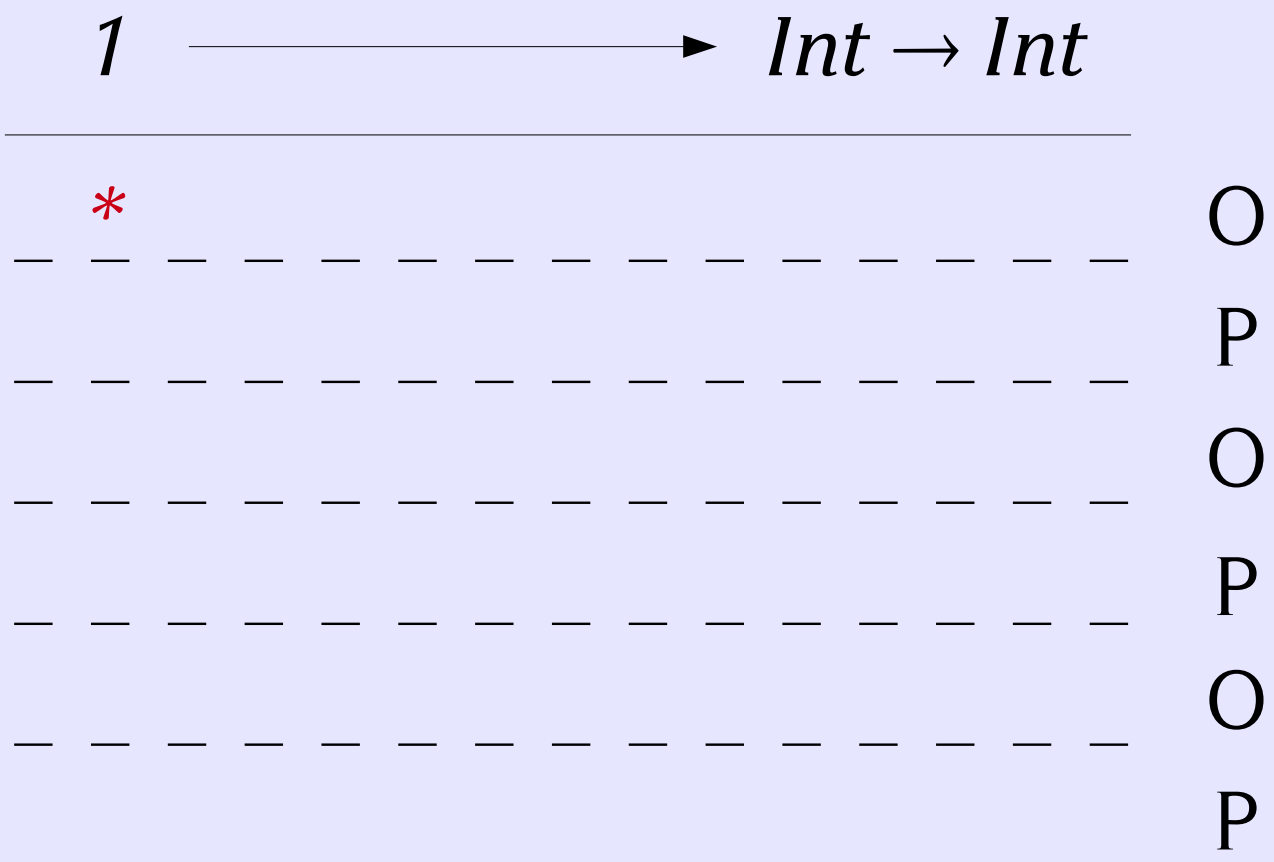

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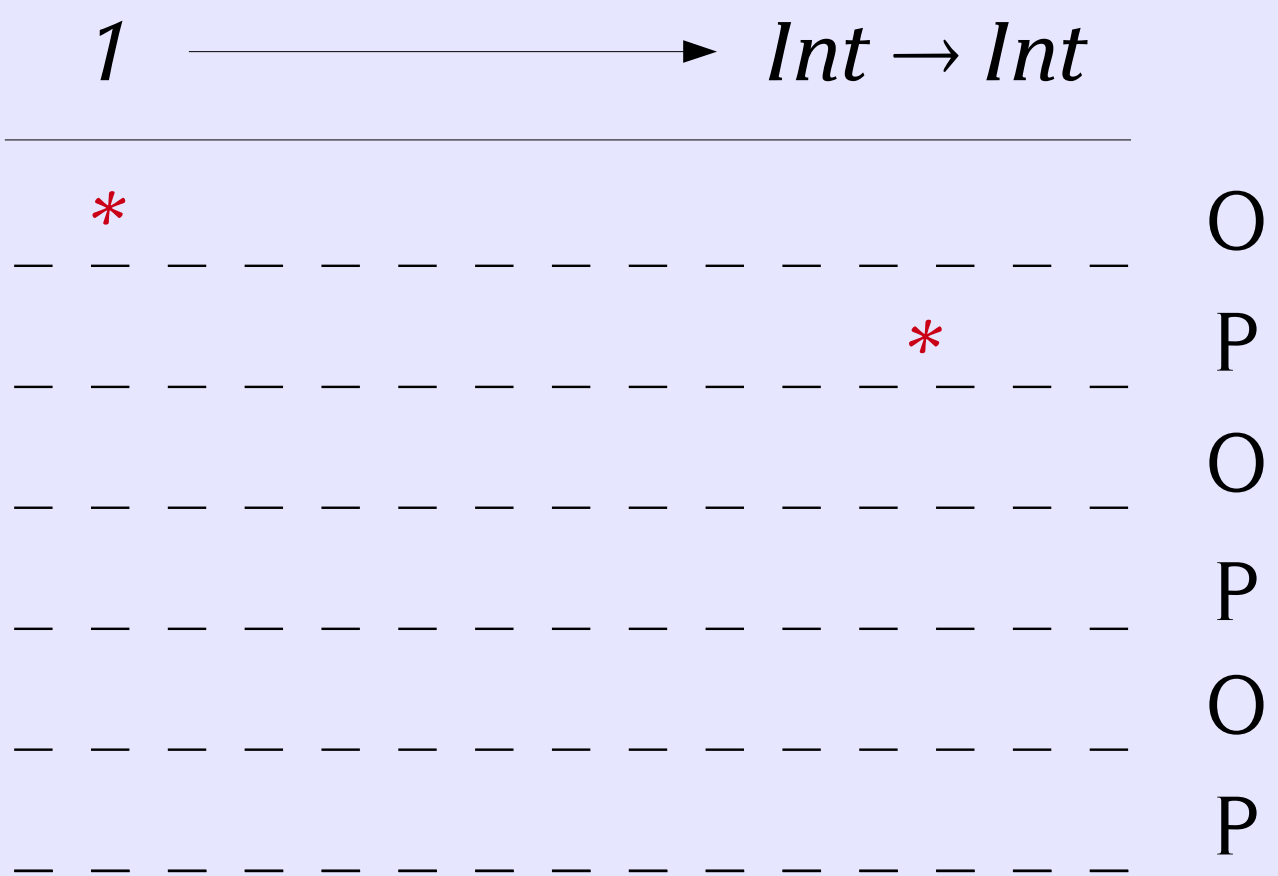

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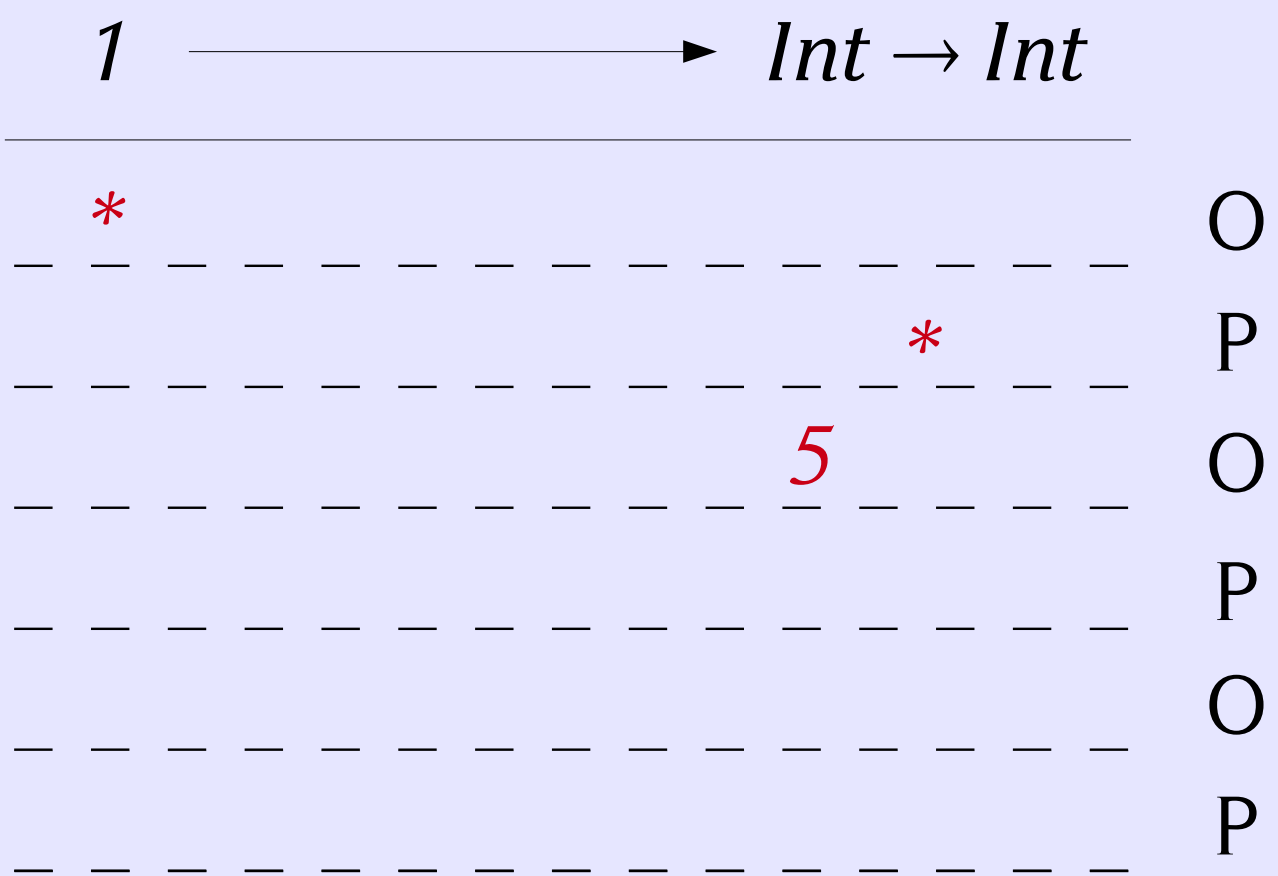

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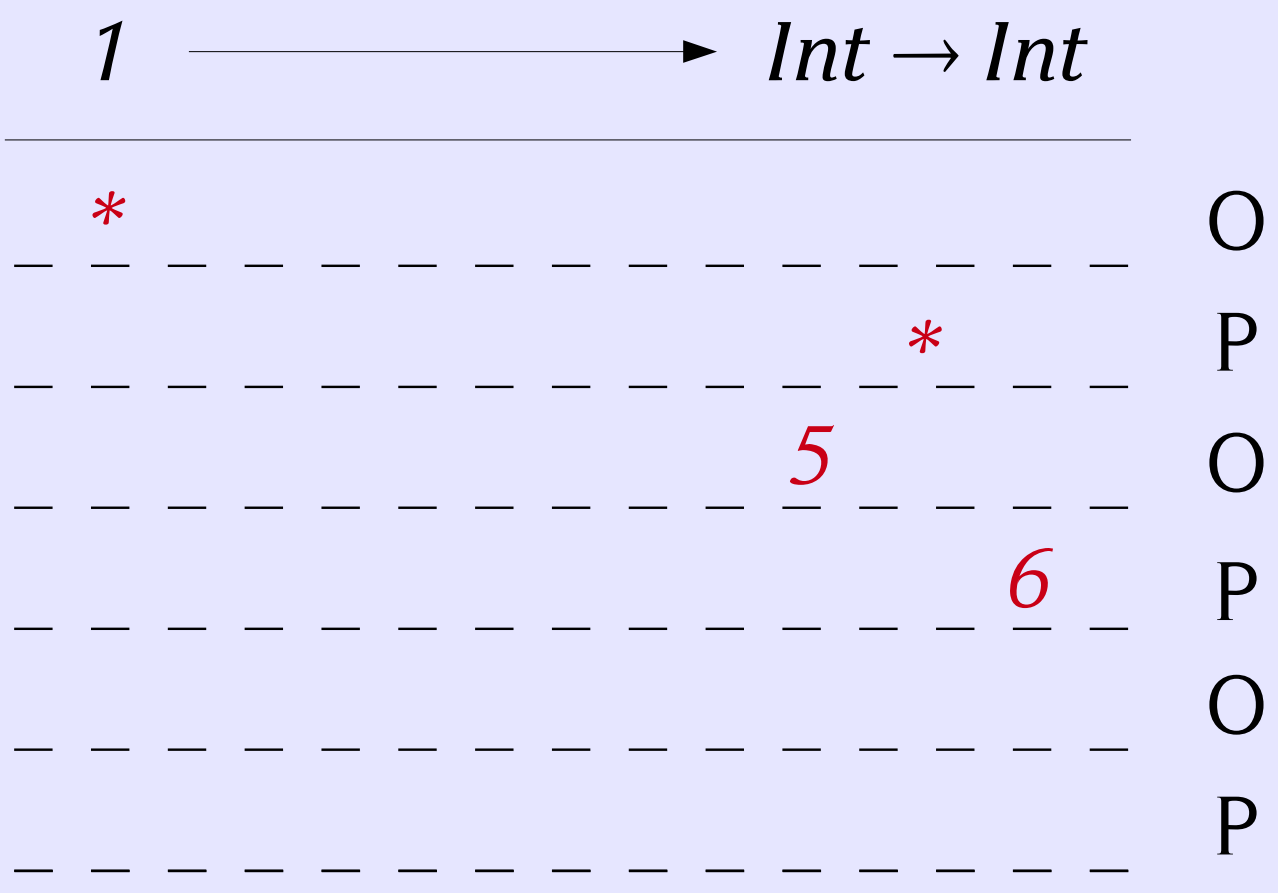

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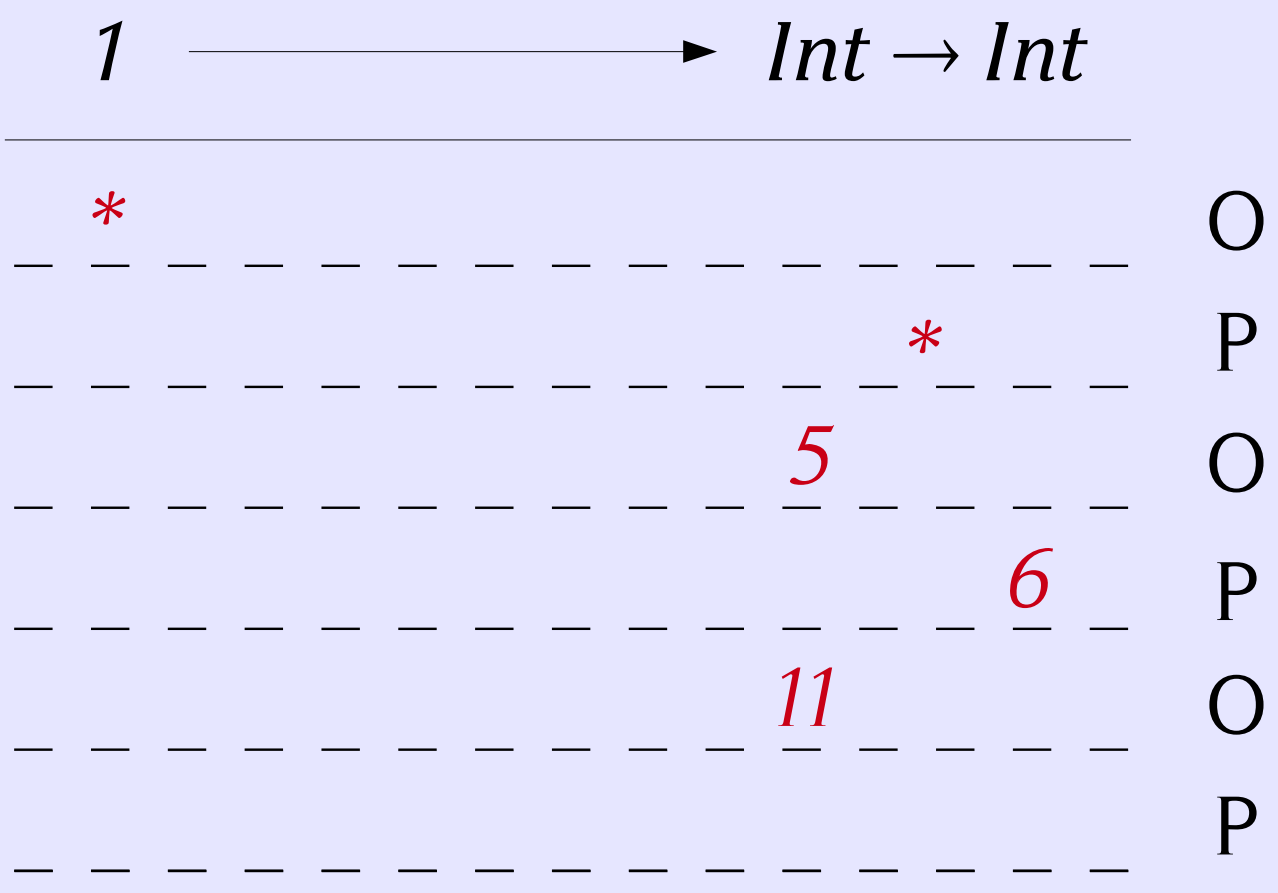

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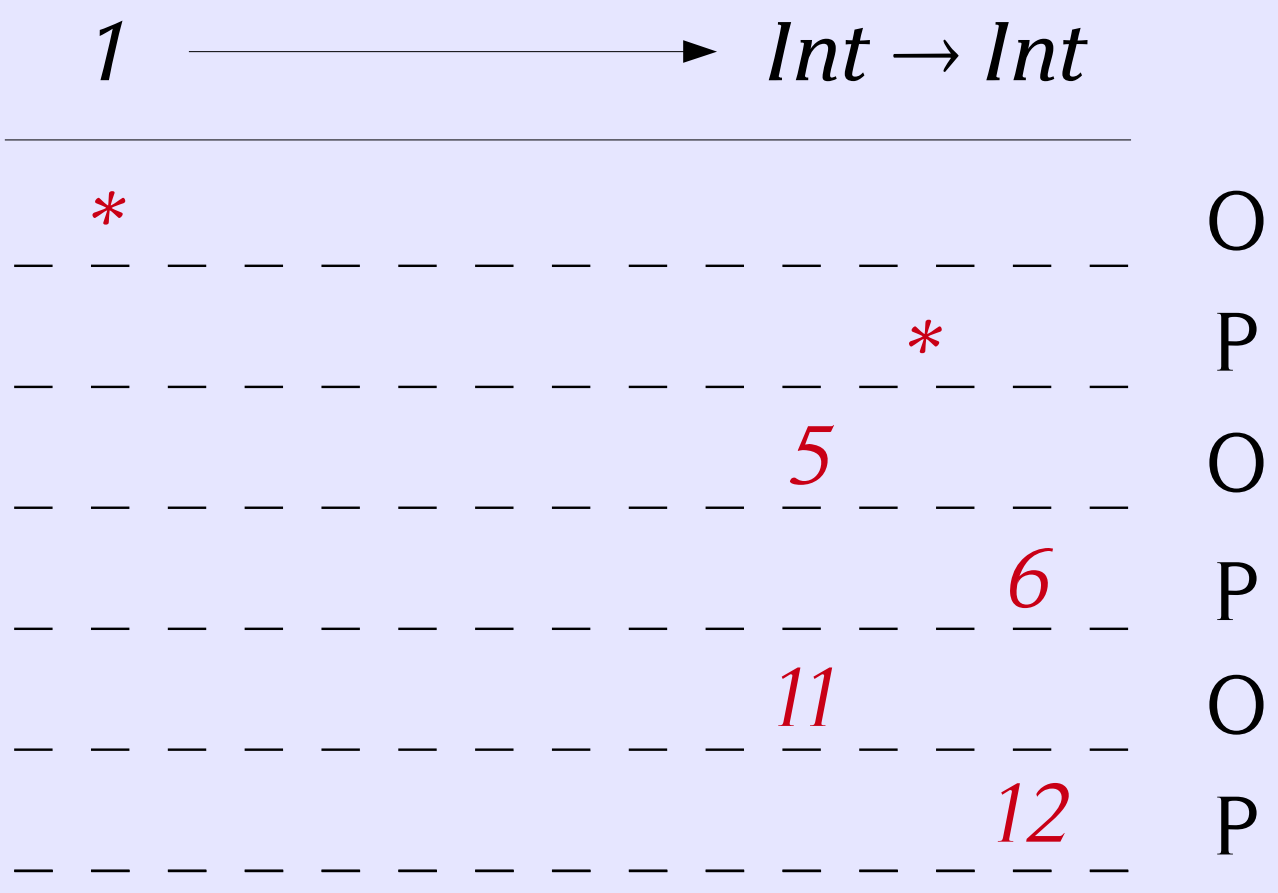

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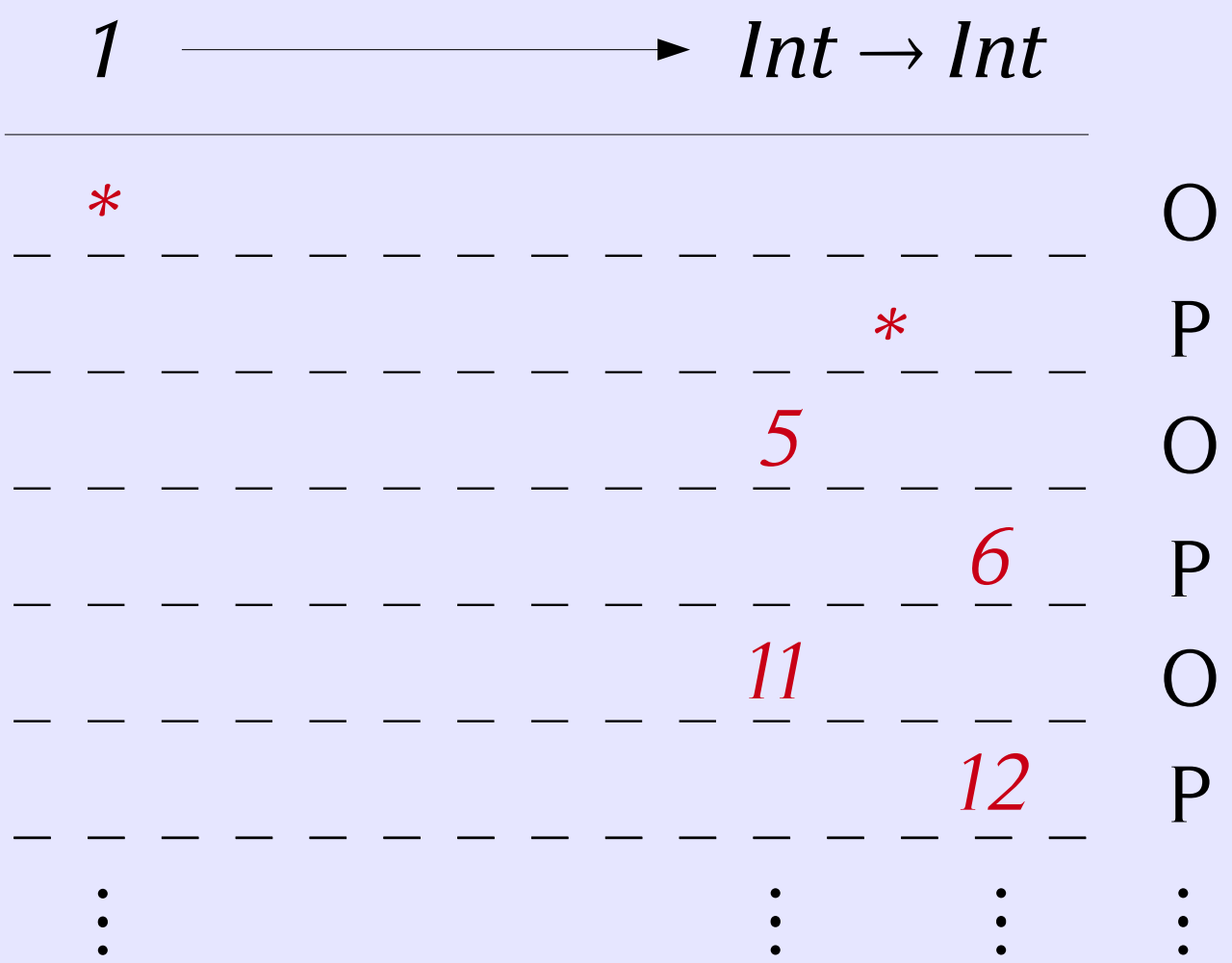

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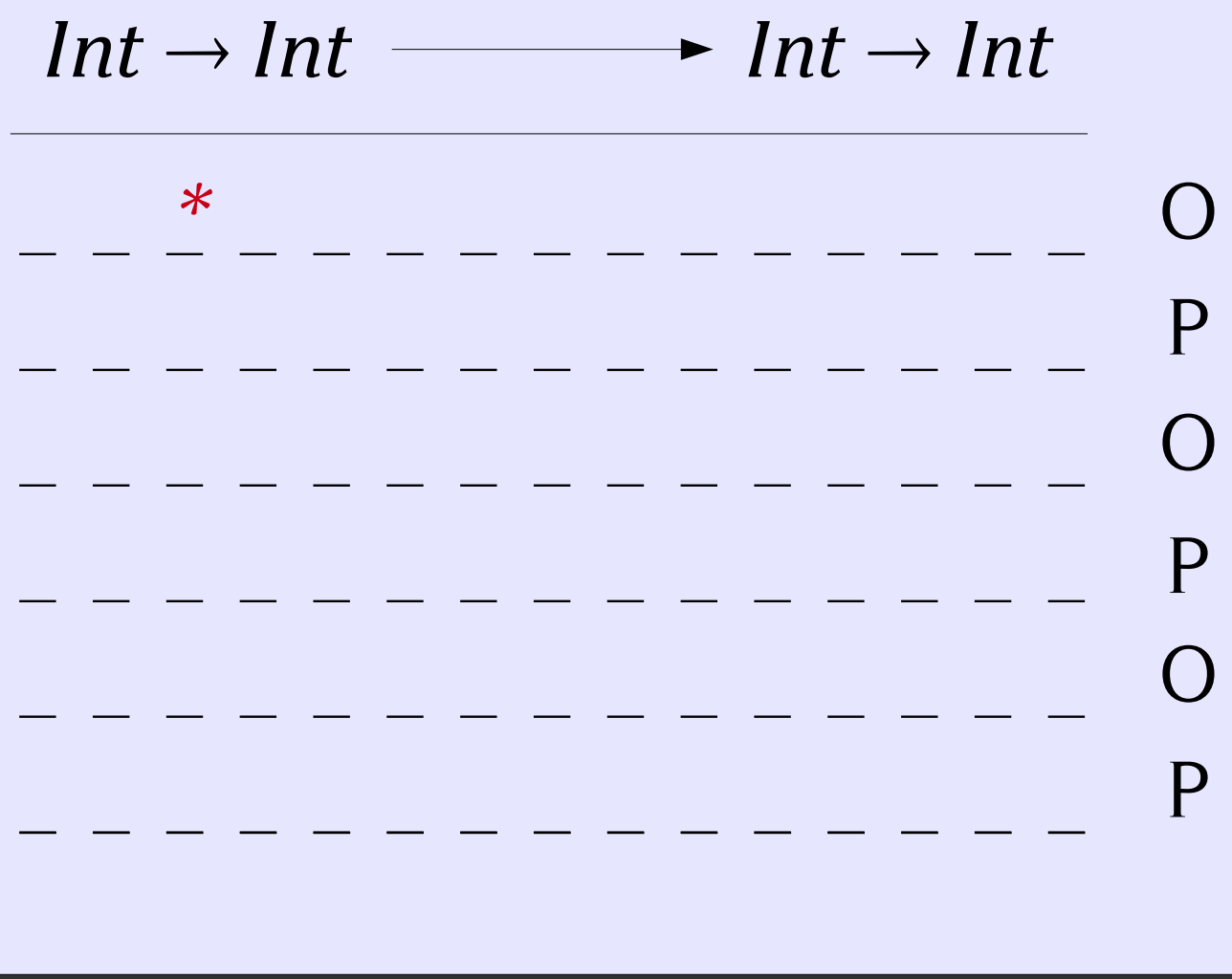

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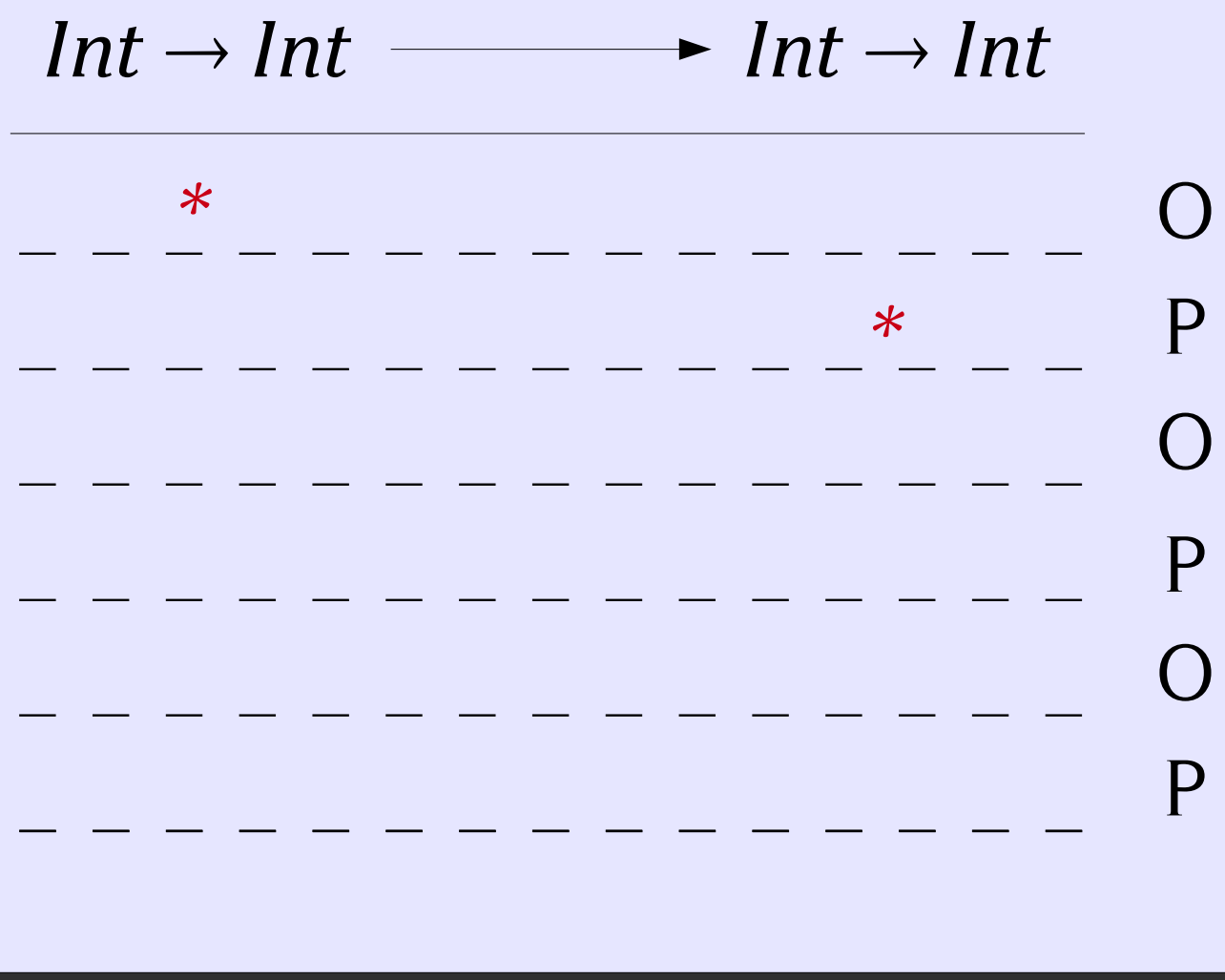

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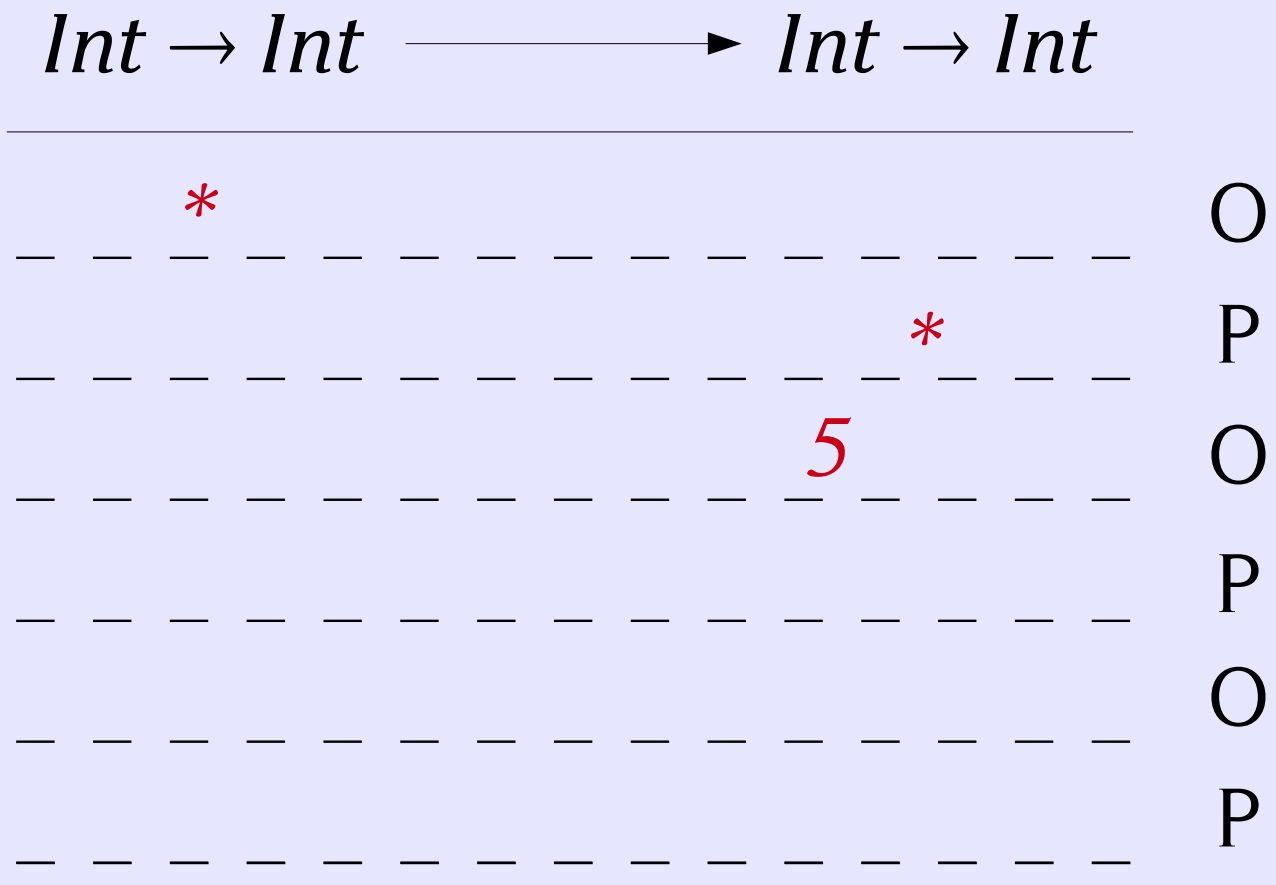

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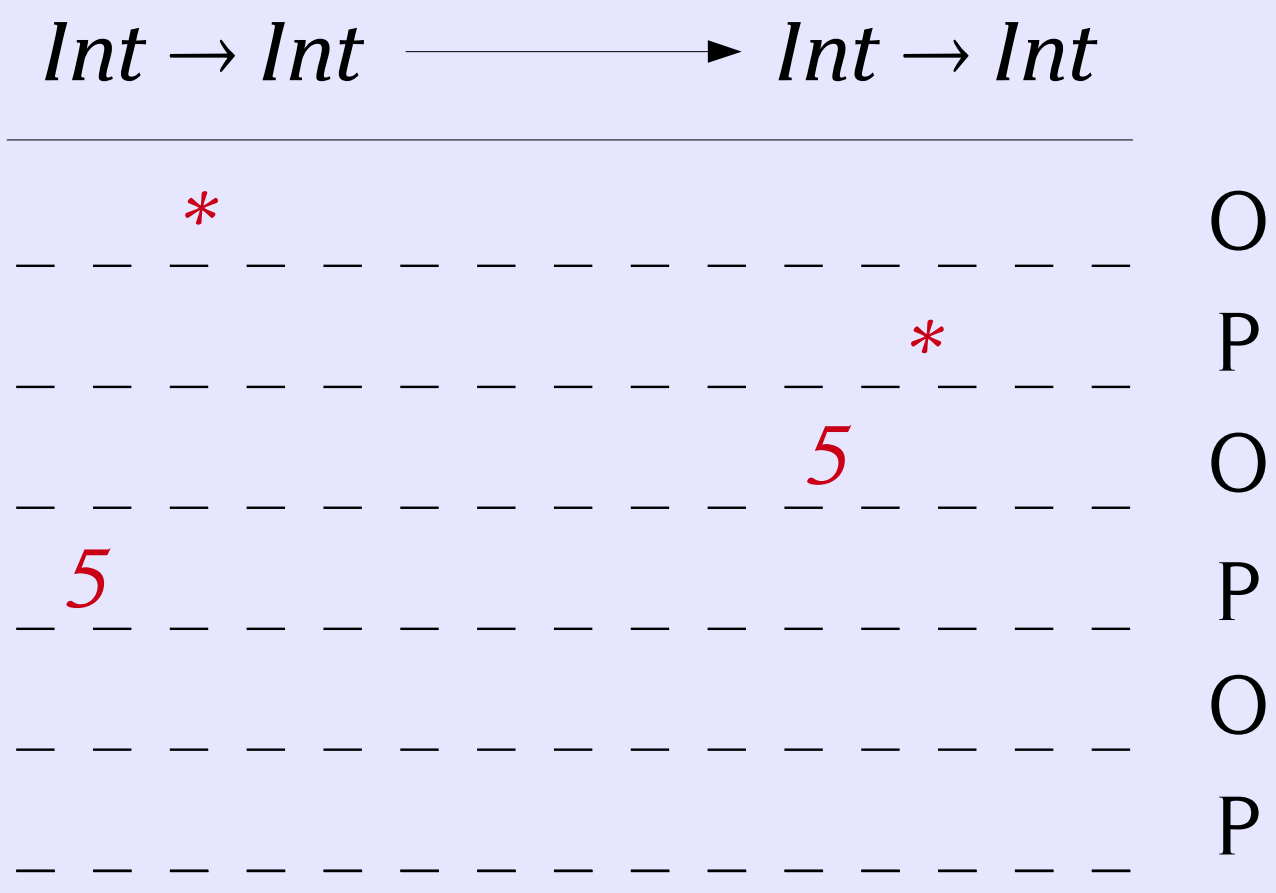

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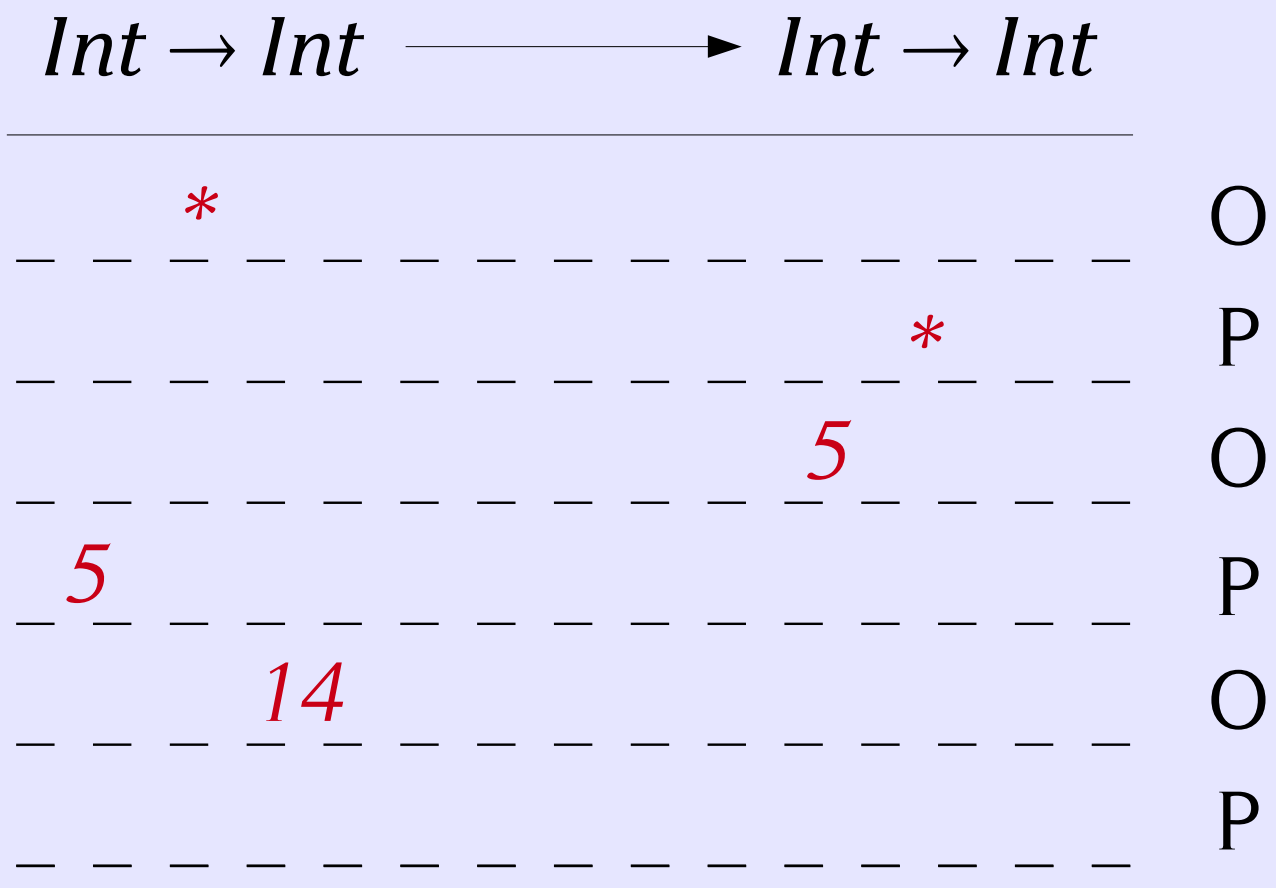

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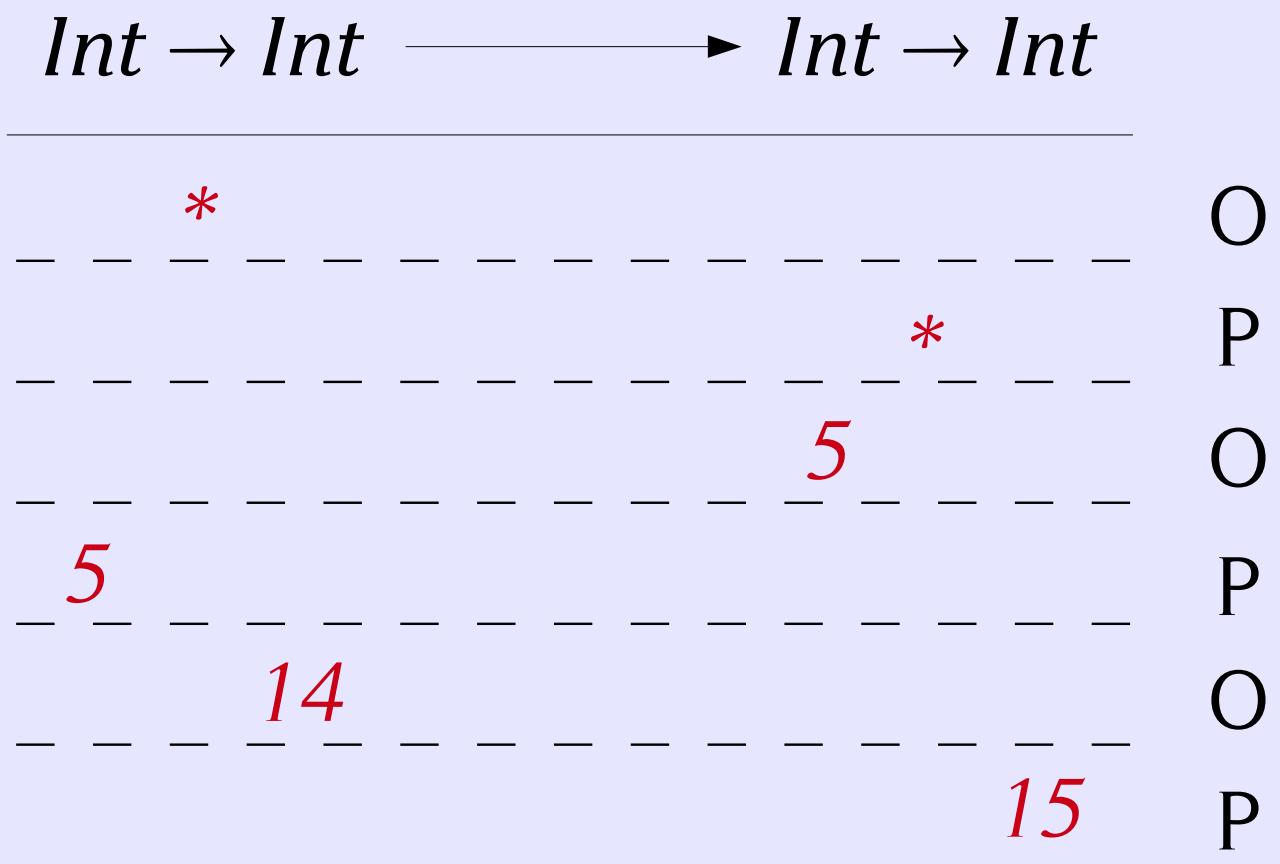

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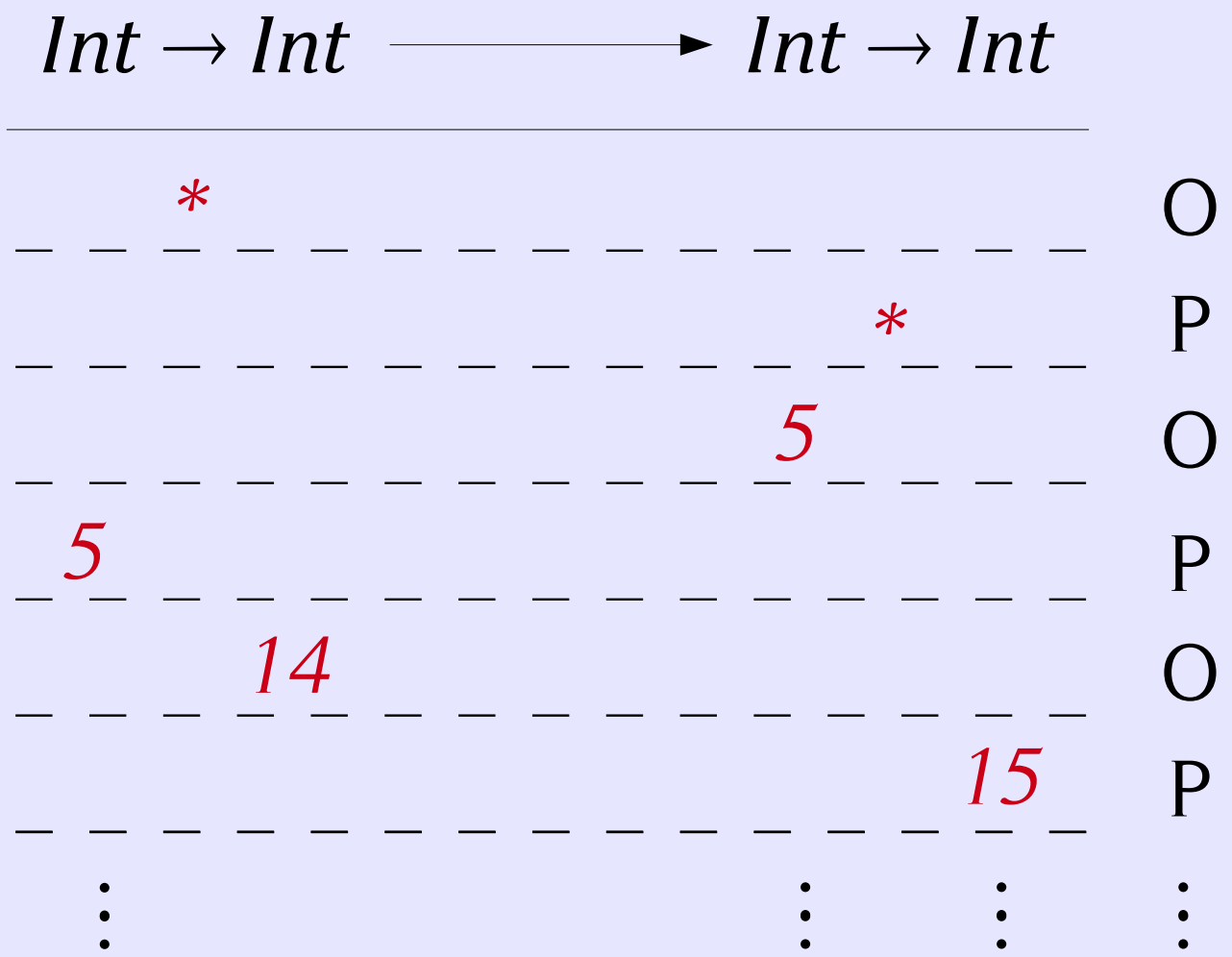

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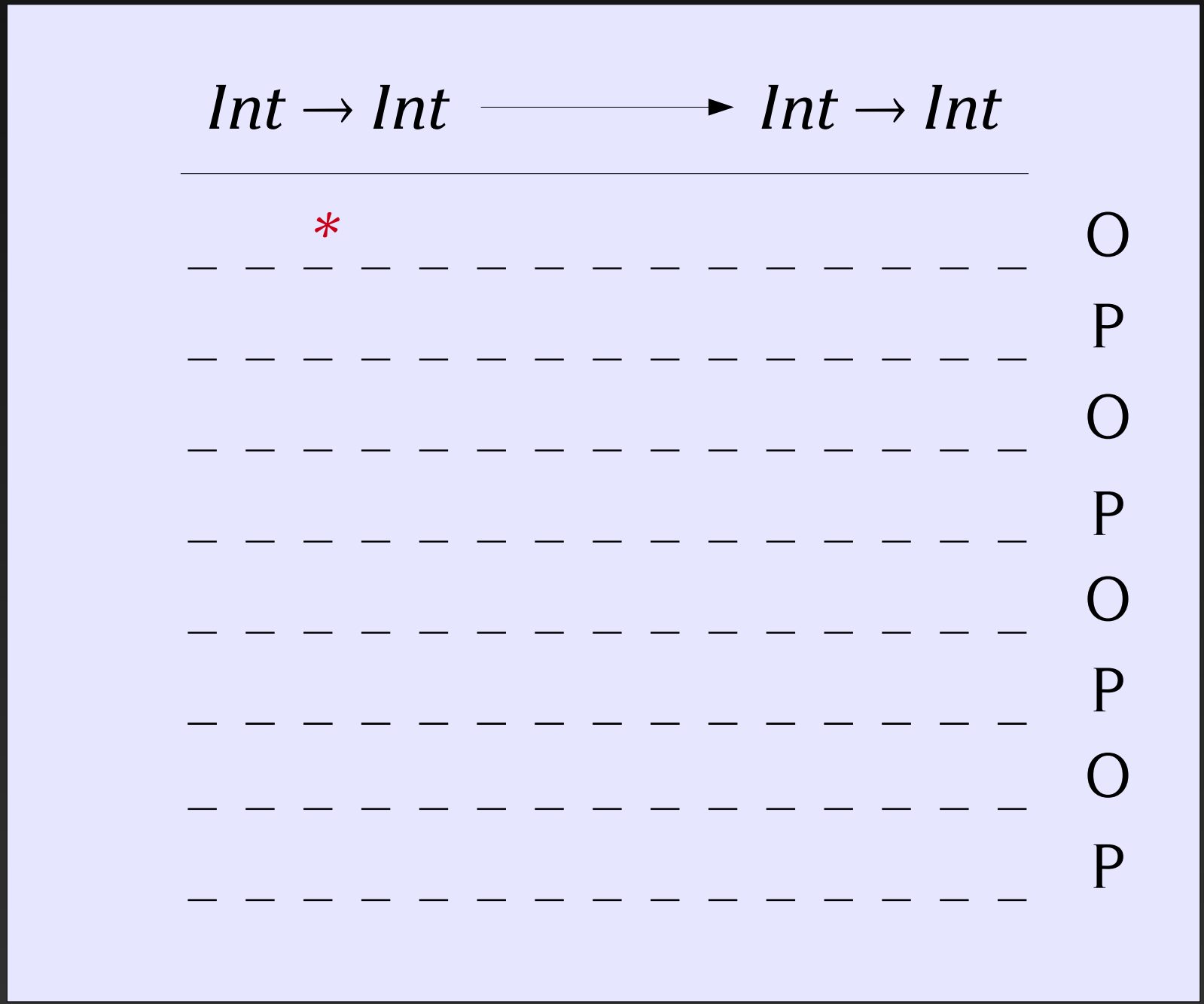

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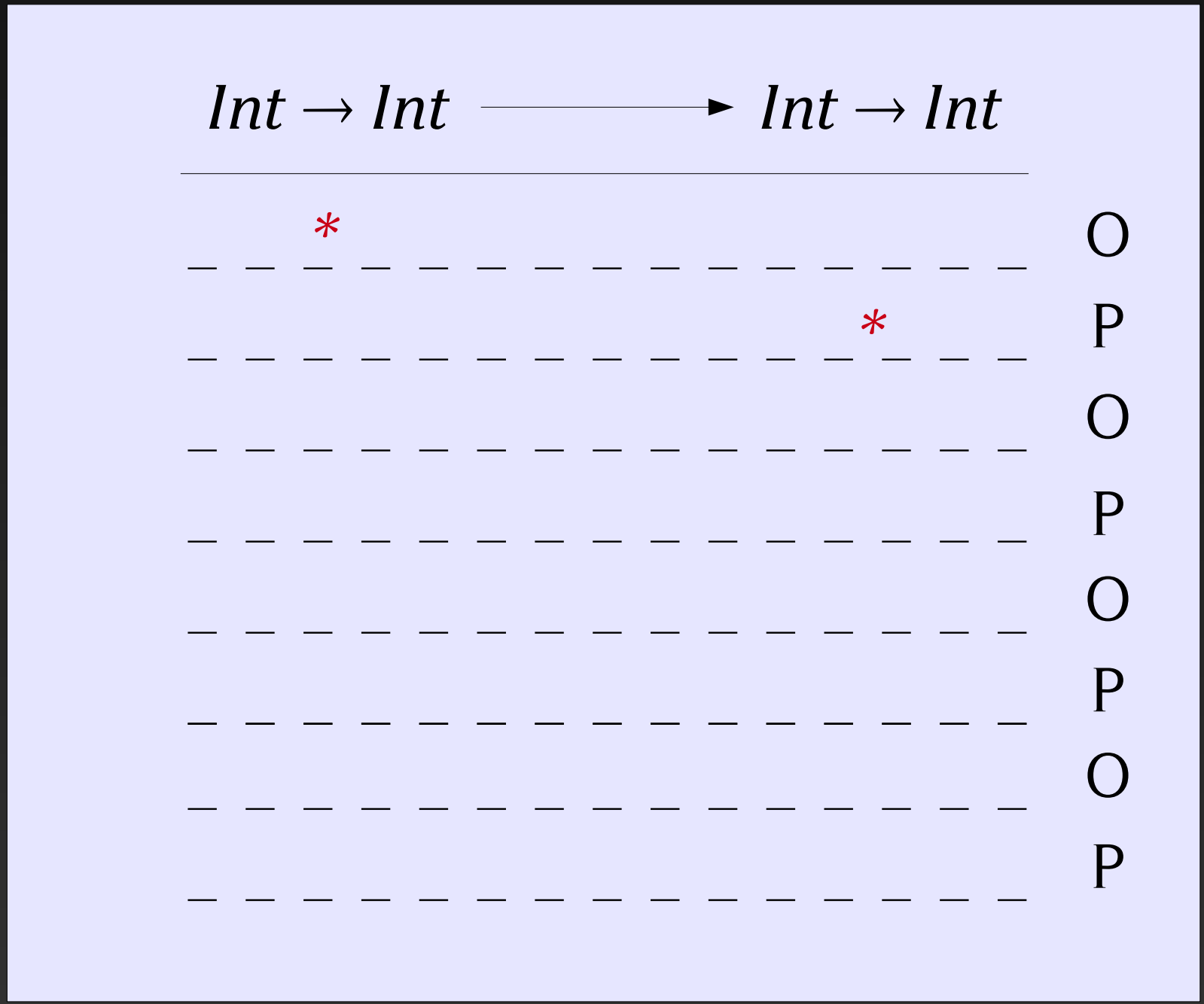

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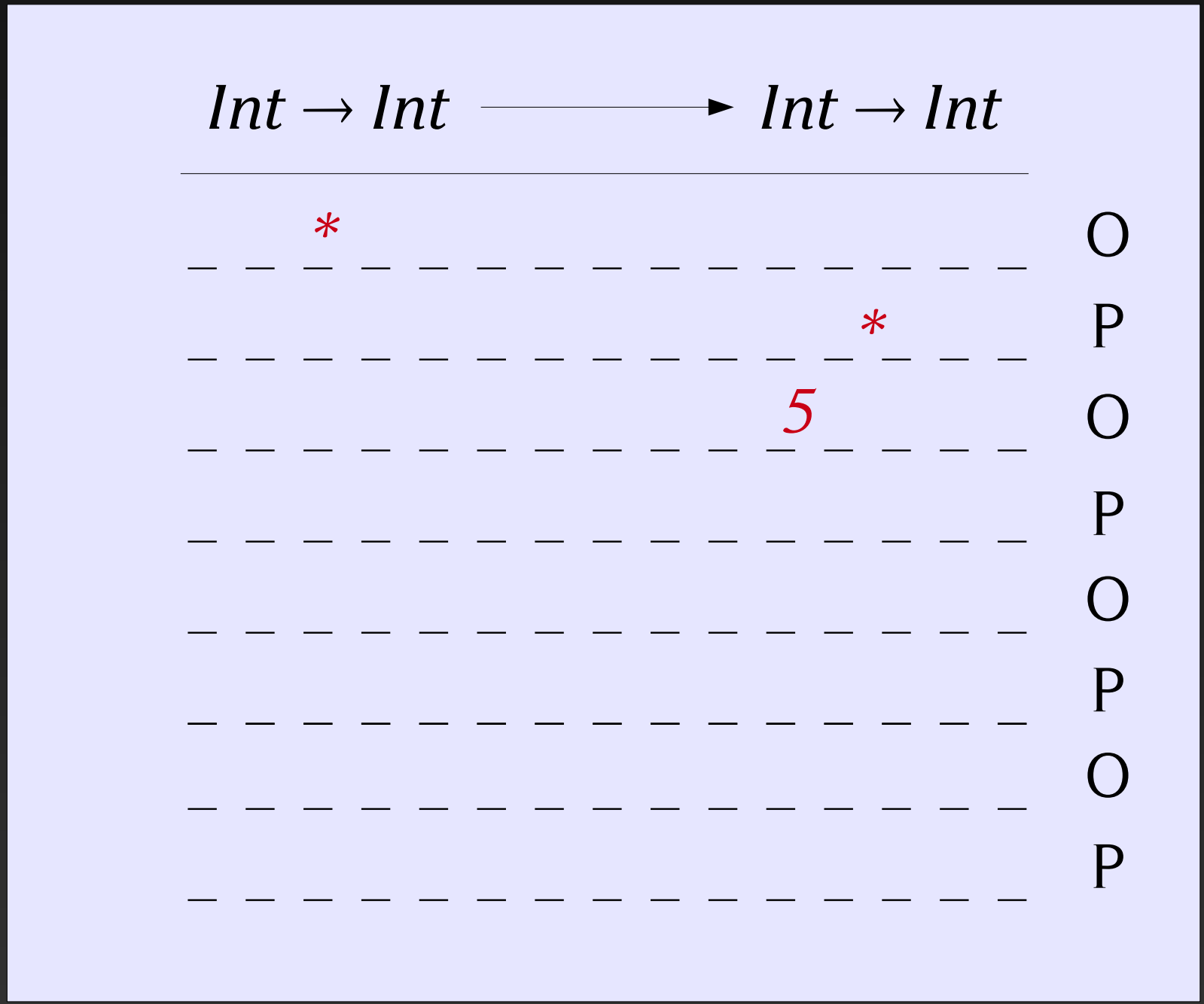

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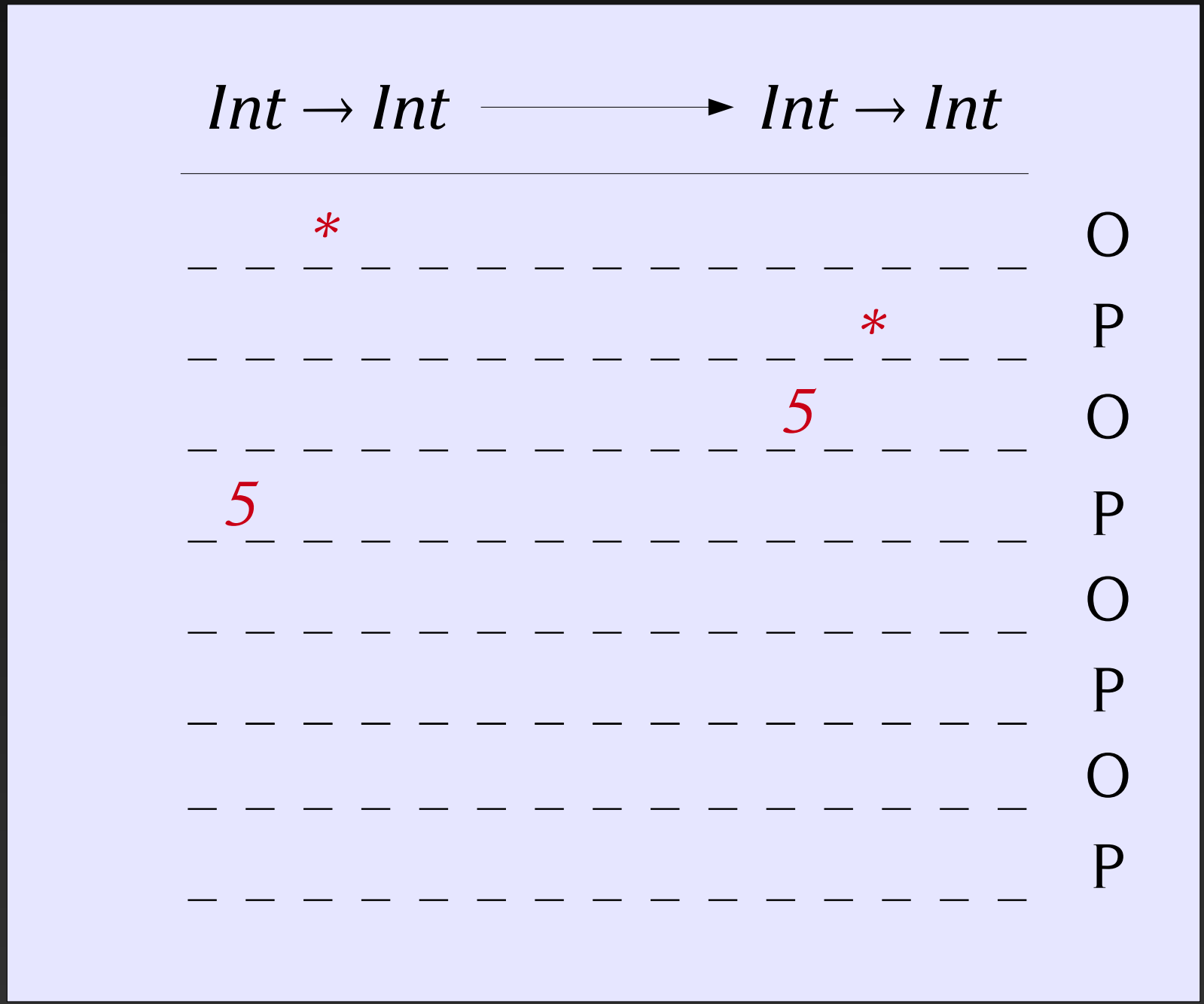

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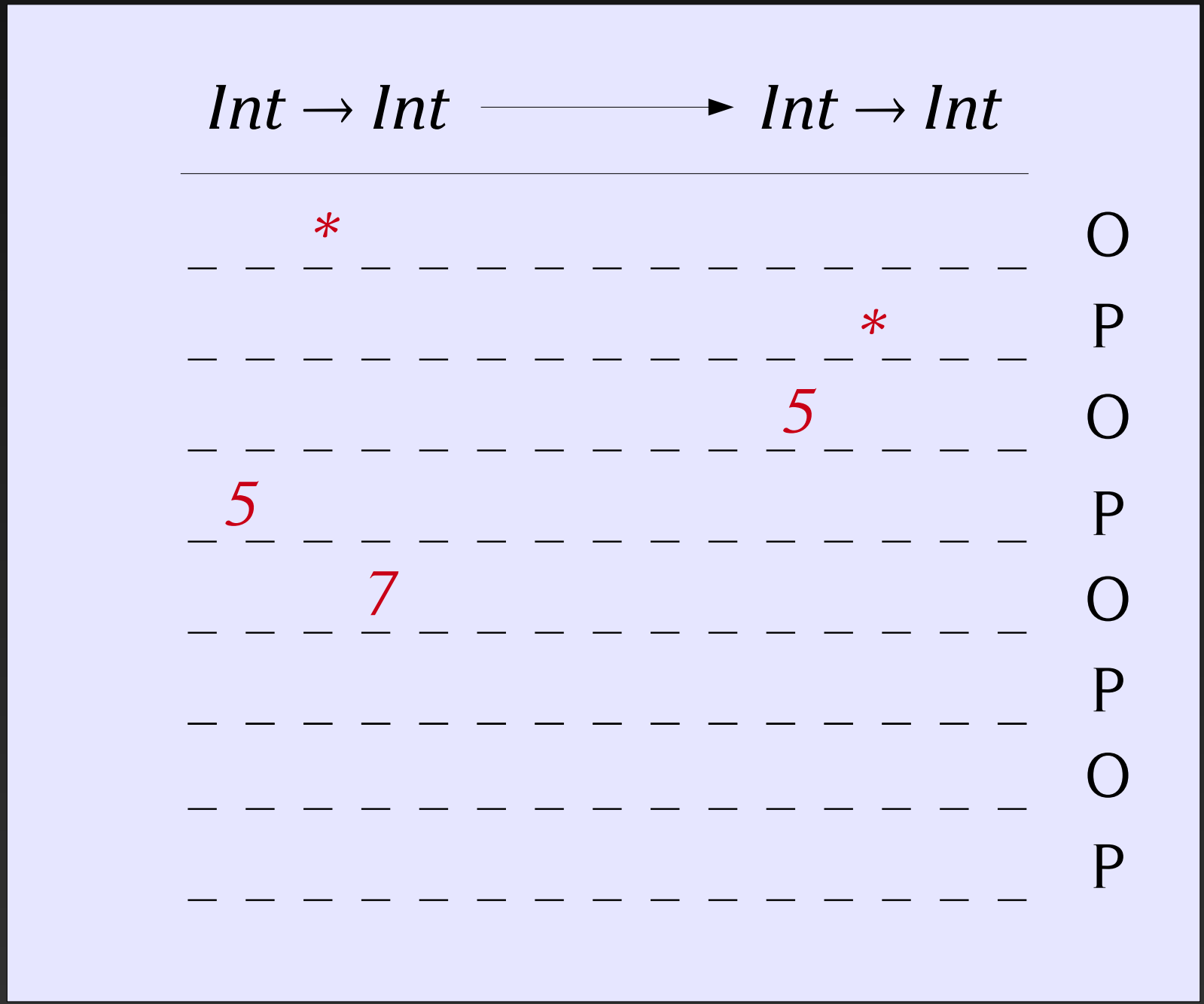

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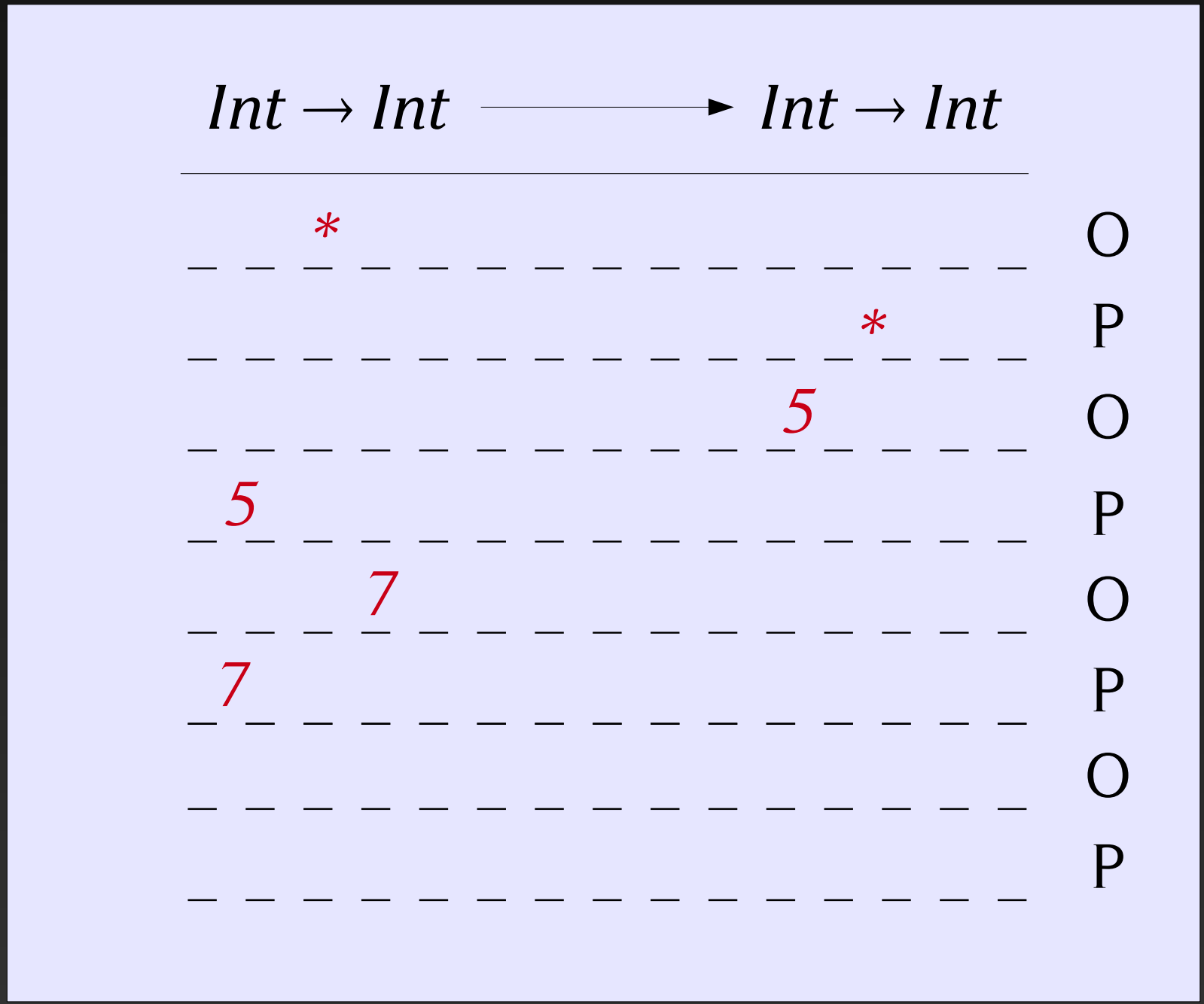

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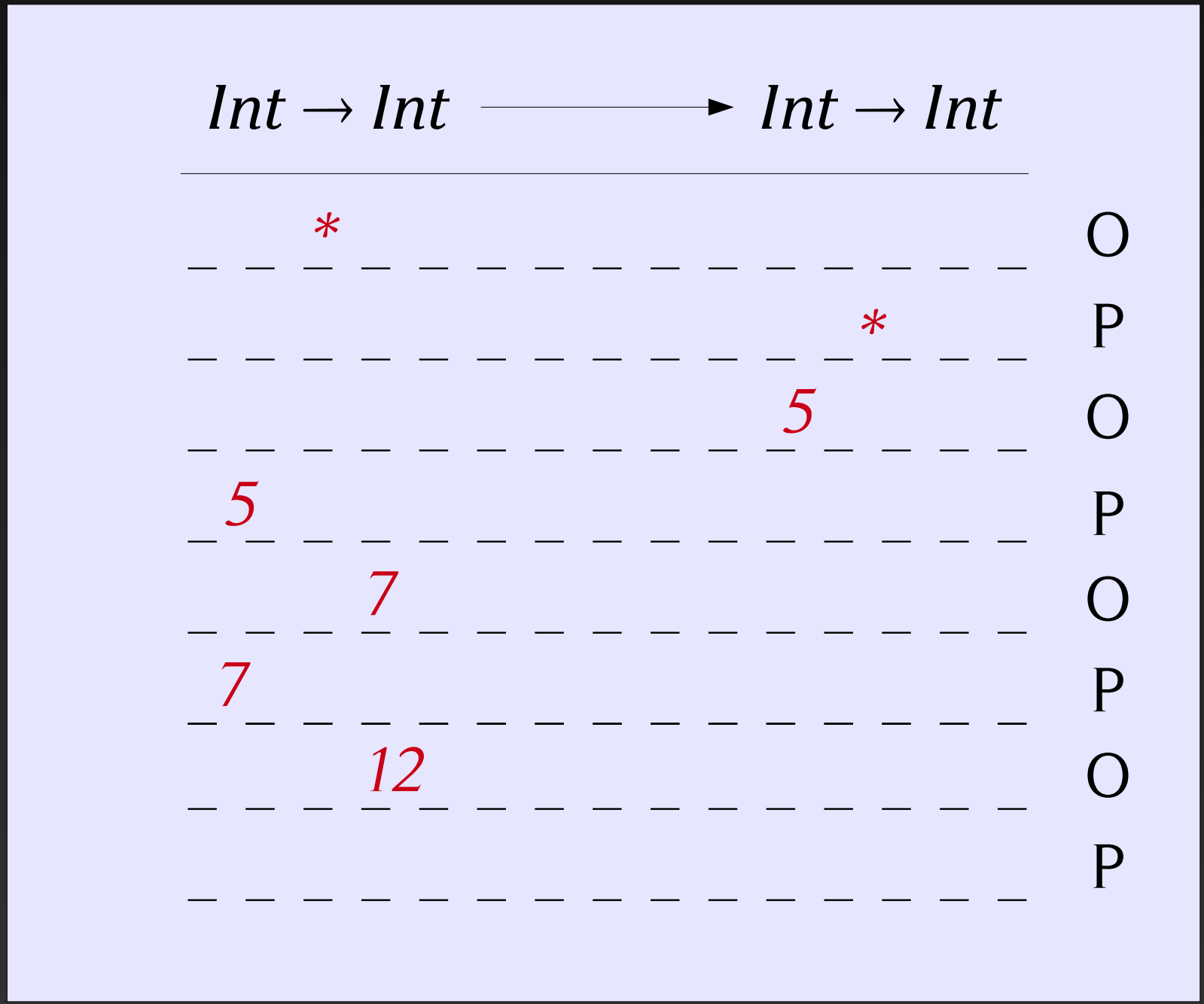

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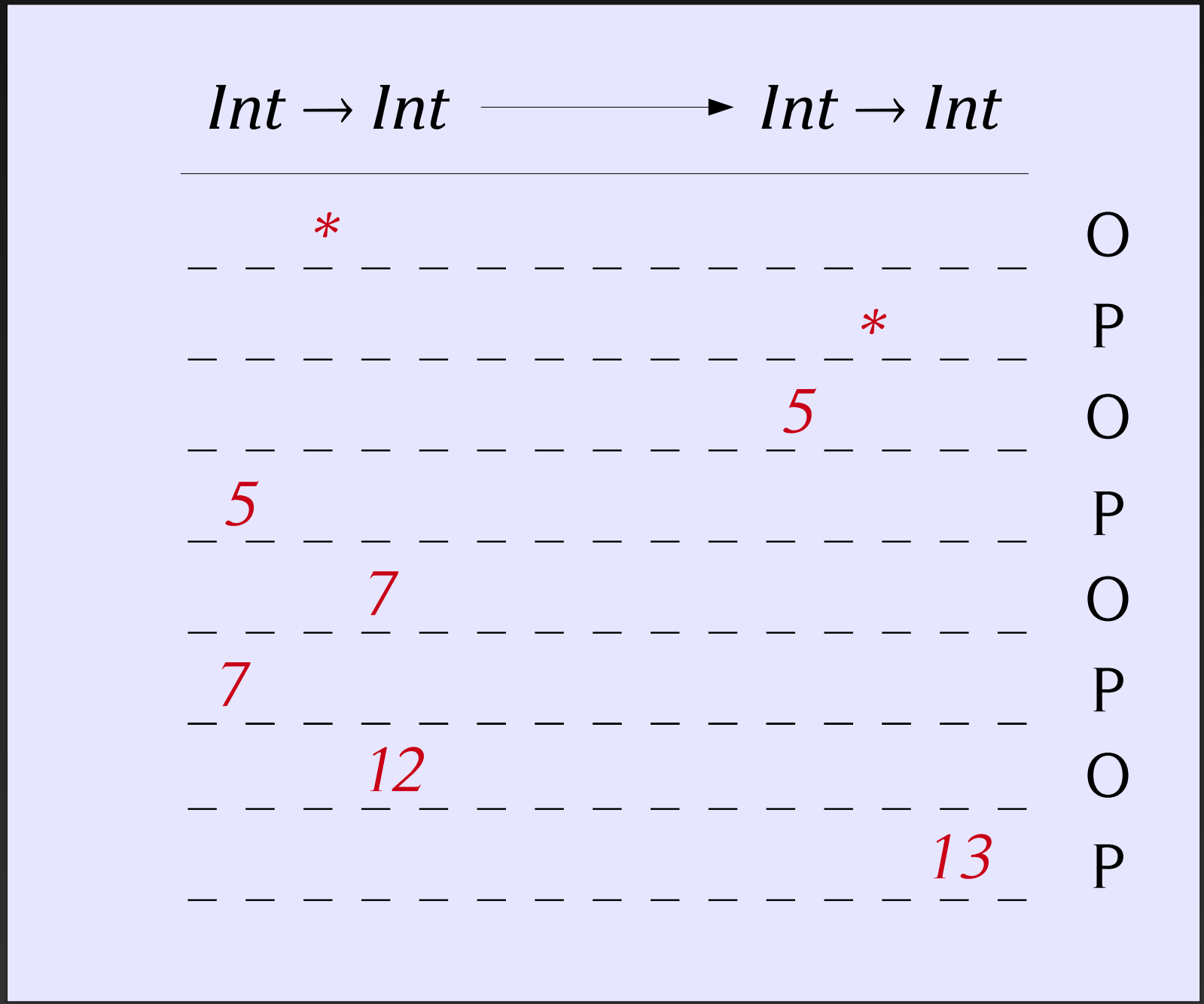

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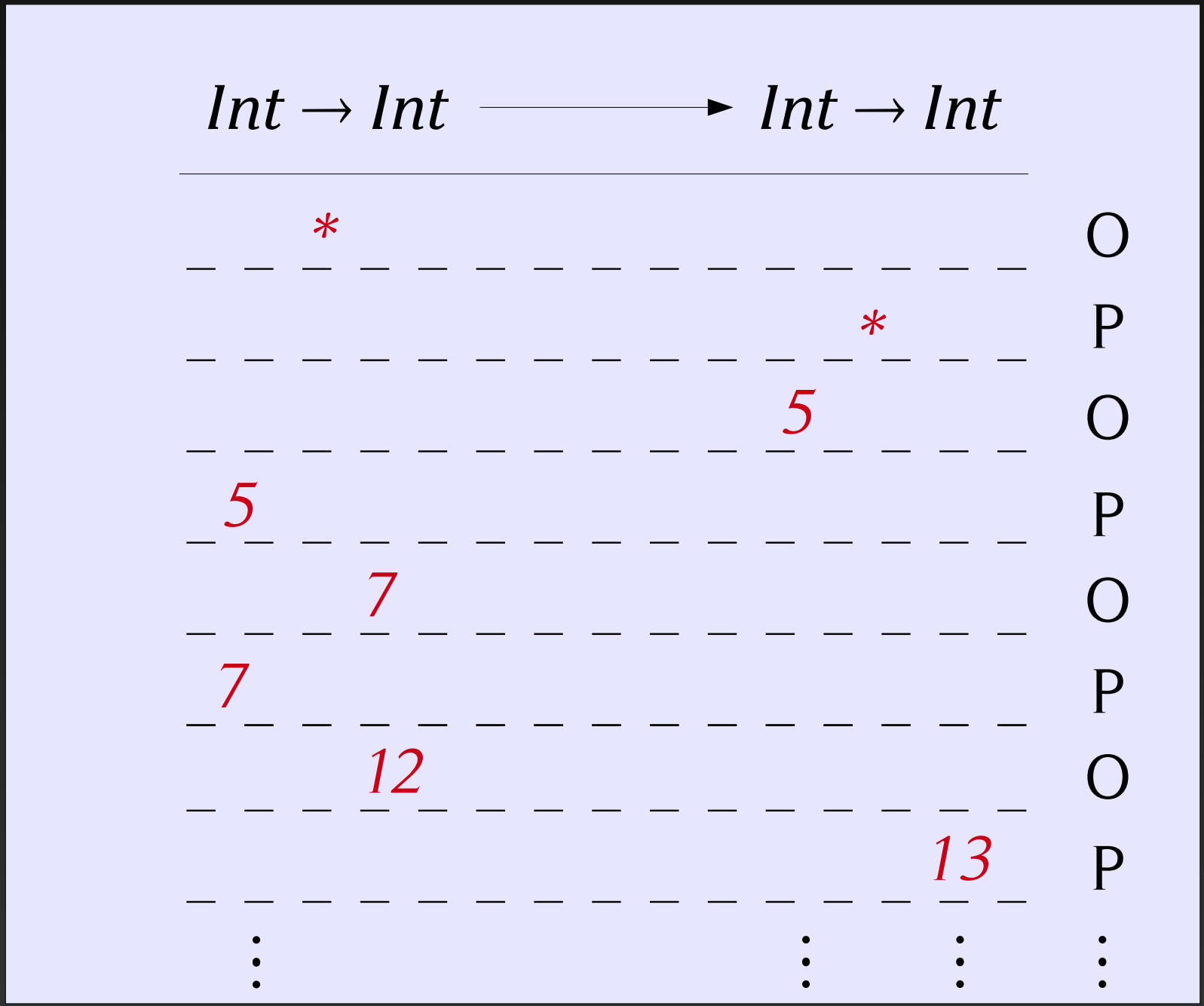

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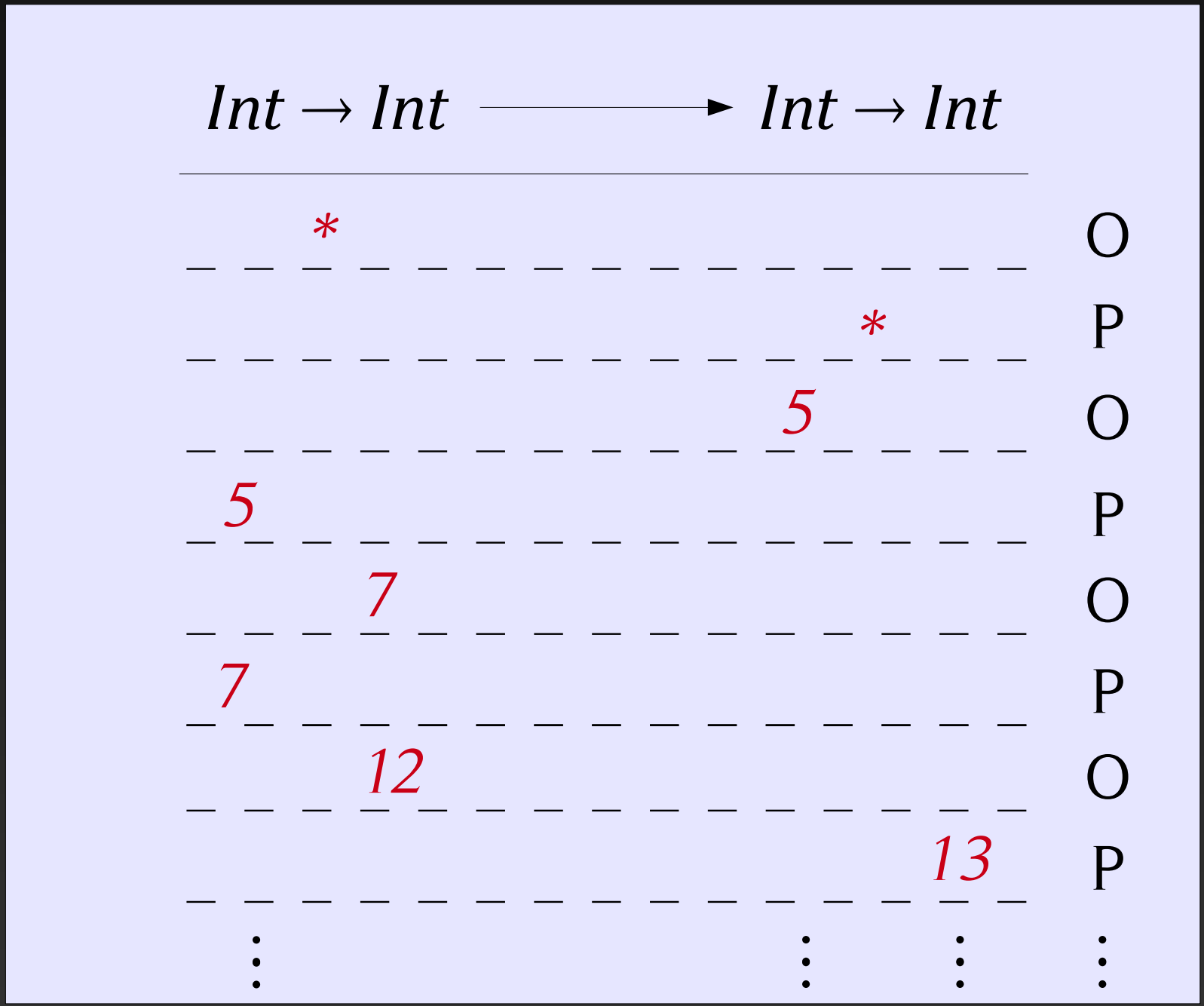

# Game Semantics

- Computation is modelled as a 2-player game between:
  - *Proponent (the program)*
  - *Opponent* (the environment)
- Qualitative games ( ≠ Game Theory)
- Programs = *strategies* for Proponent

# Composition

$\vdash \lambda x.x+1 : \mathtt{int} \longrightarrow \mathtt{int}$

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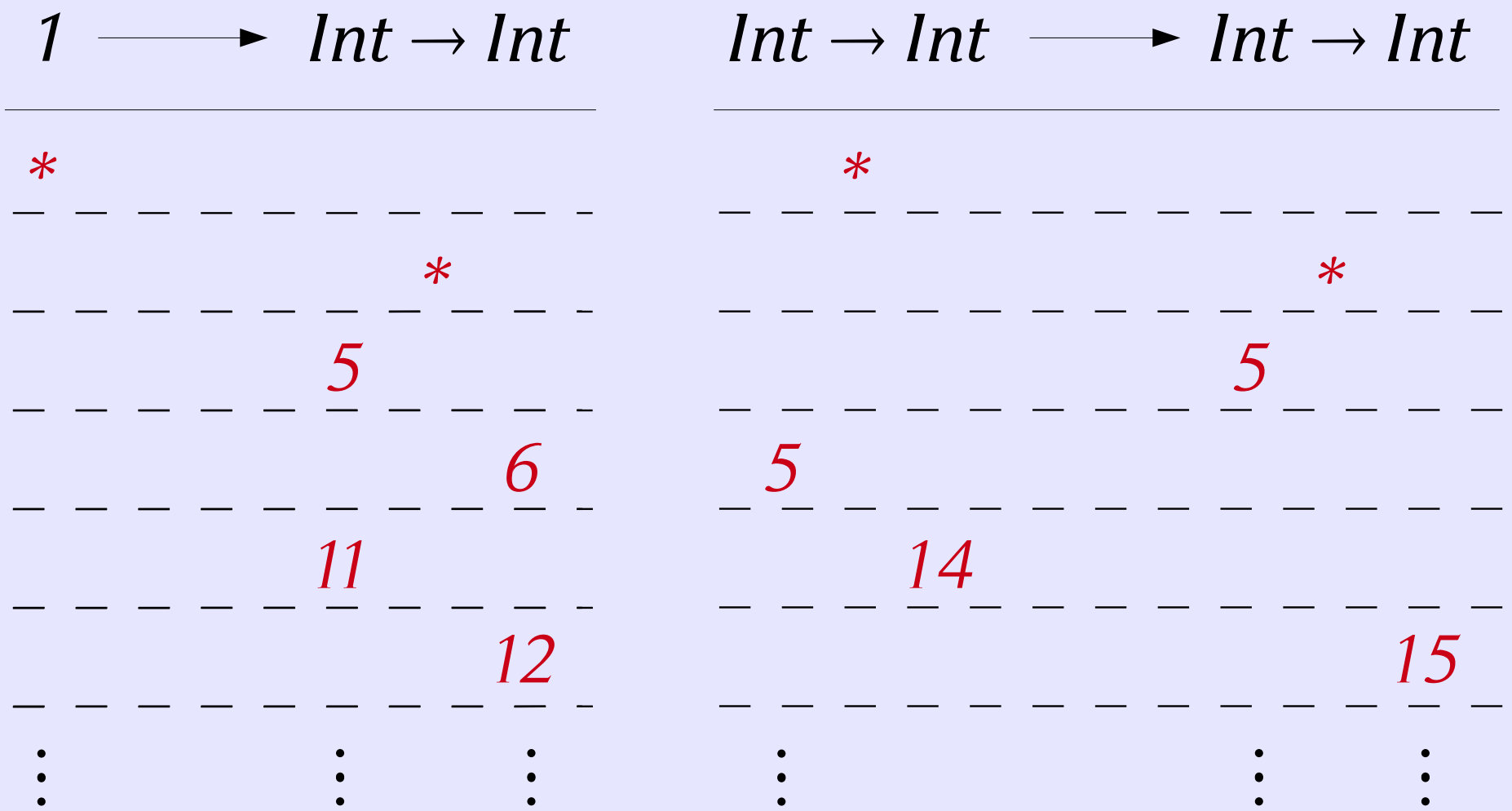

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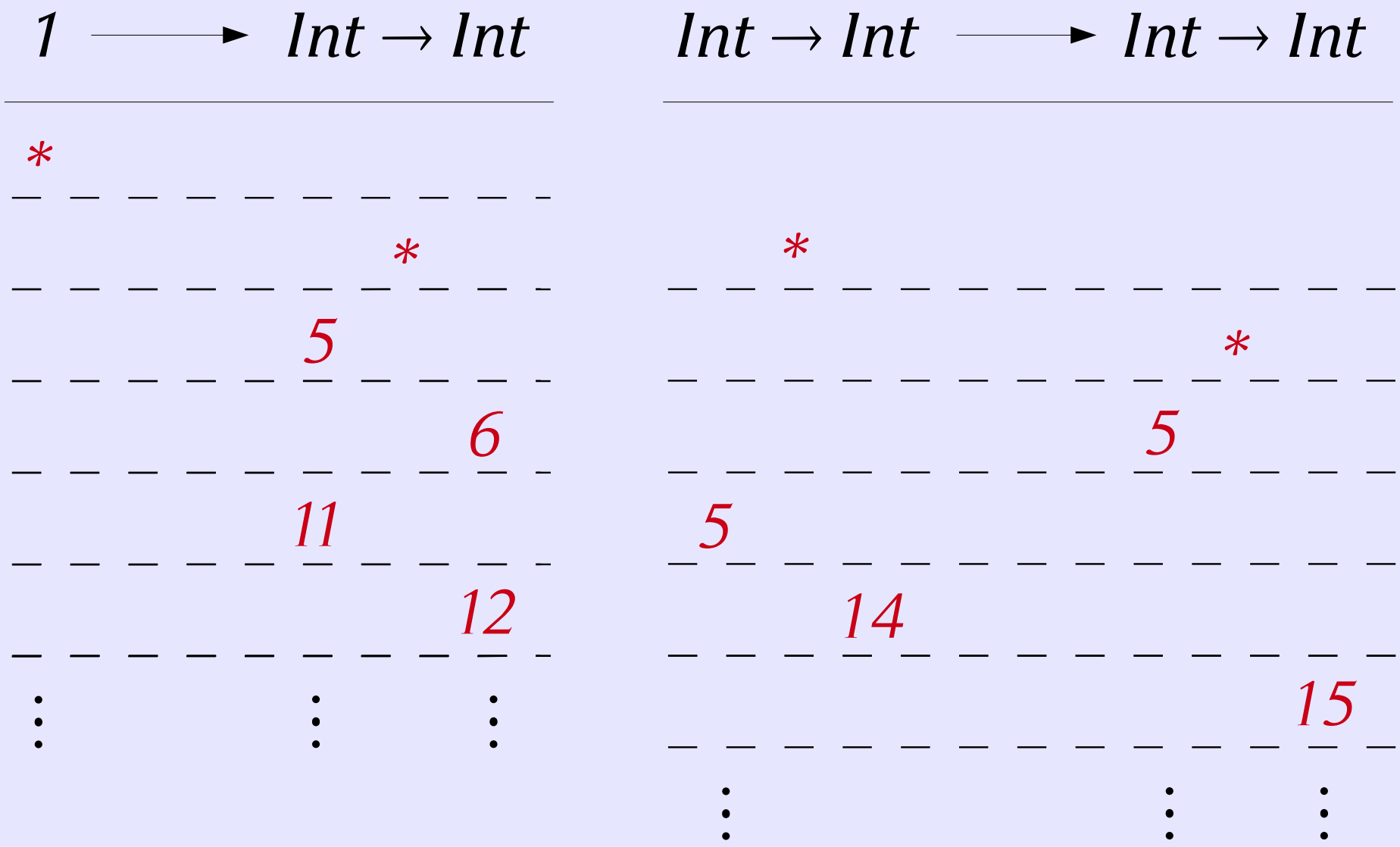

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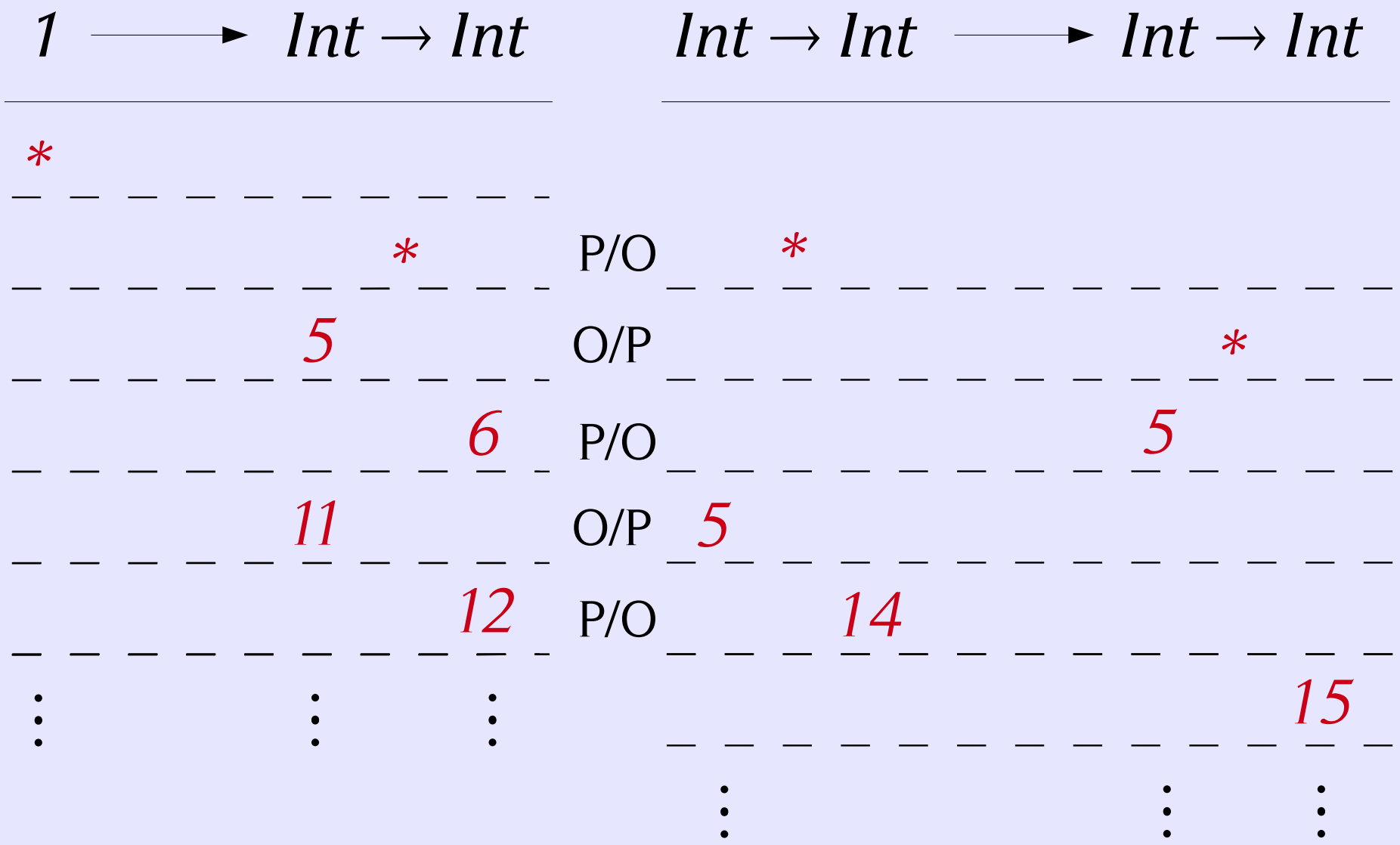

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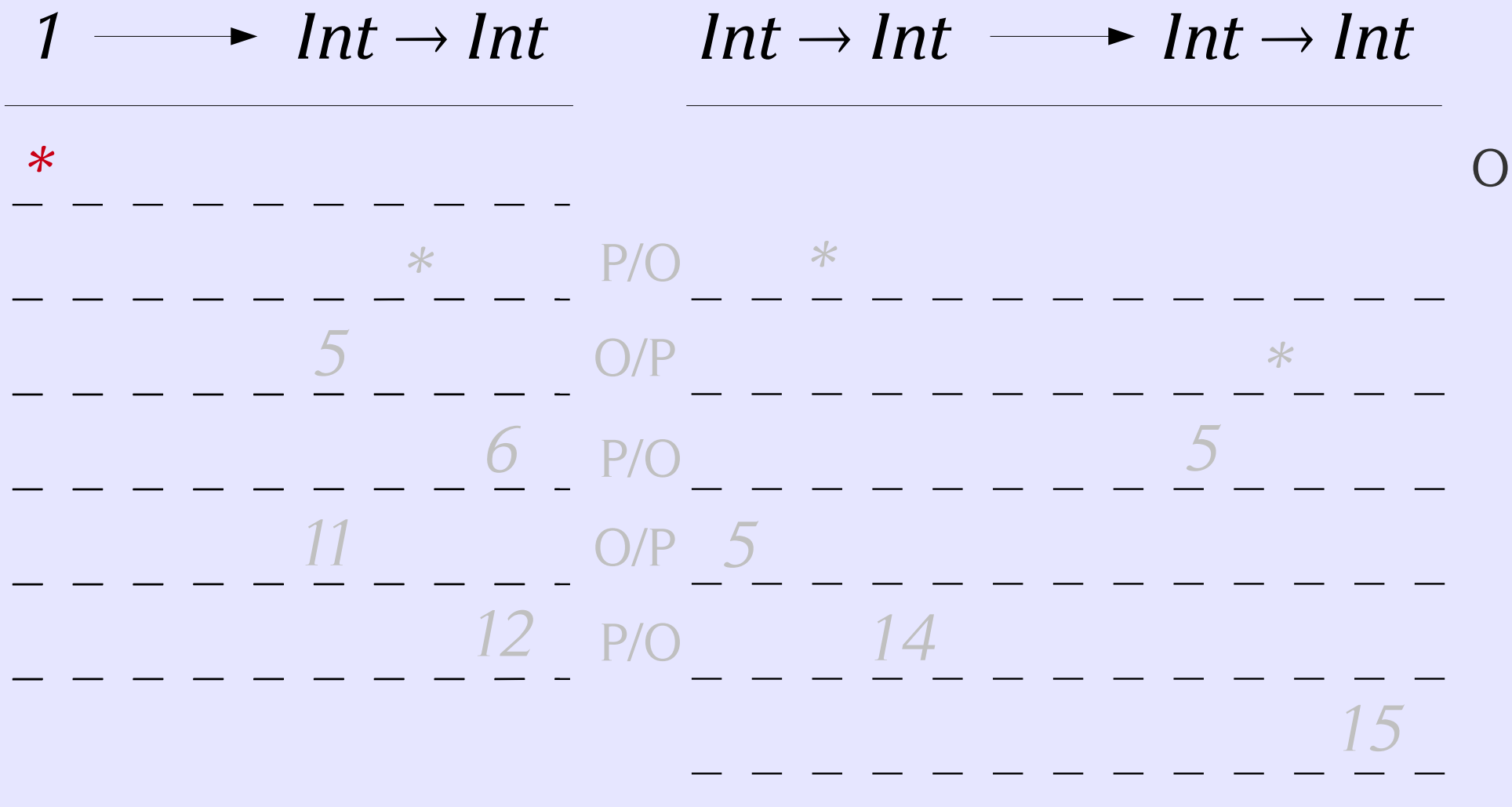

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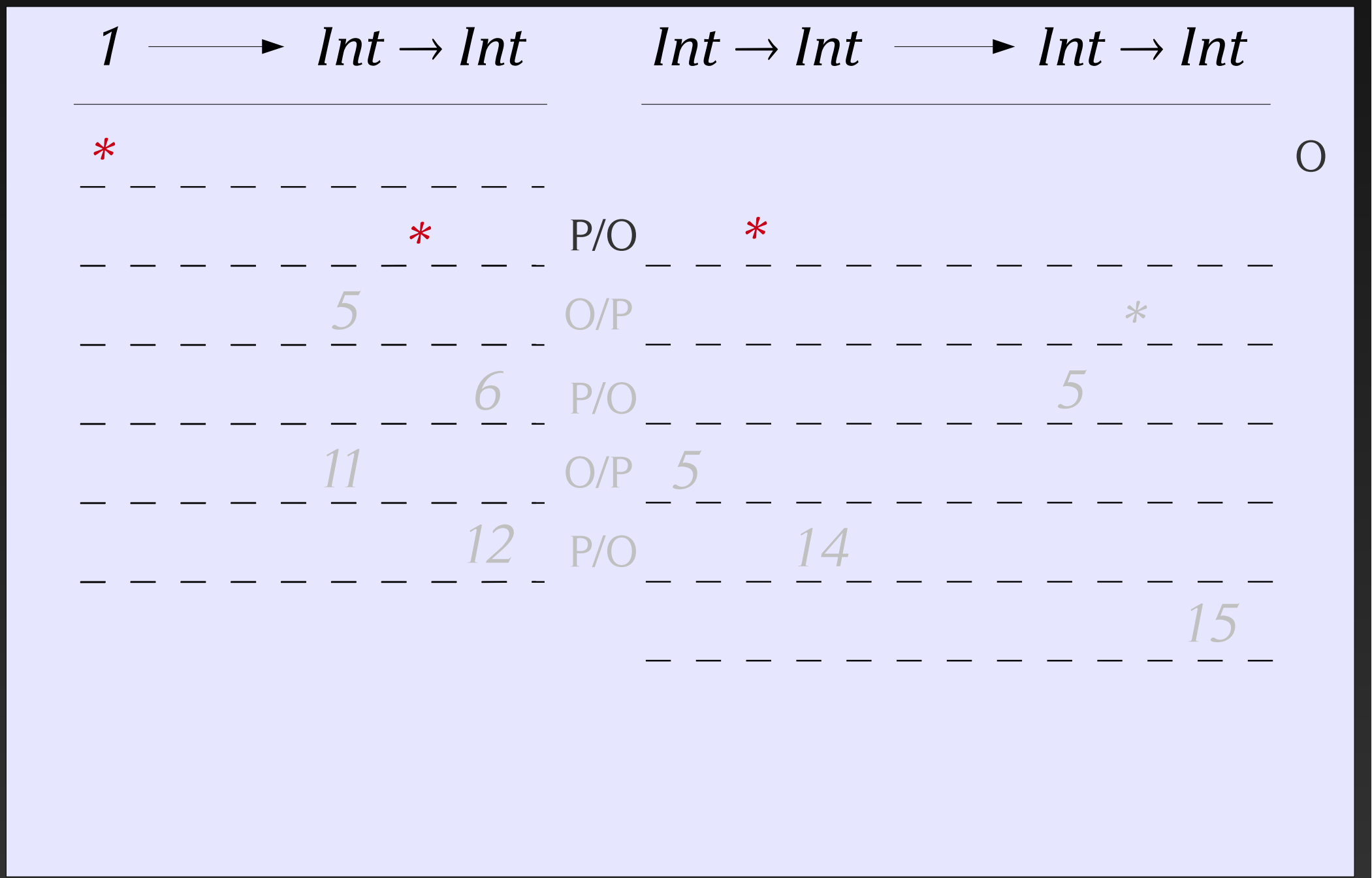

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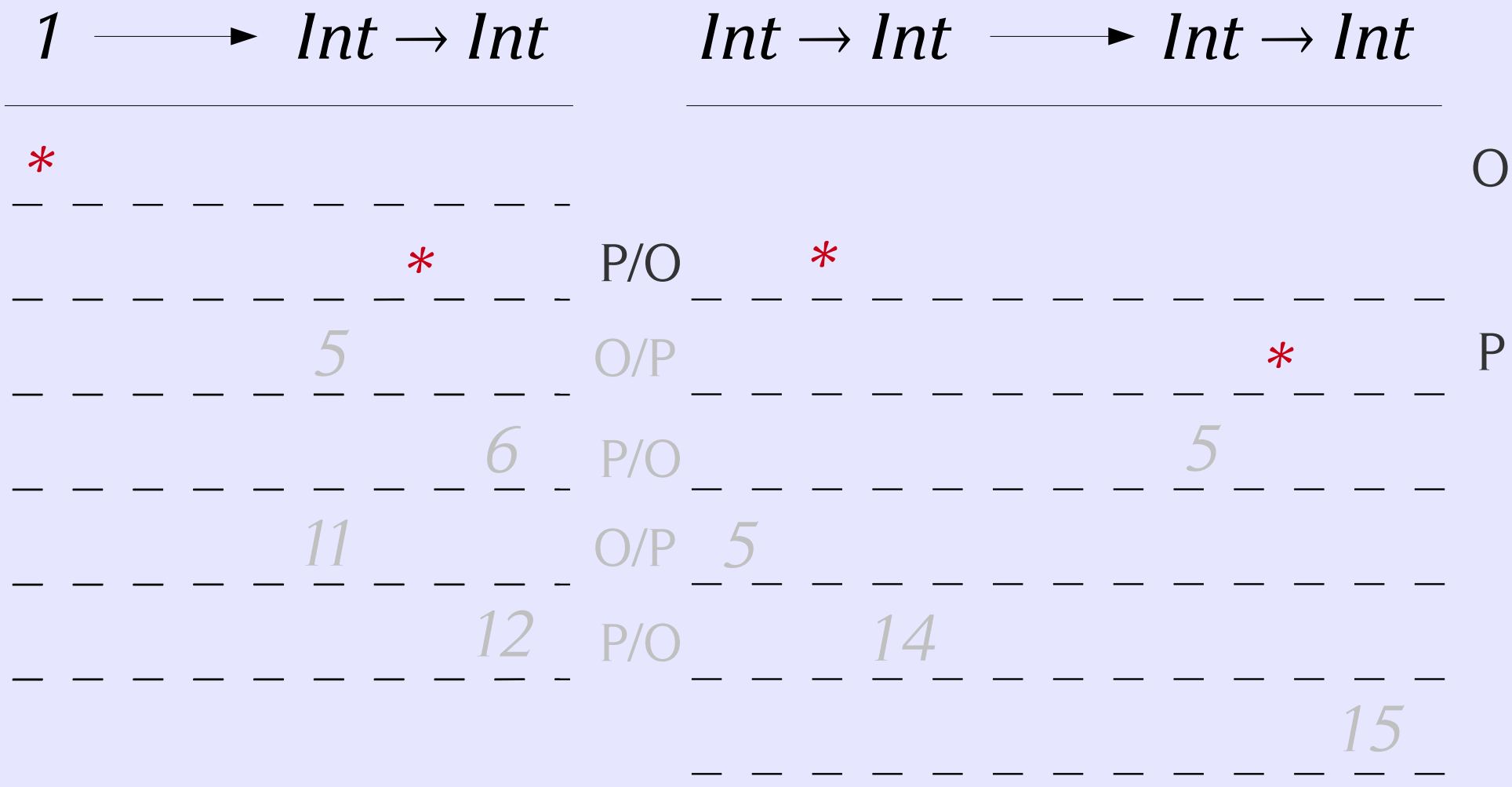

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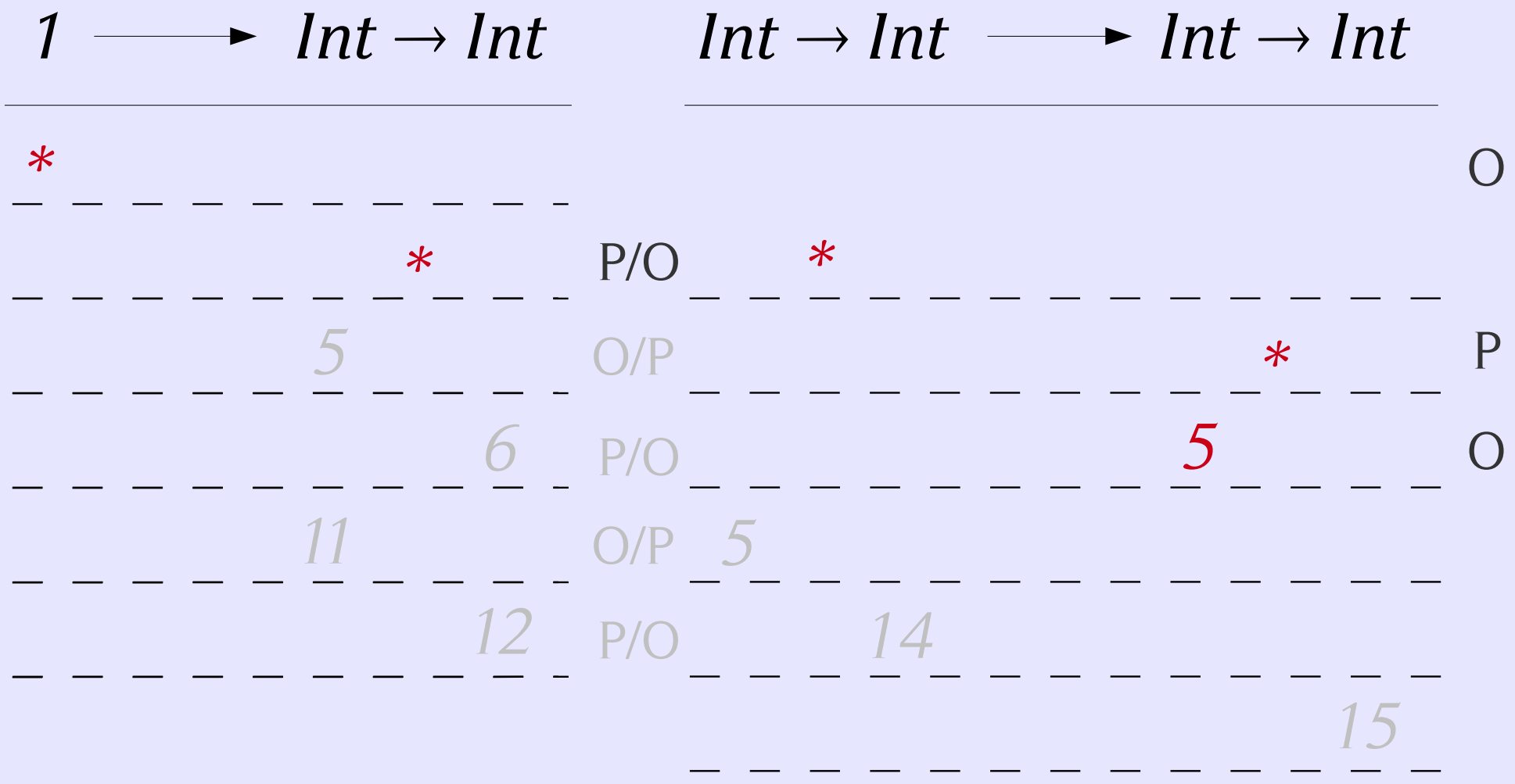

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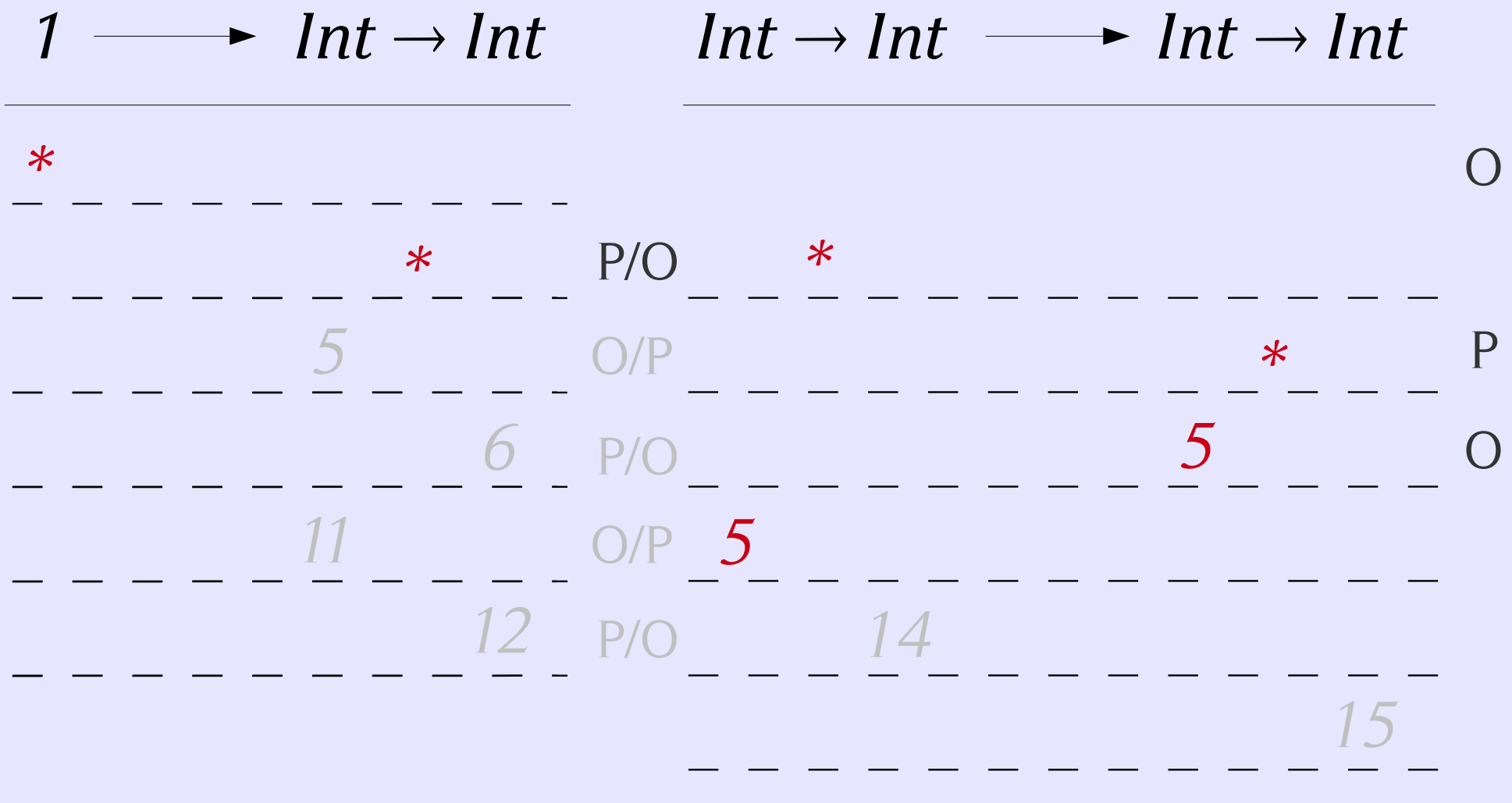

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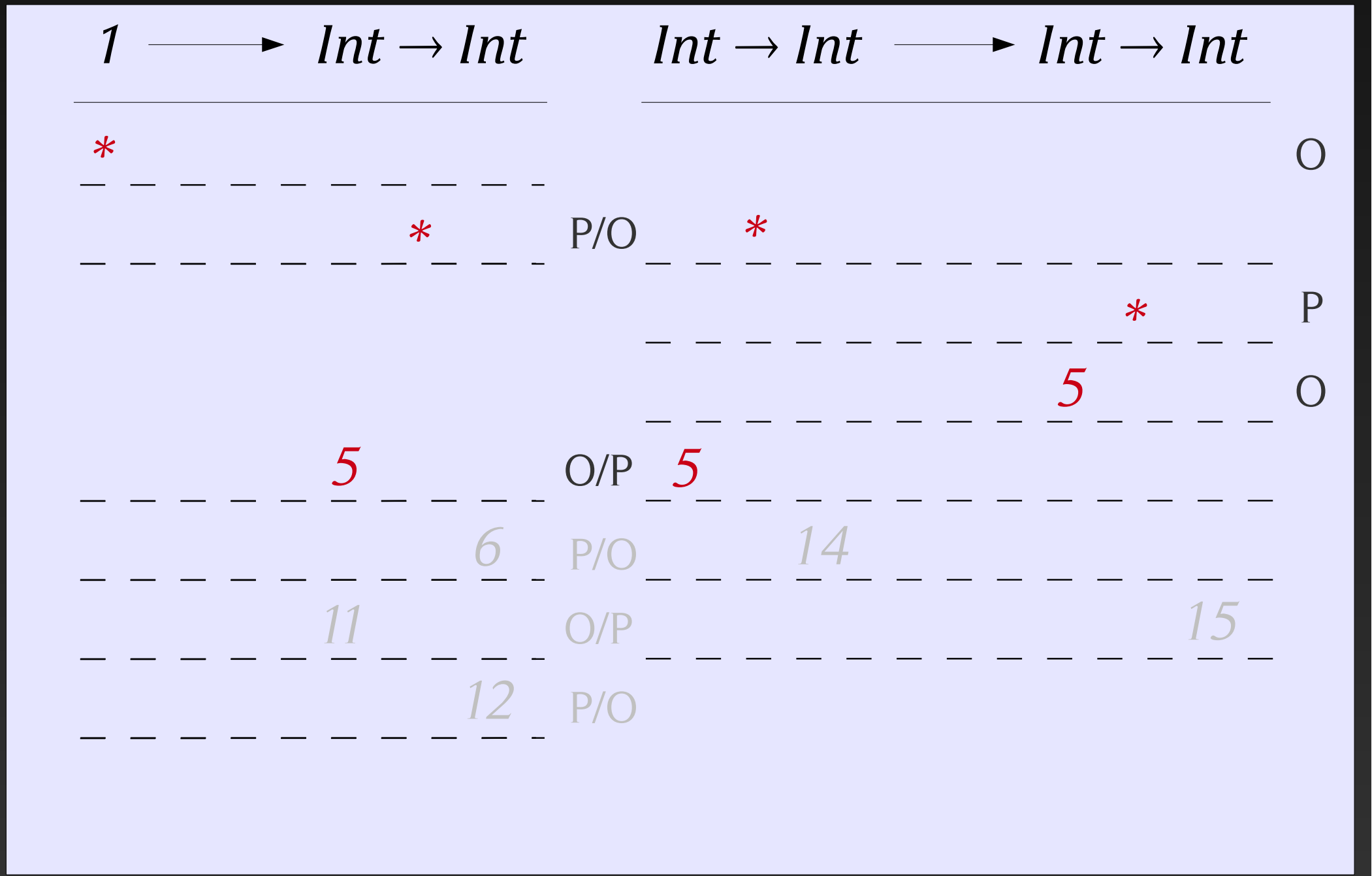

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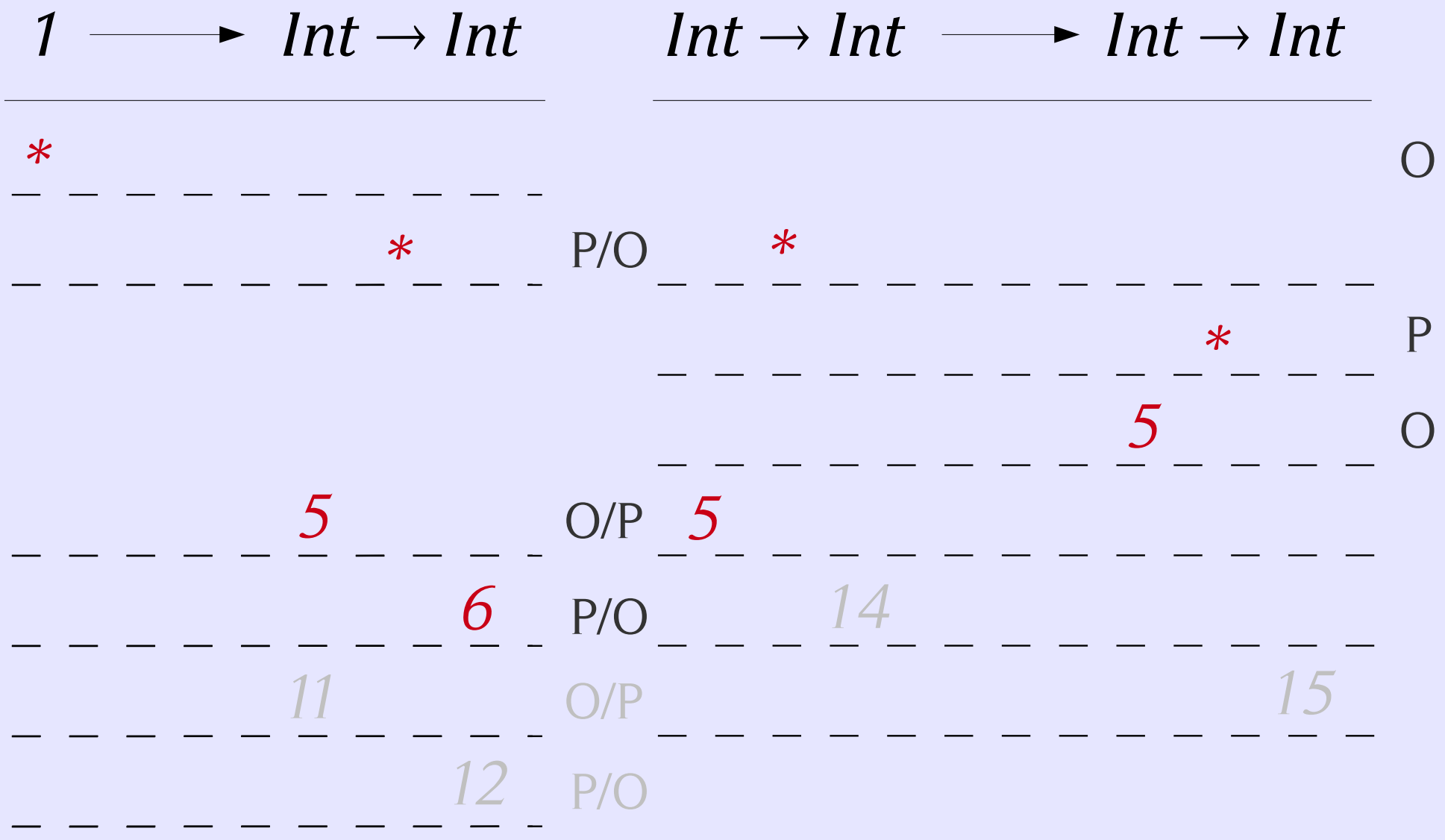

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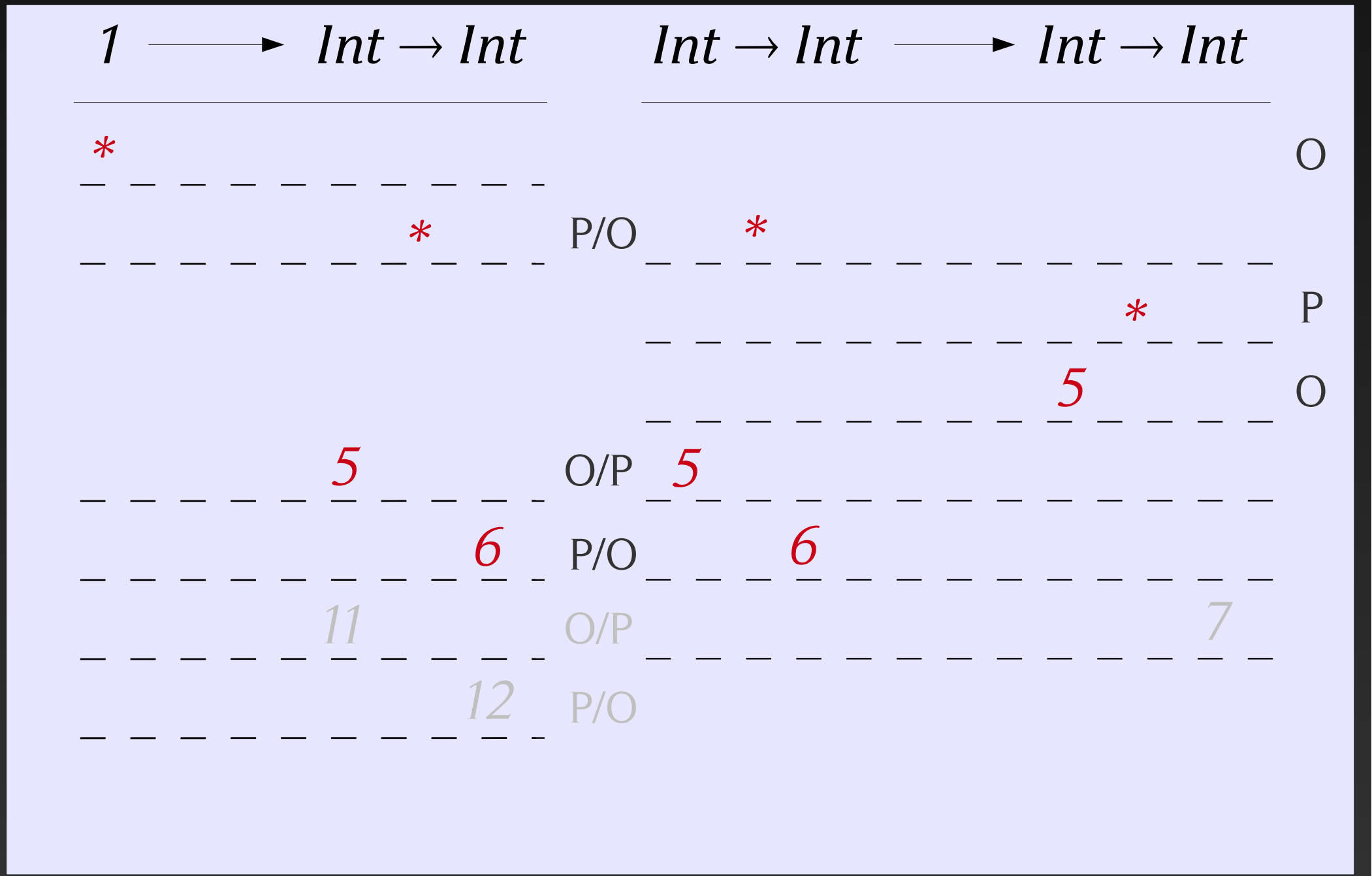

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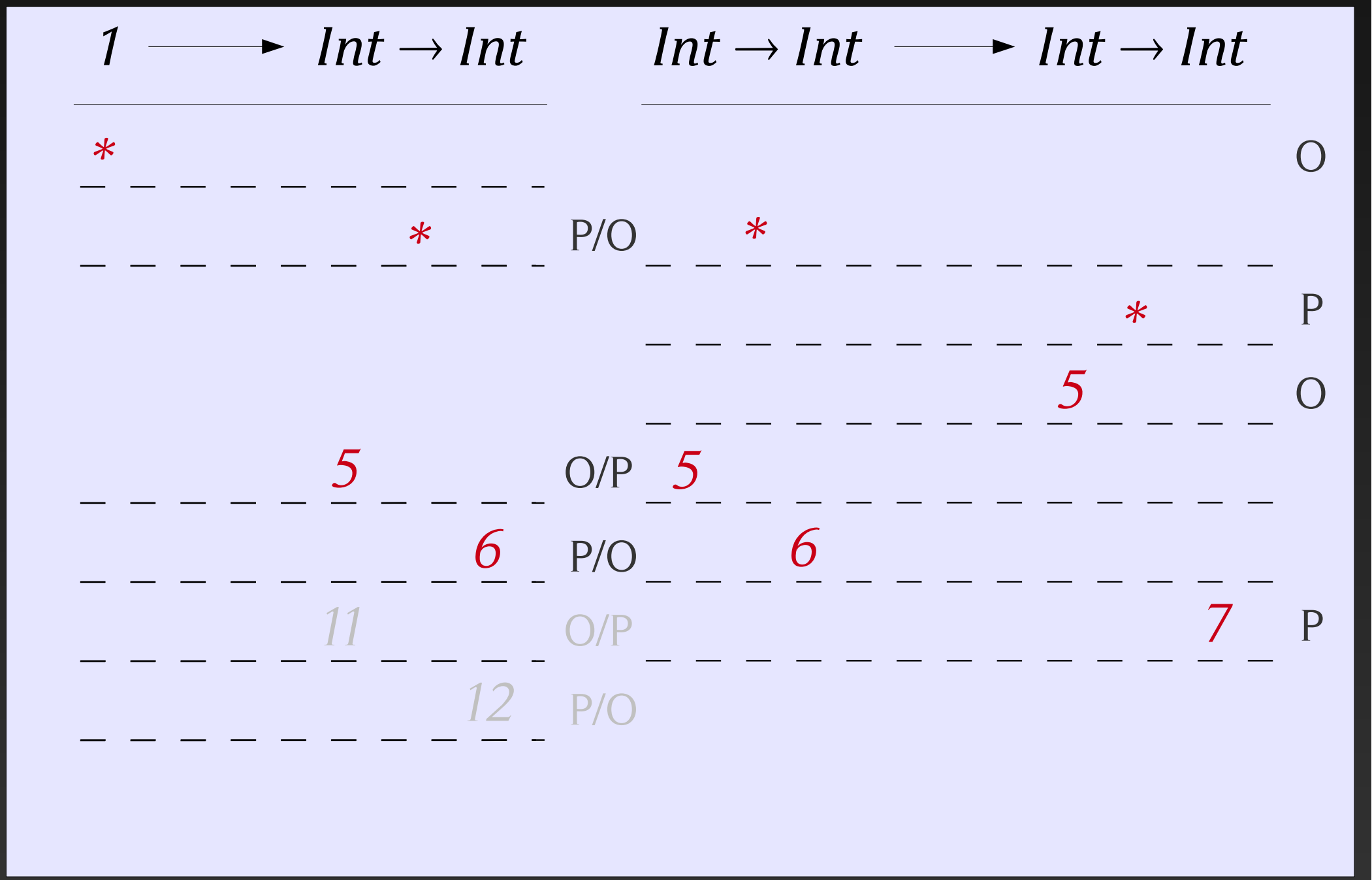

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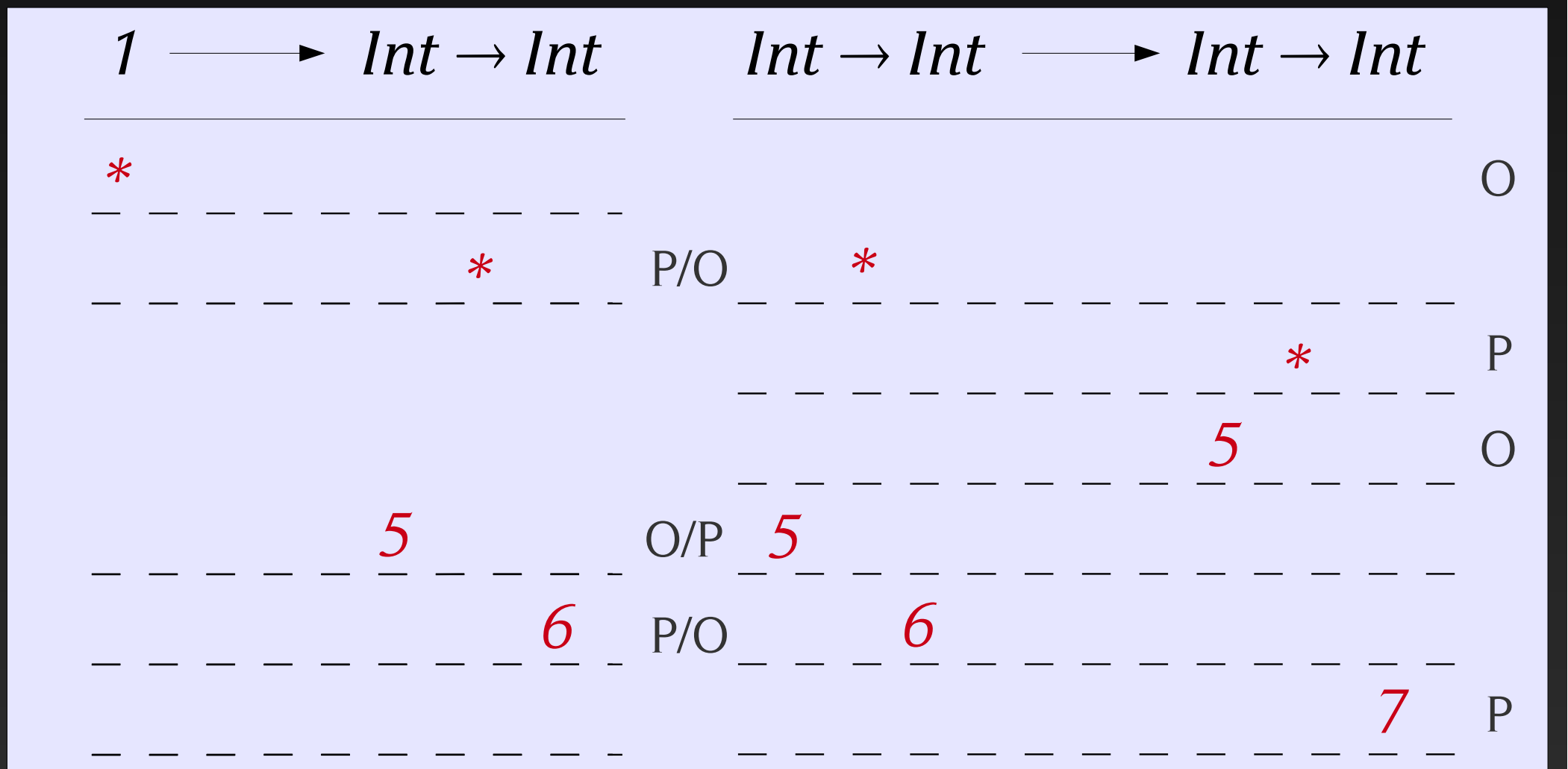

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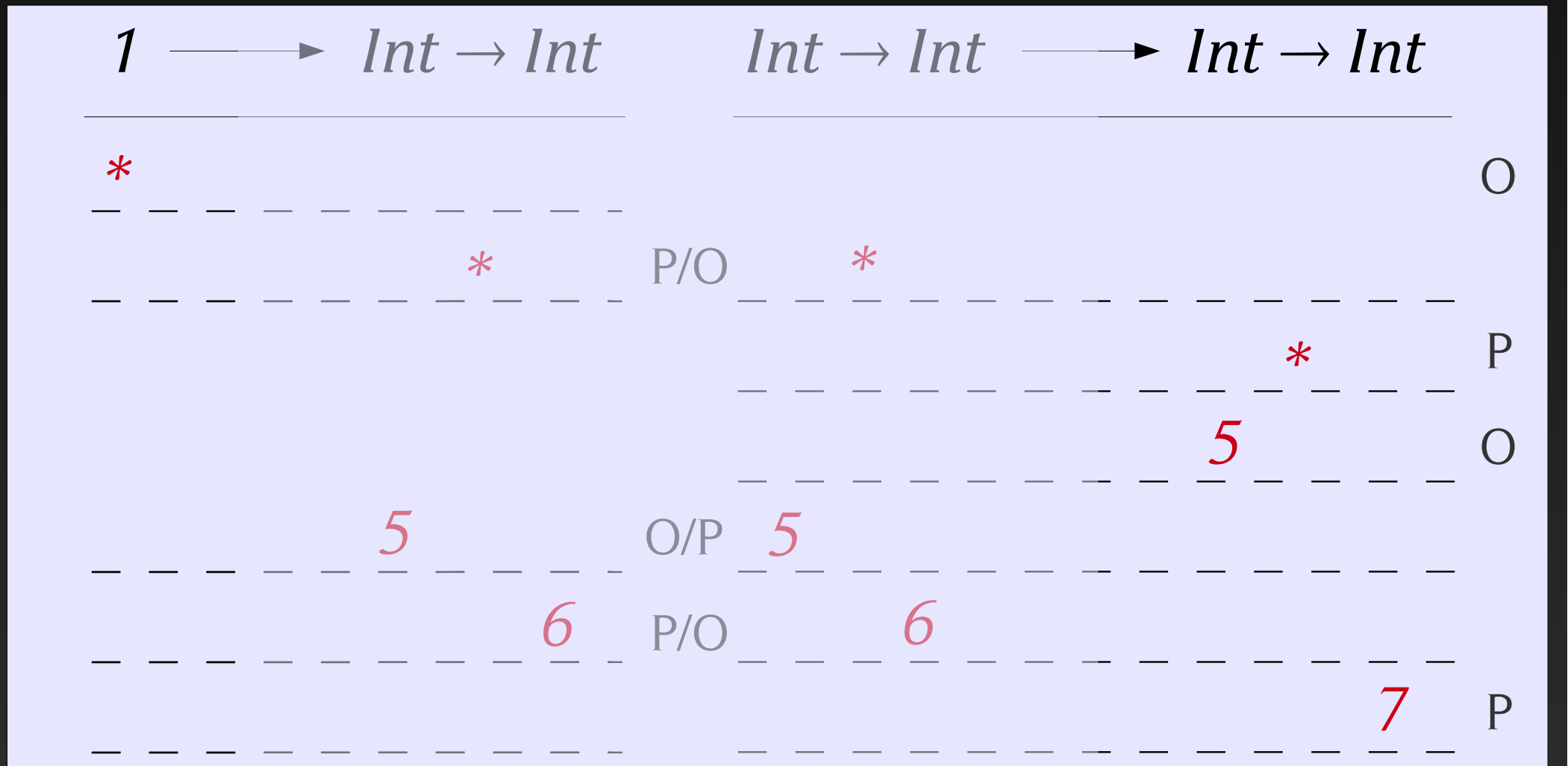

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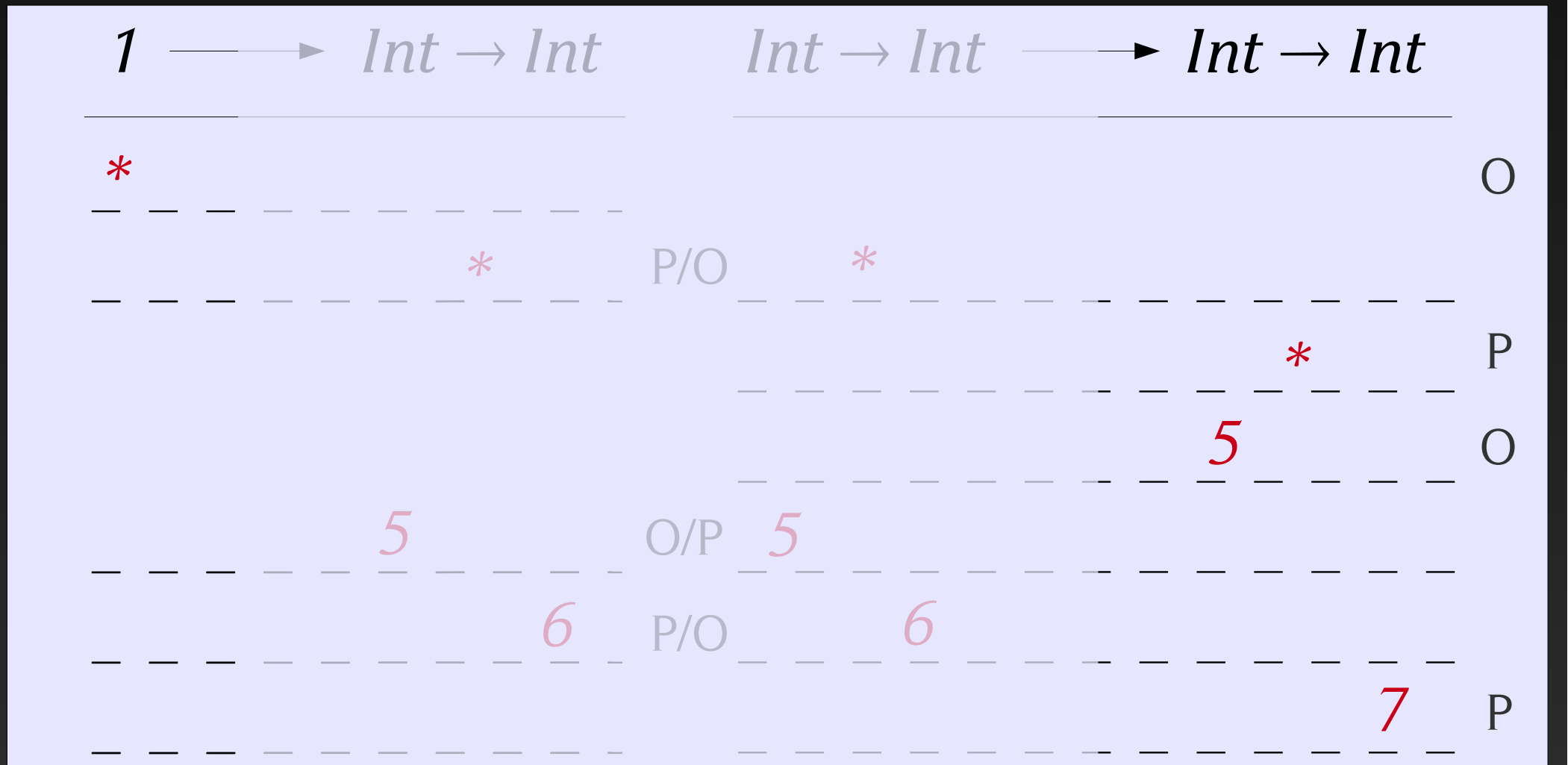

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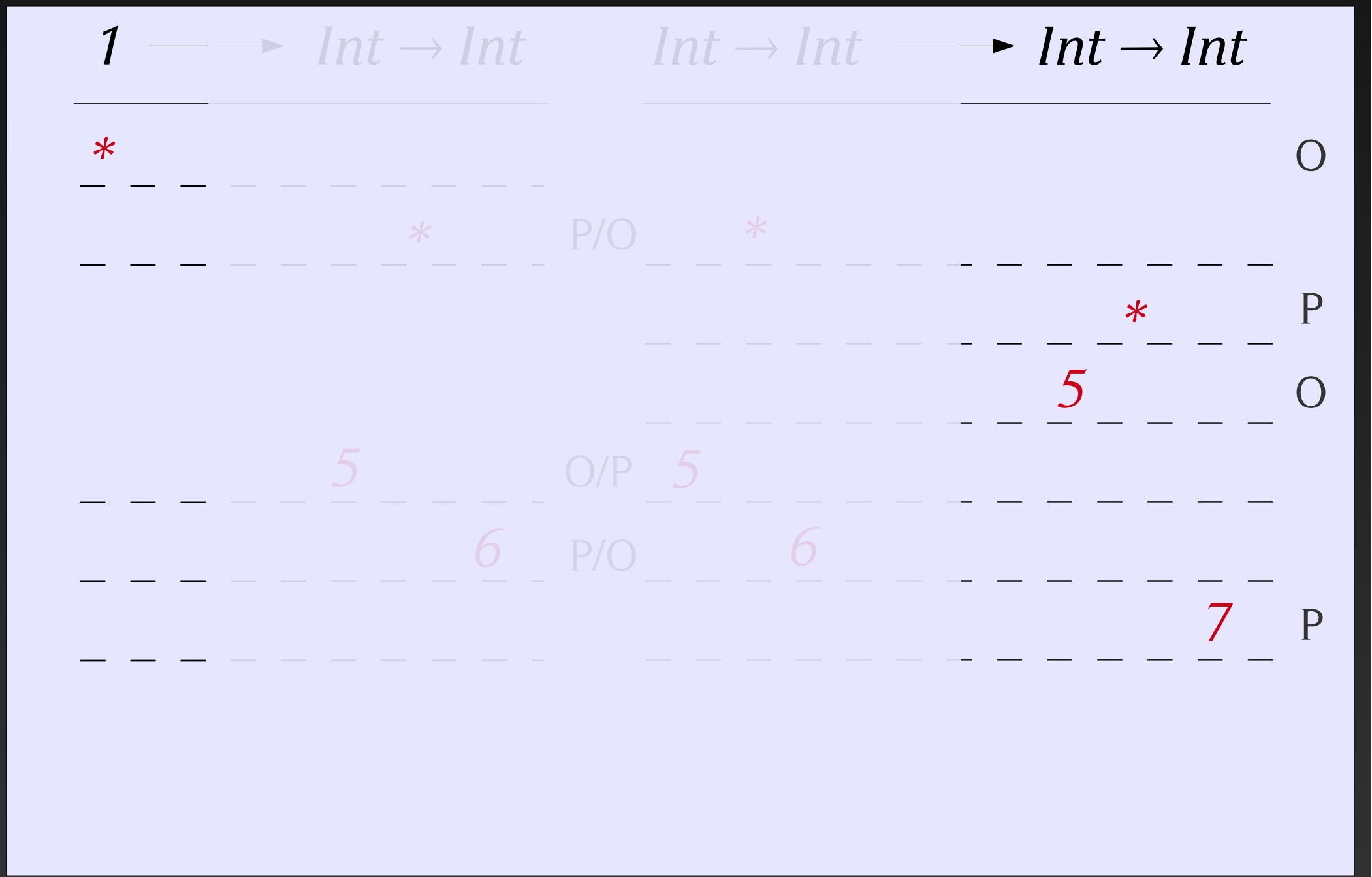

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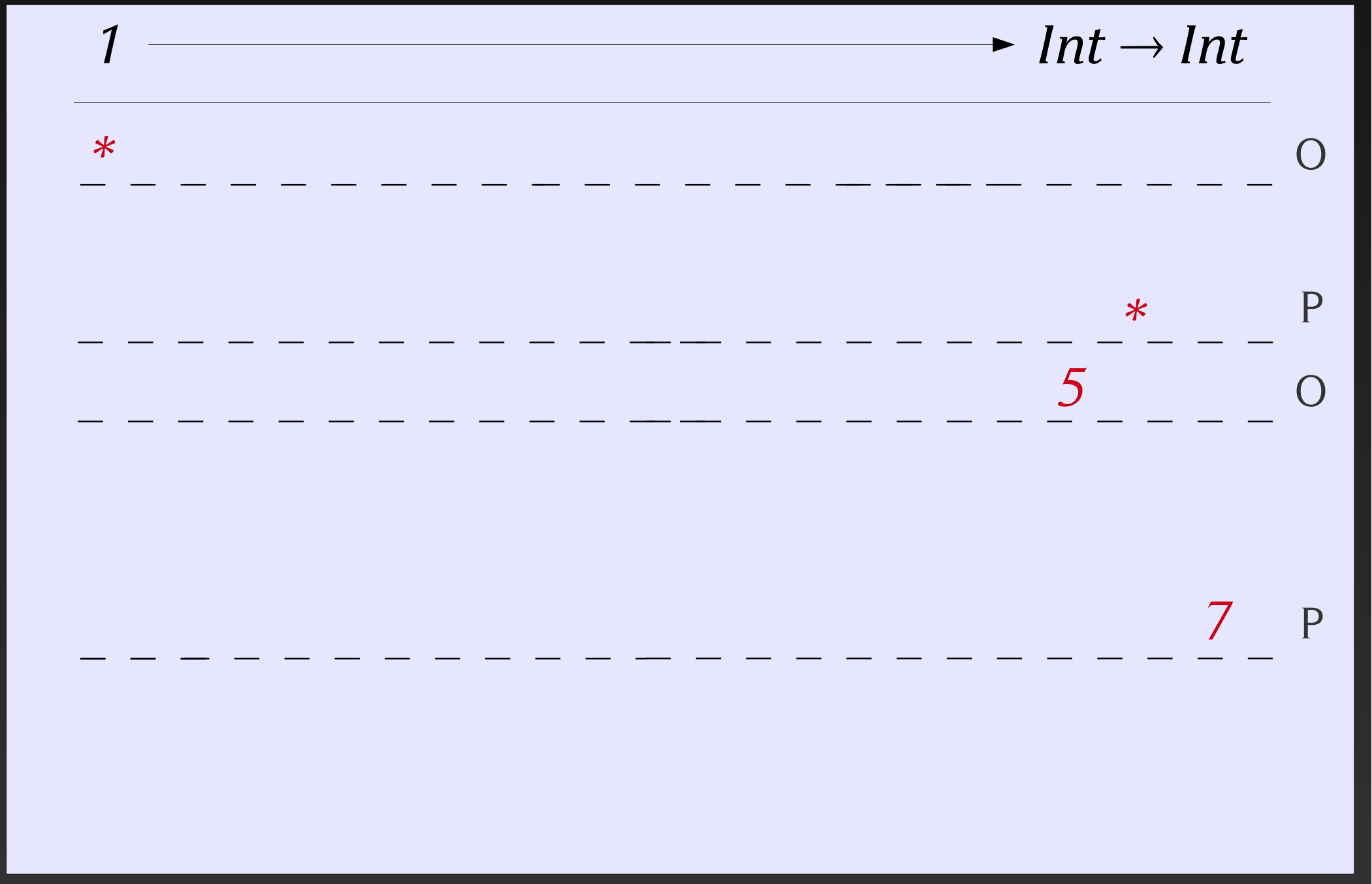

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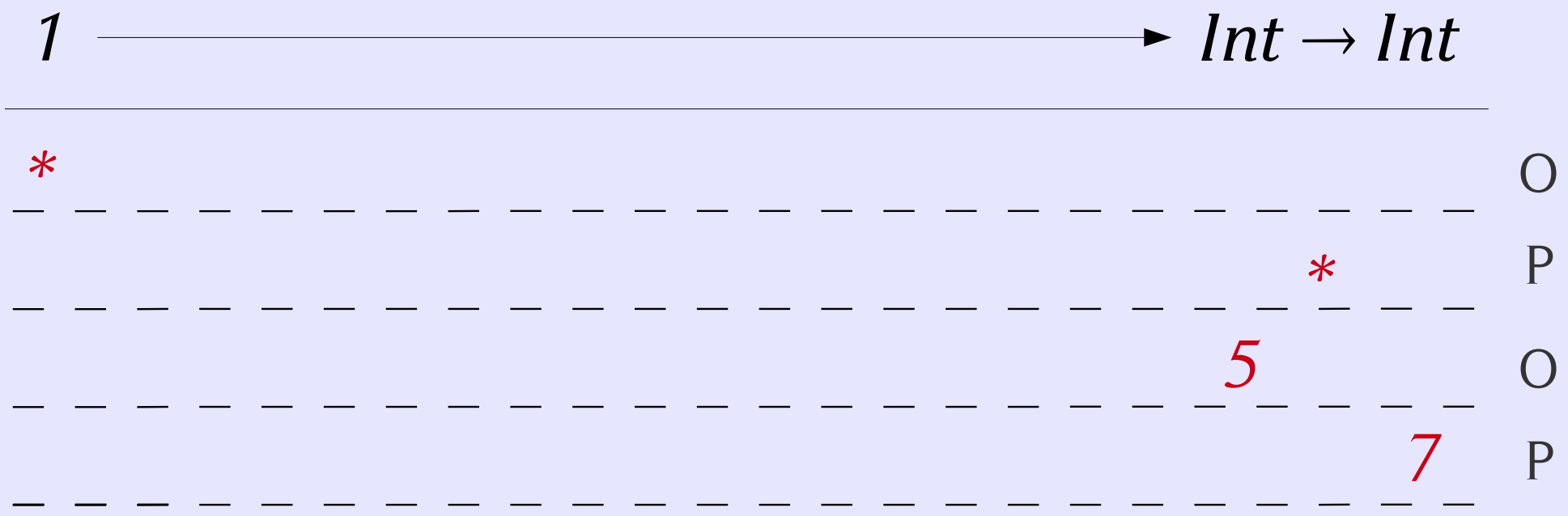

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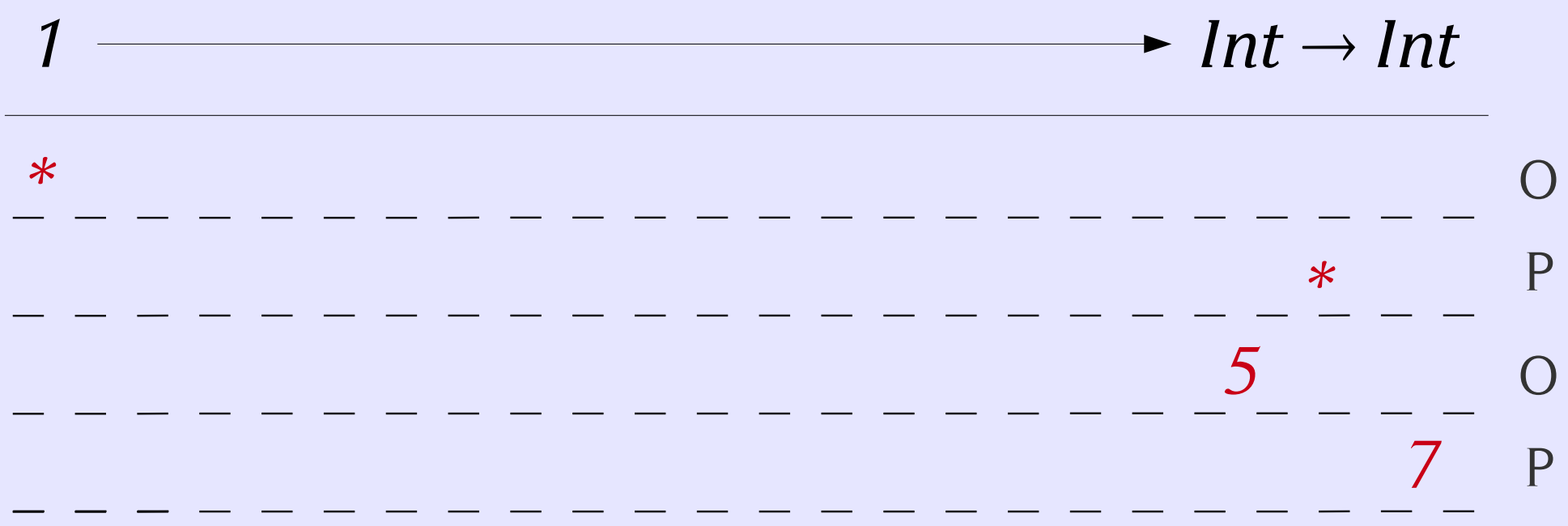


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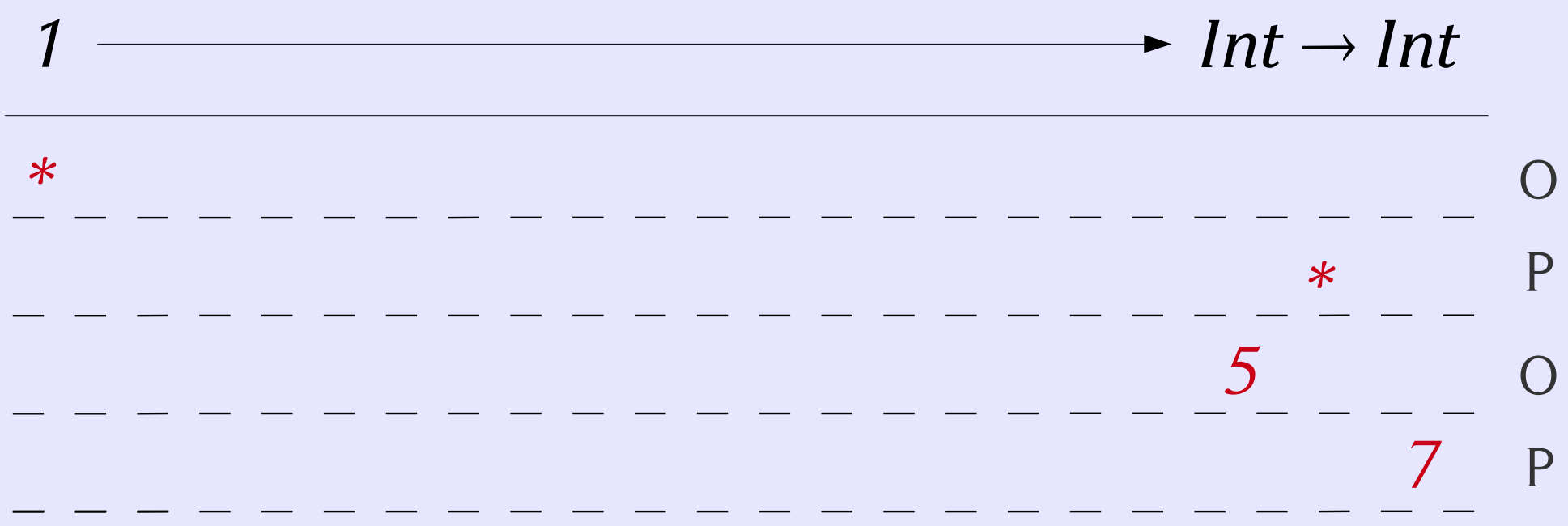


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$$A \xrightarrow{\sigma} B \xrightarrow{\tau} C \quad = \quad A \xrightarrow{\sigma;\tau} C$$

# Game Semantics

- Computation is modelled as a 2-player game between:
  - *Opponent* (the environment)
  - *Proponent* (the program)
- Qualitative games ( ≠ Game Theory)
- Programs = *strategies* for Proponent
- Families (i.e. *categories*) of games

# From PCF to nominal games

*Full Abstraction for PCF* (early 90's)

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Nominal game semantics (2004-)

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# Nominal sets

Foundation for maths with names and name-binding

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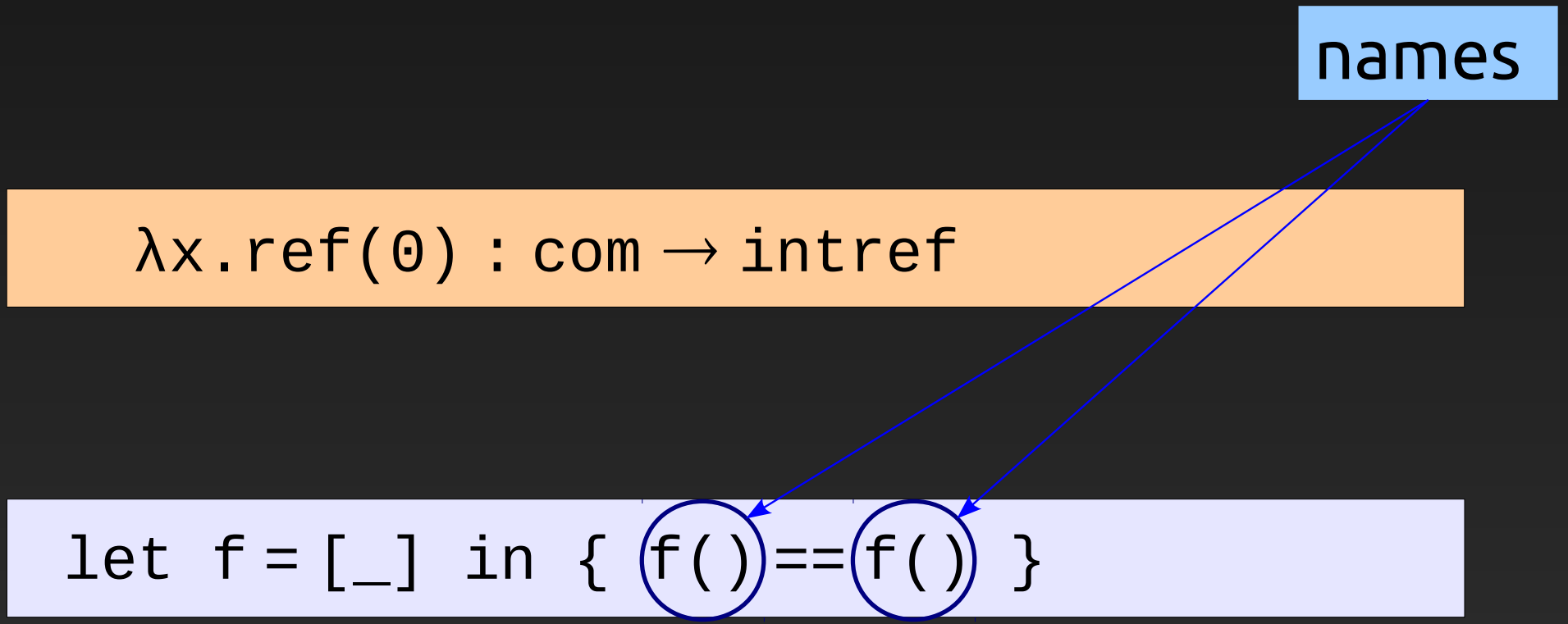
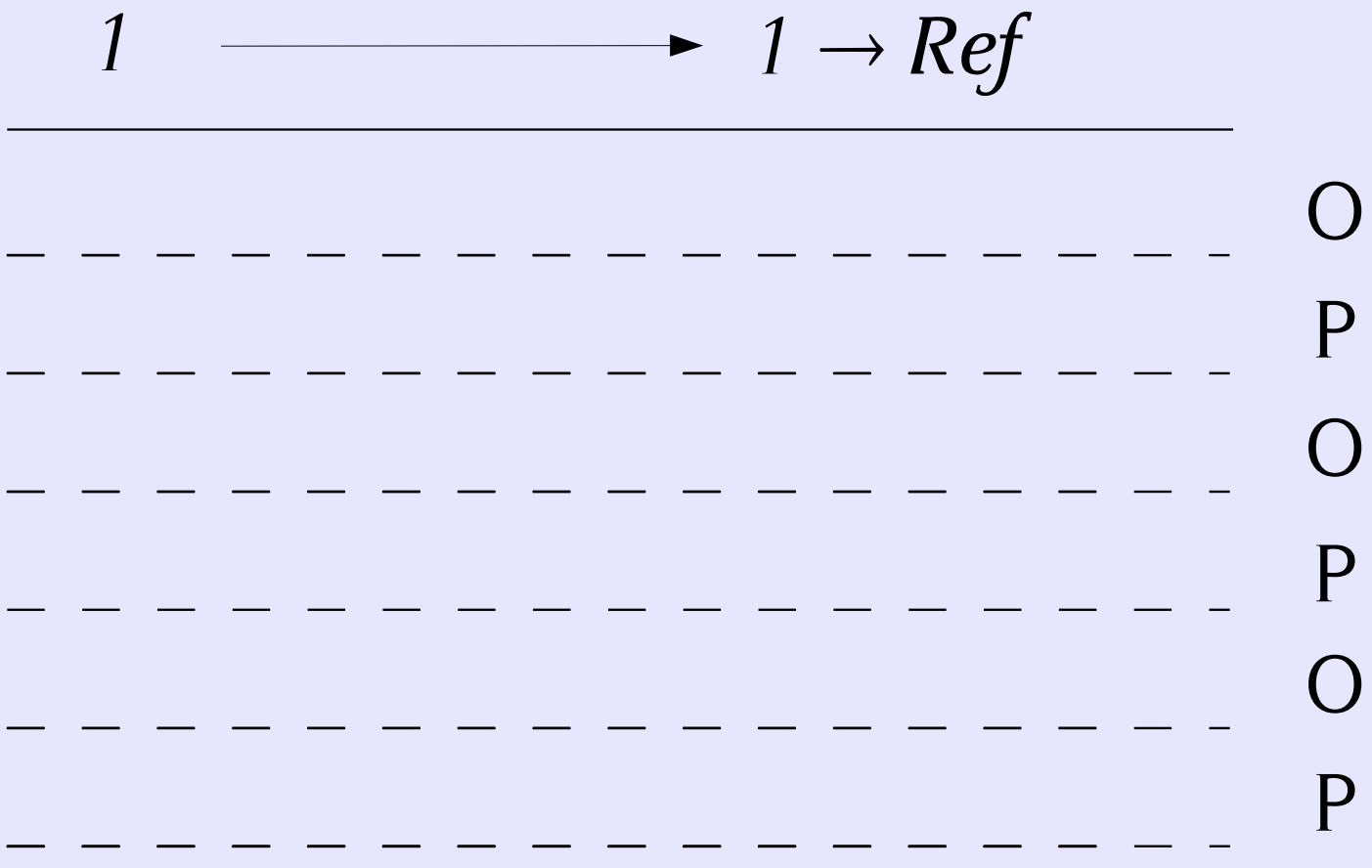
## Games in nominal sets



# Examples

$\lambda x.\mathtt{ref}() : \mathtt{com} \to \mathtt{ref}$



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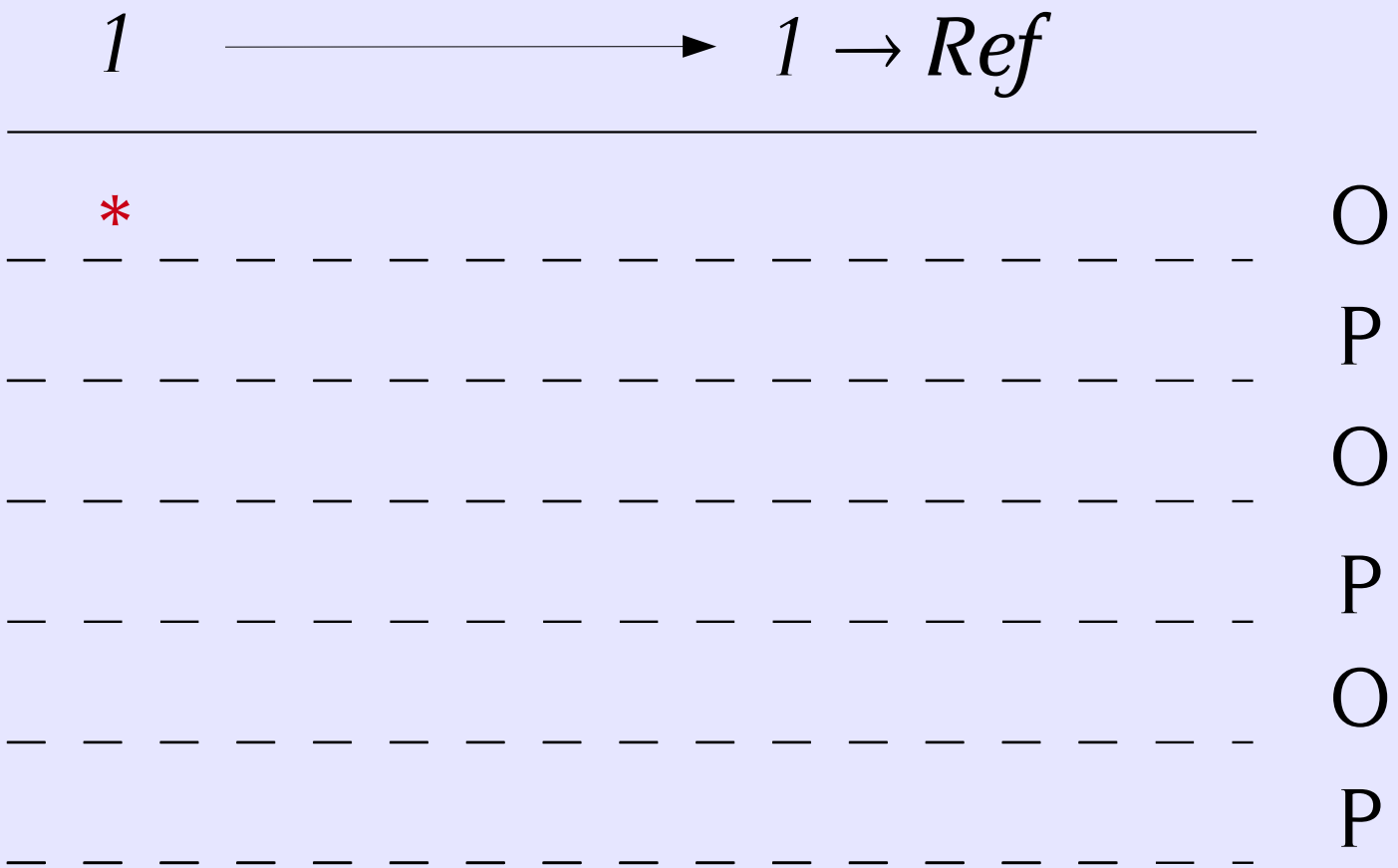

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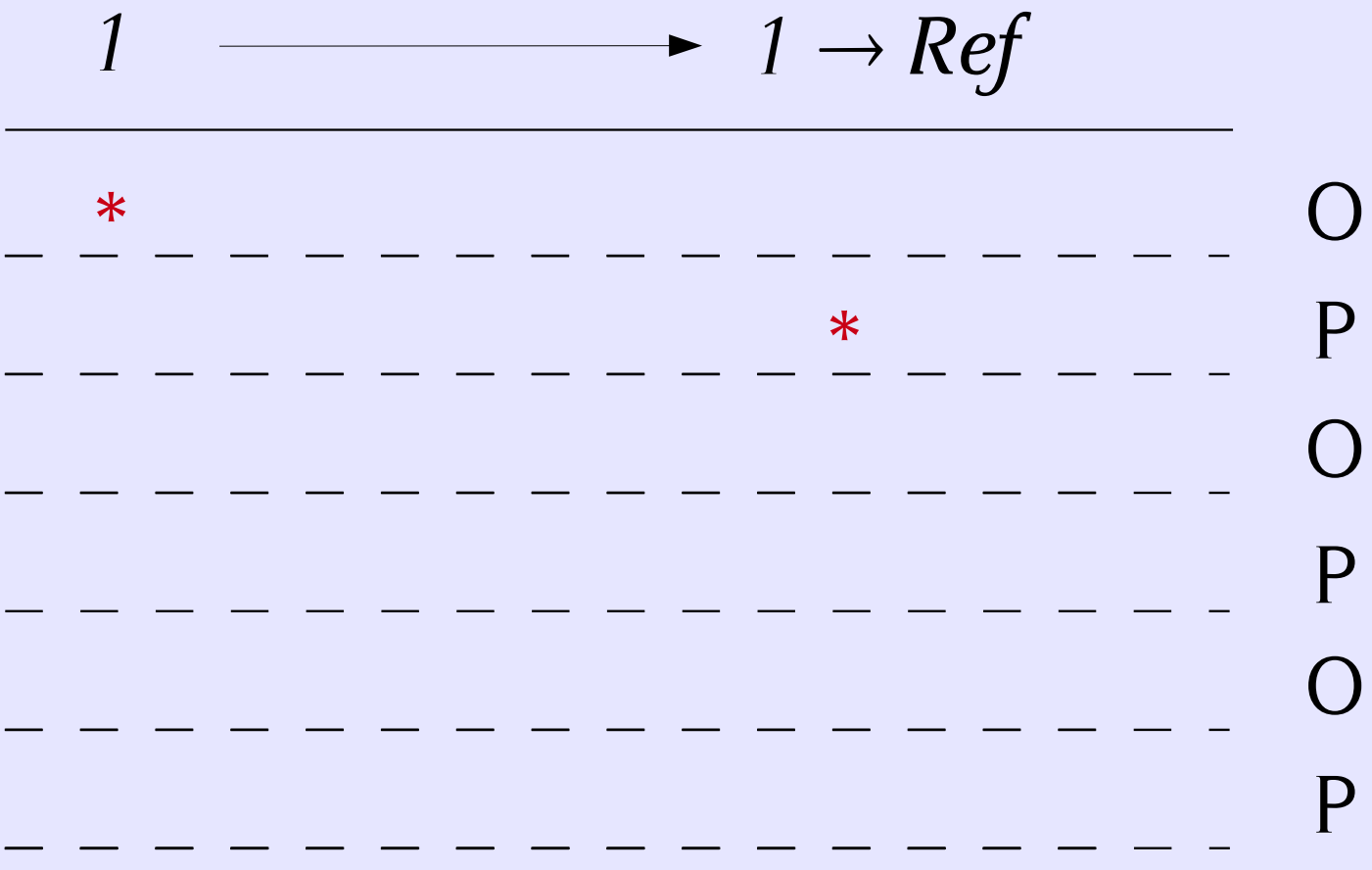

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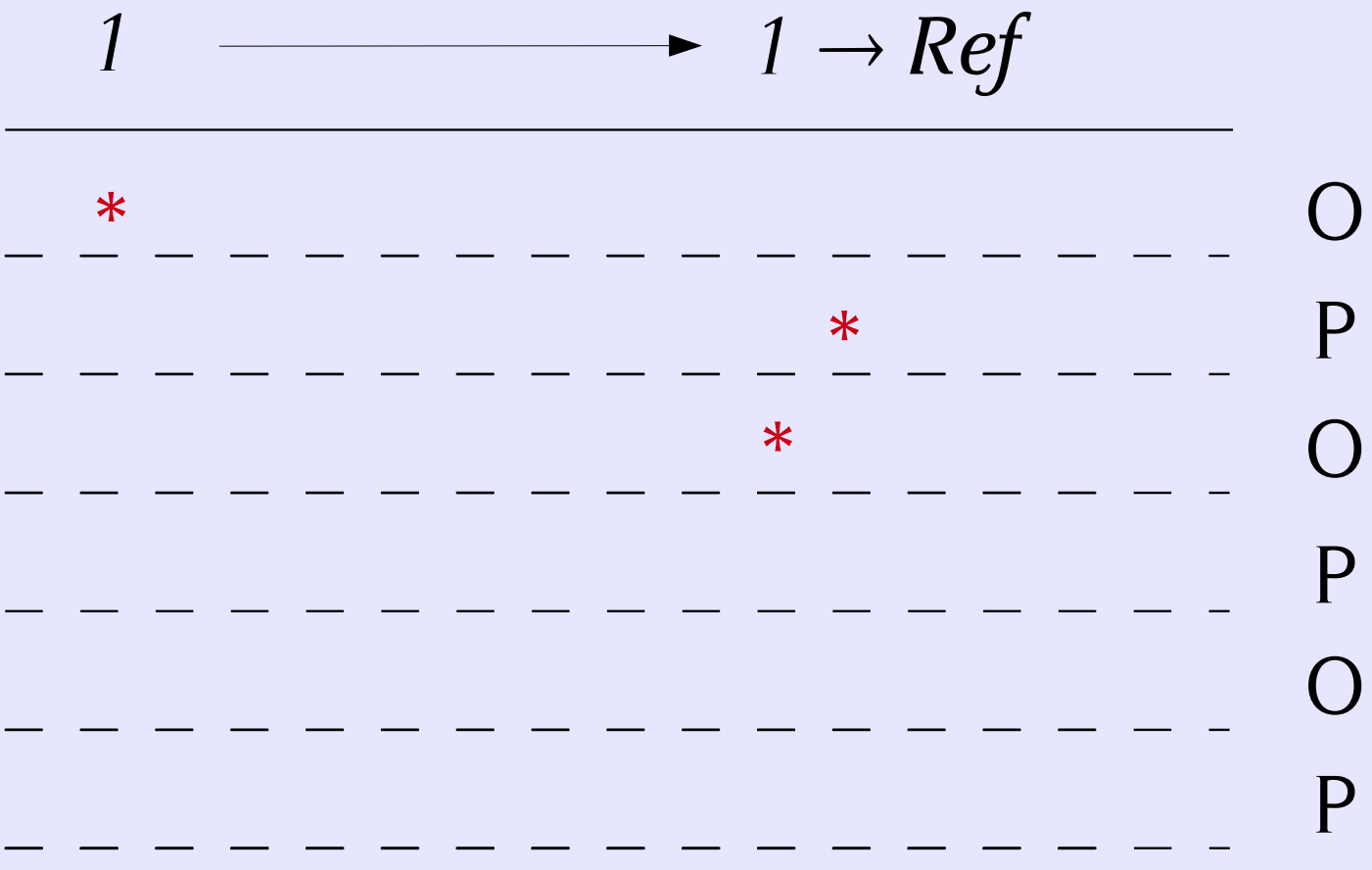

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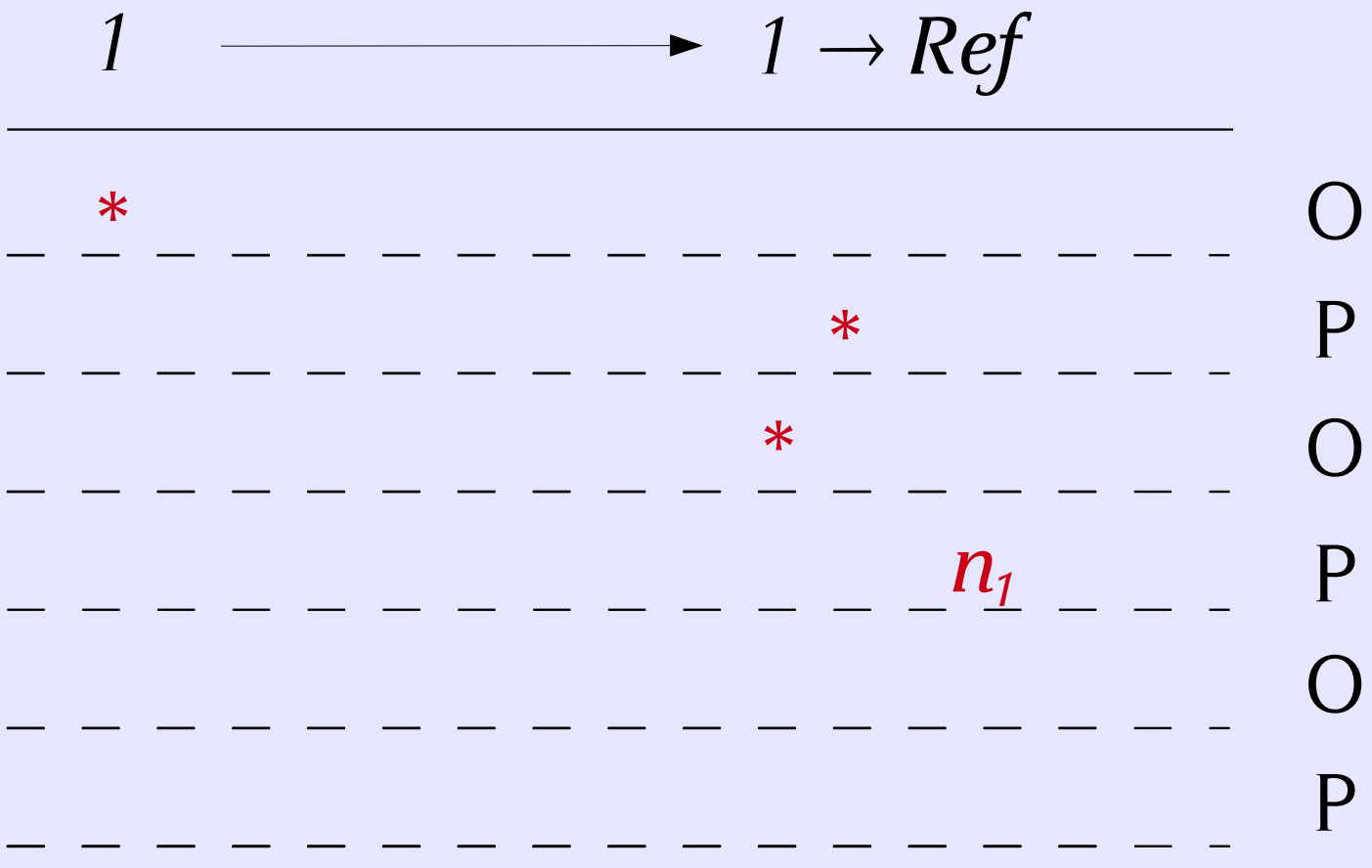

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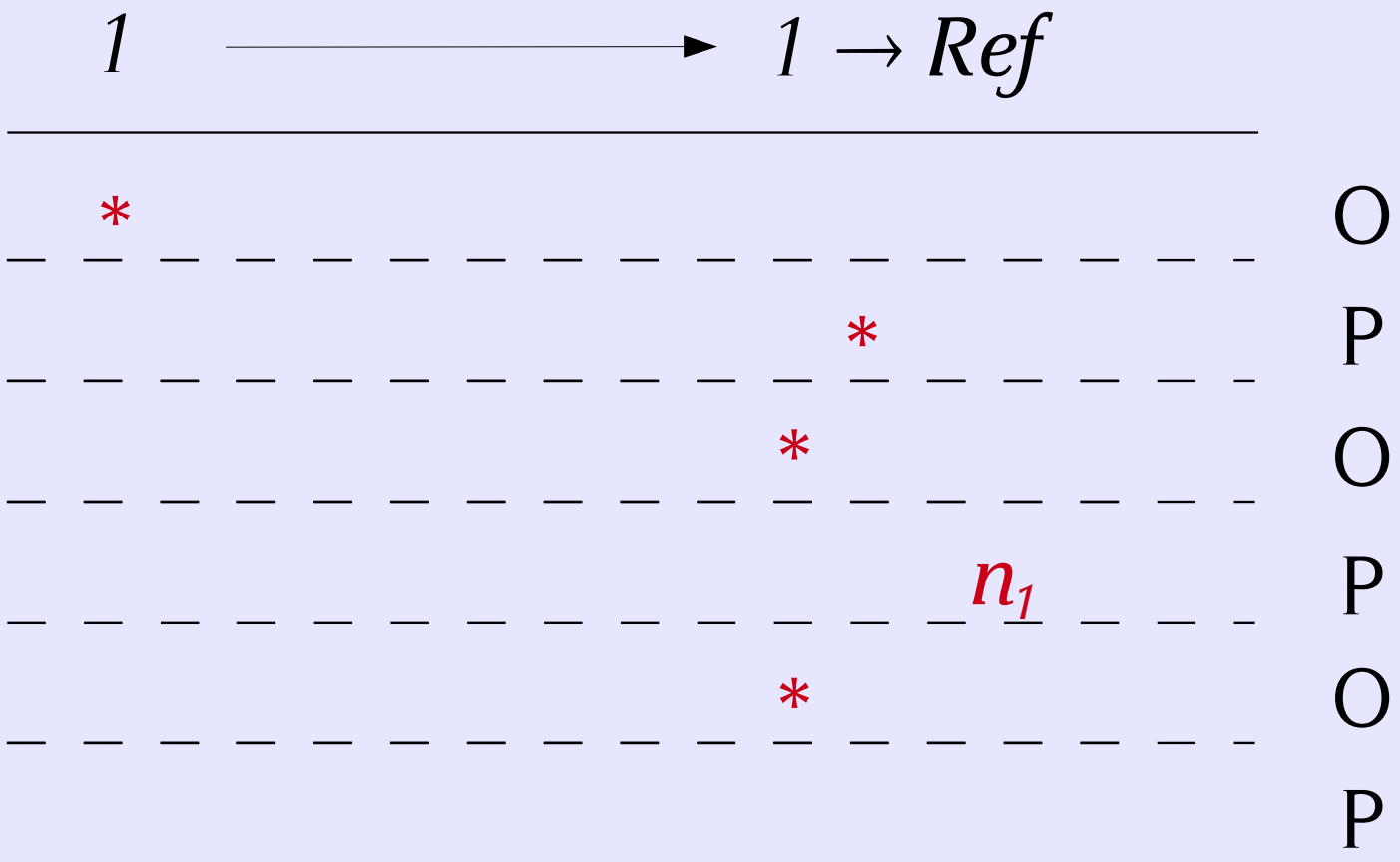

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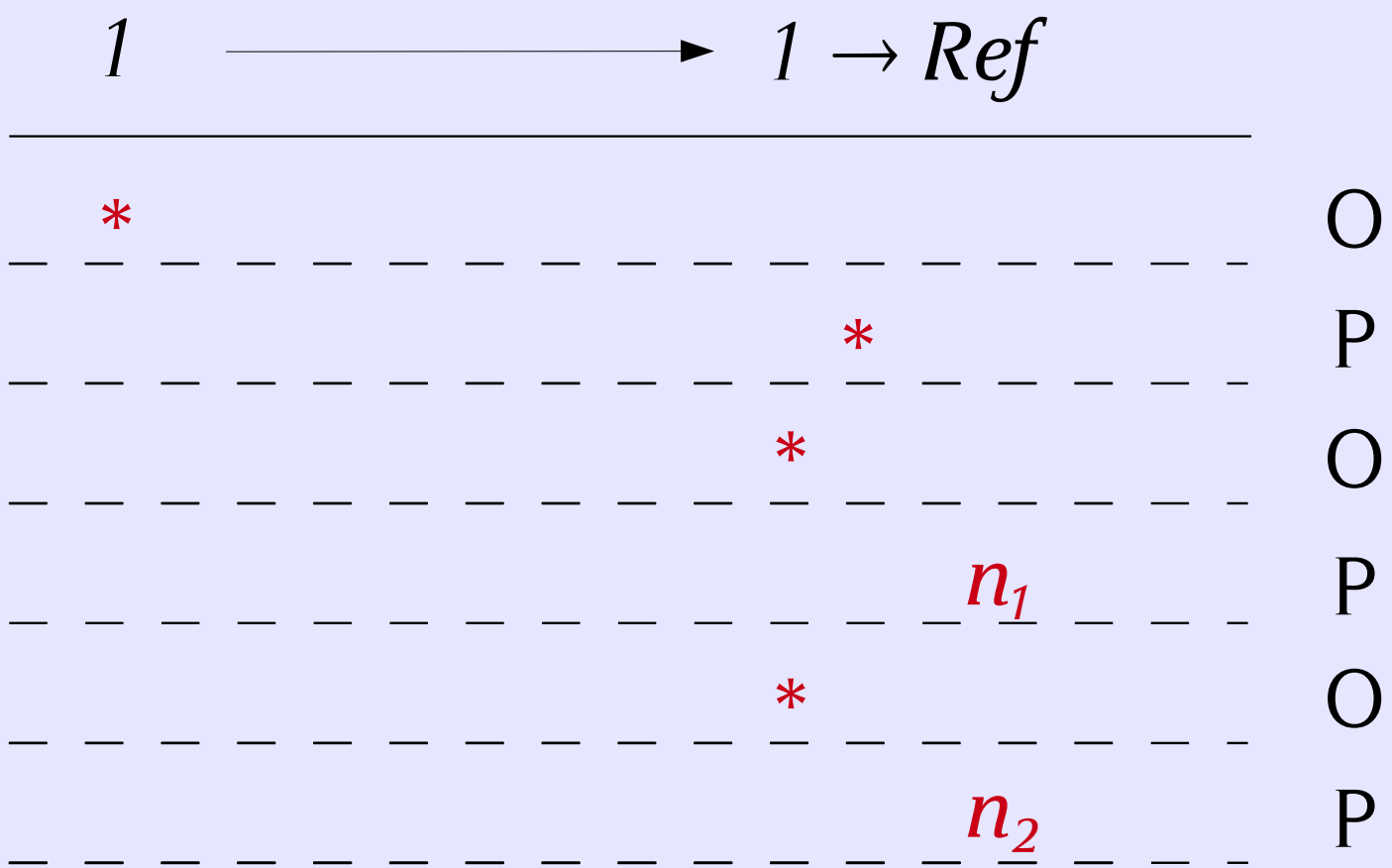

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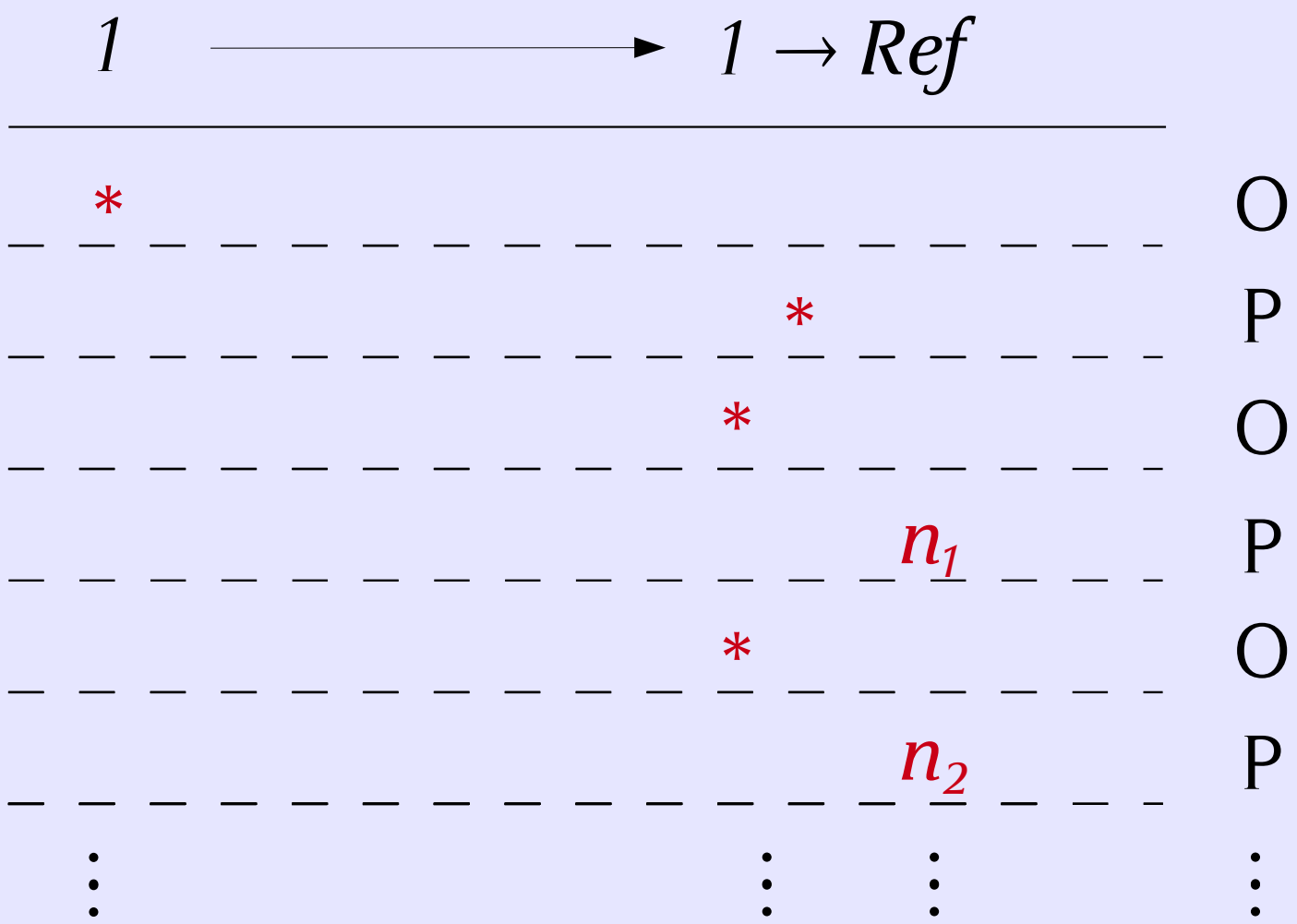

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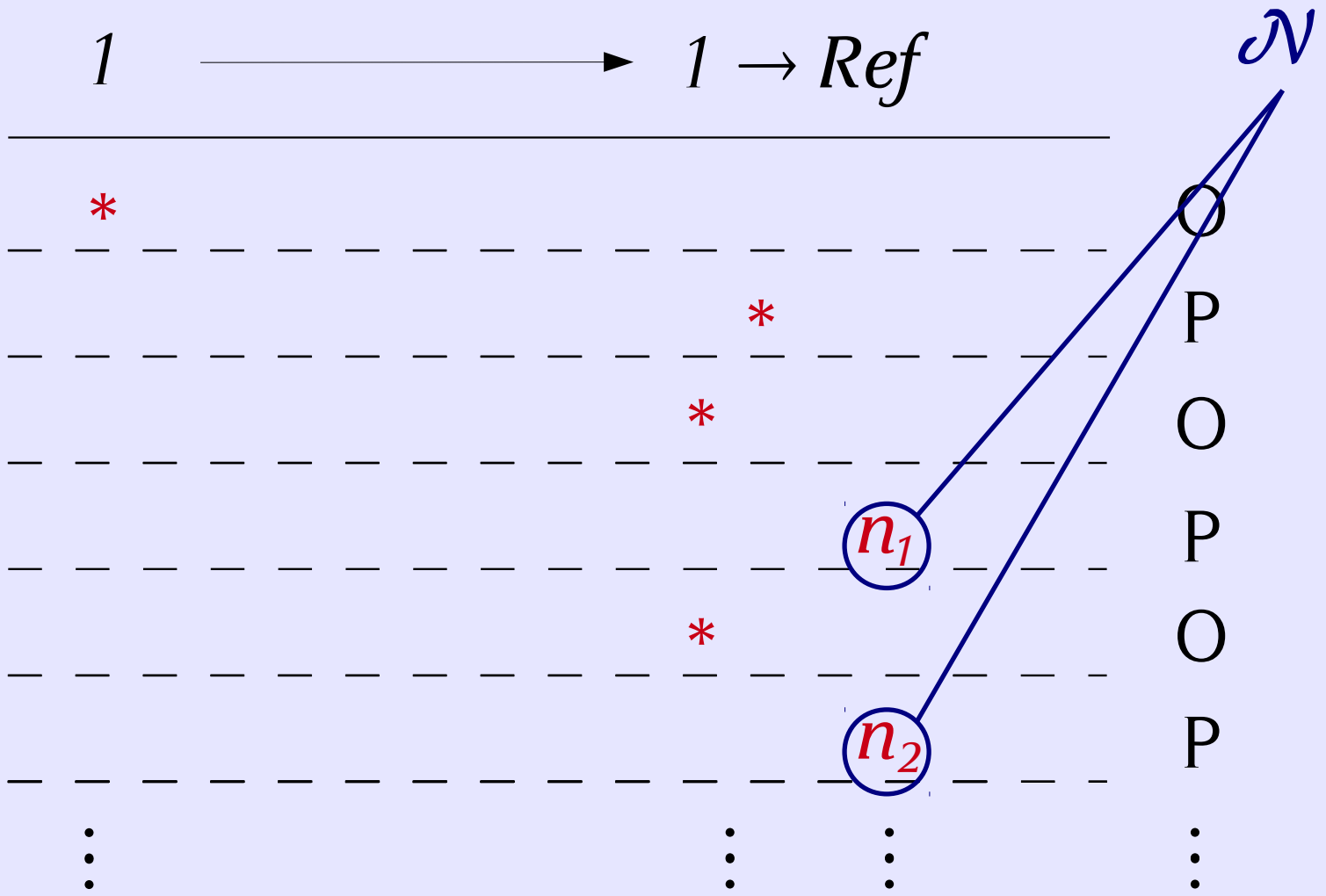

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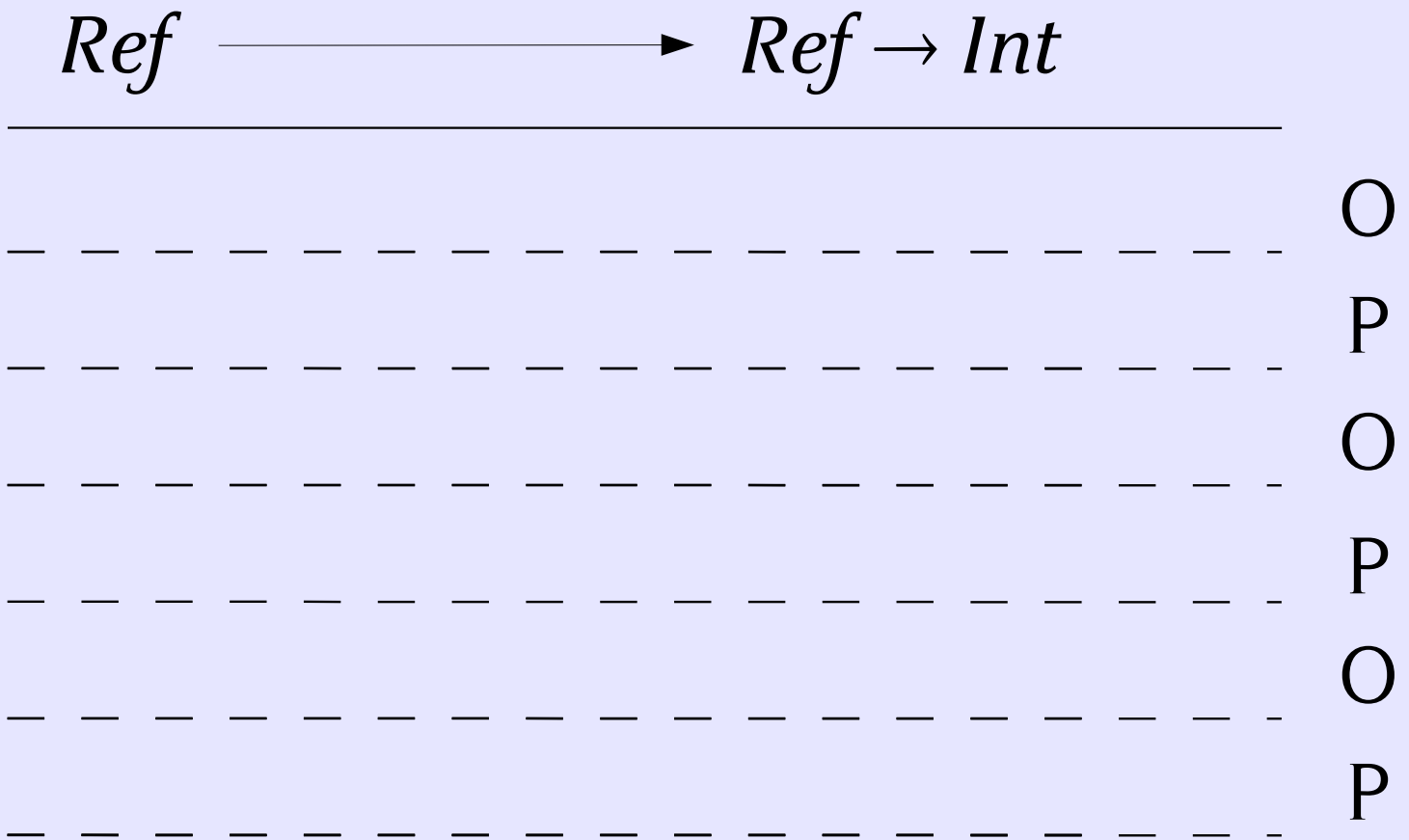
$\texttt{x : ref} \vdash \texttt{λy.(x == y) : ref} \rightarrow \texttt{int}$



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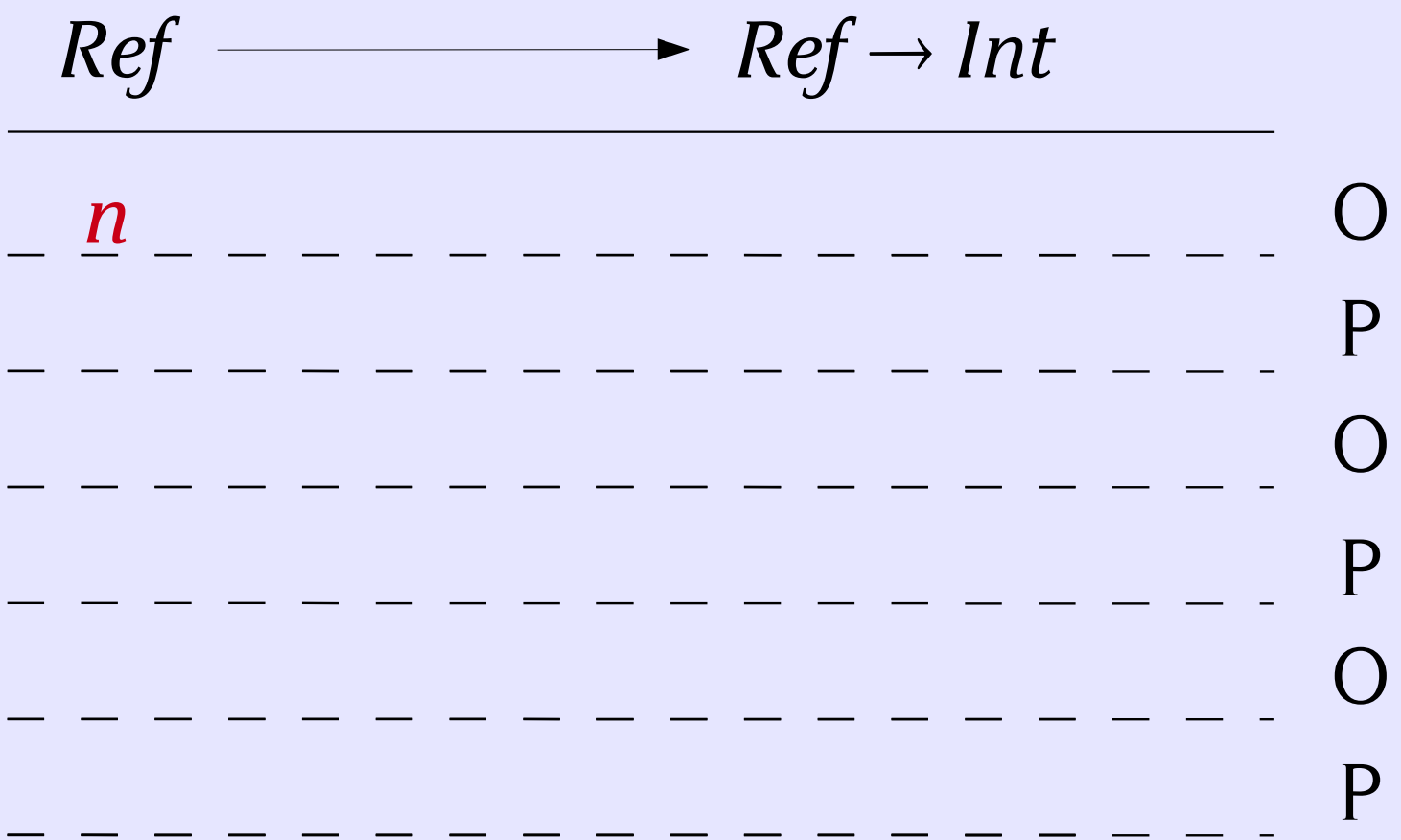

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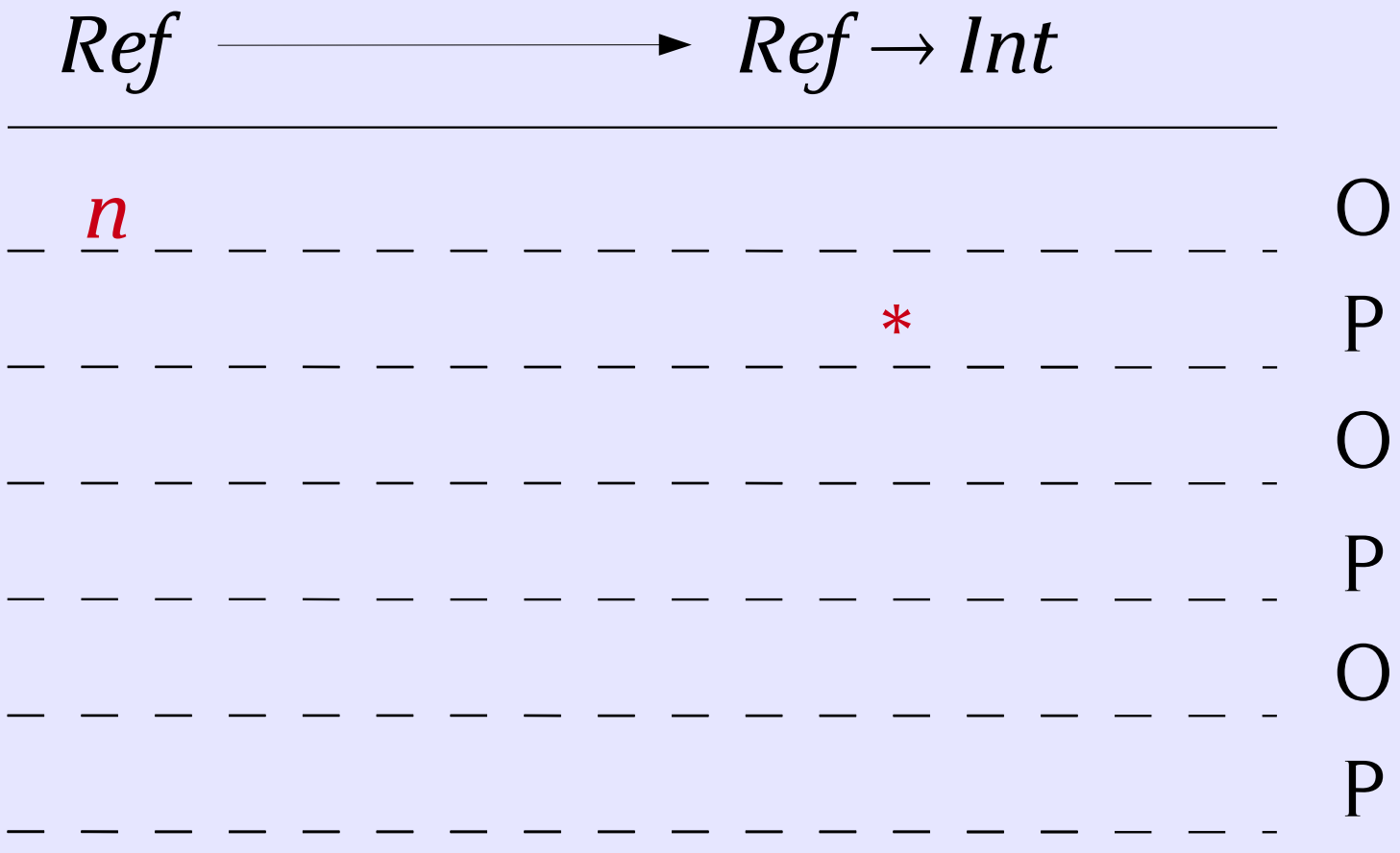

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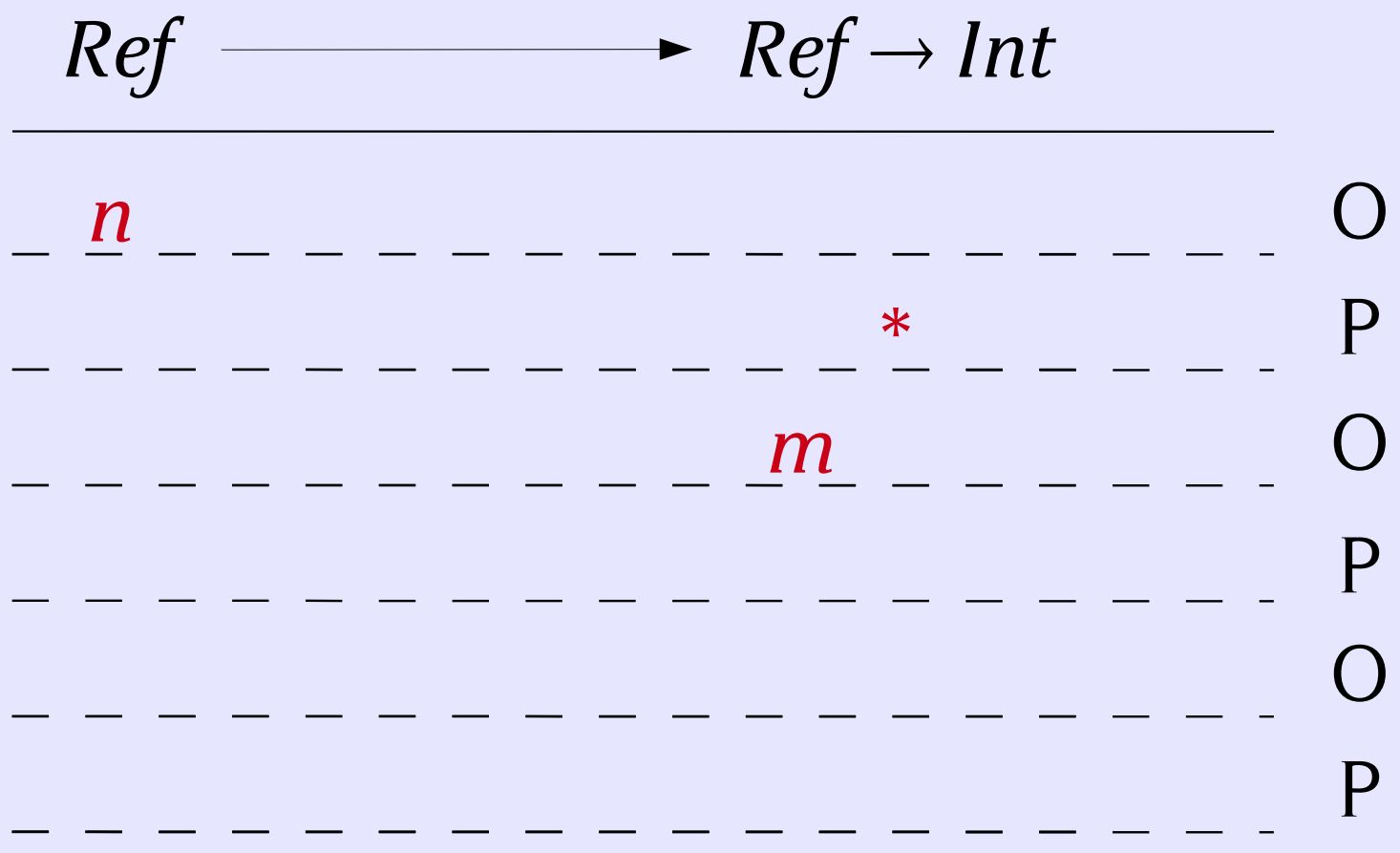

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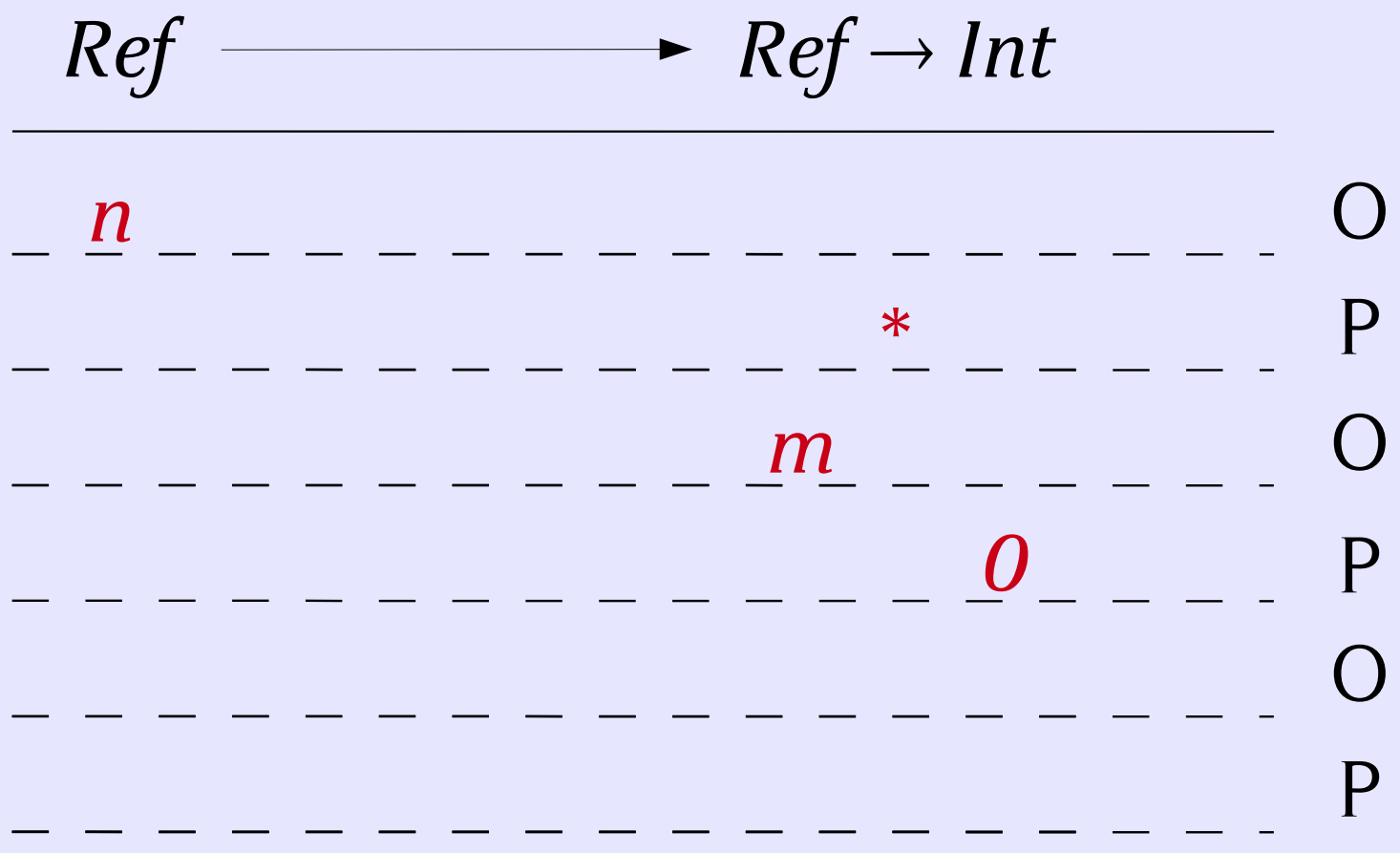

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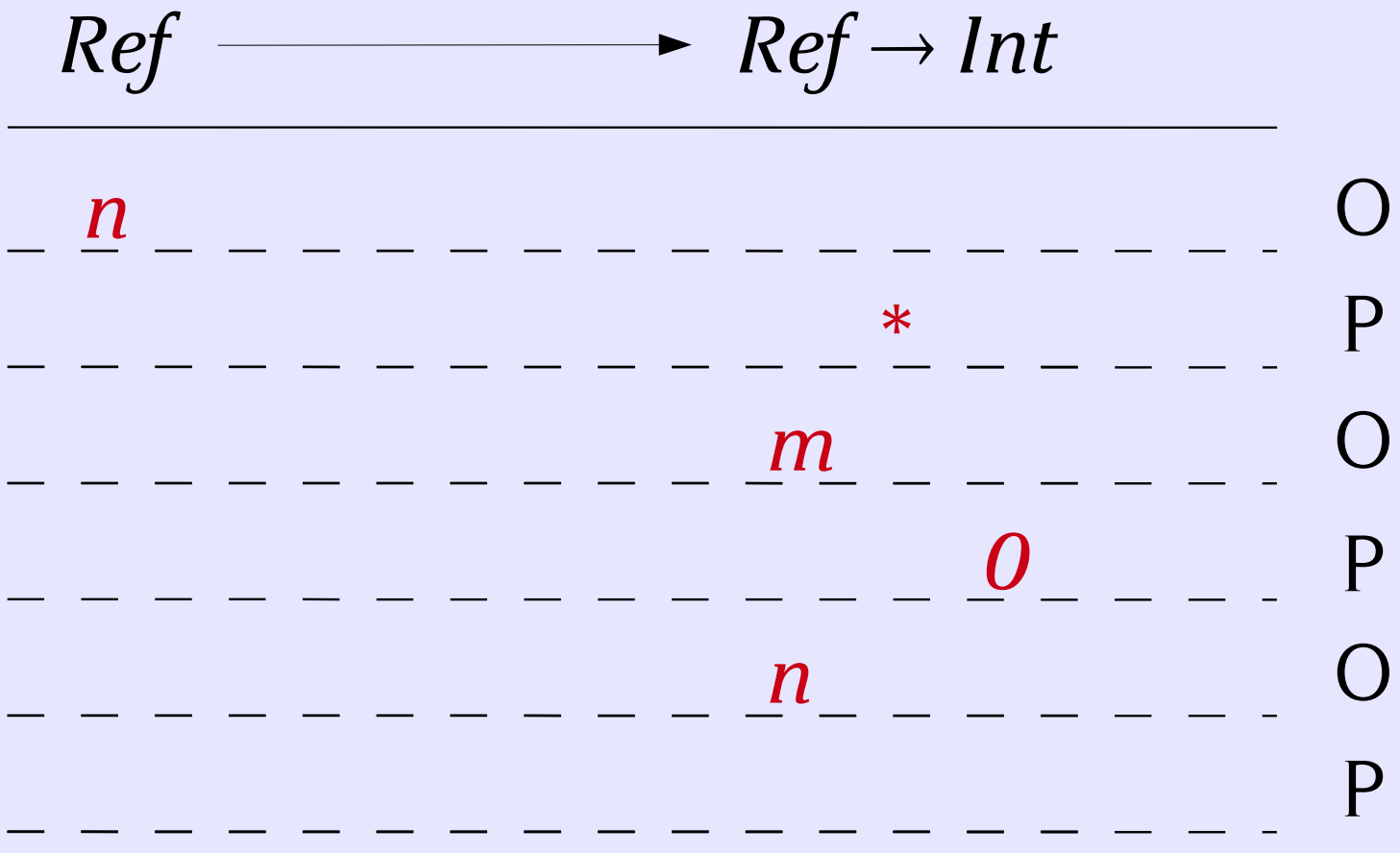

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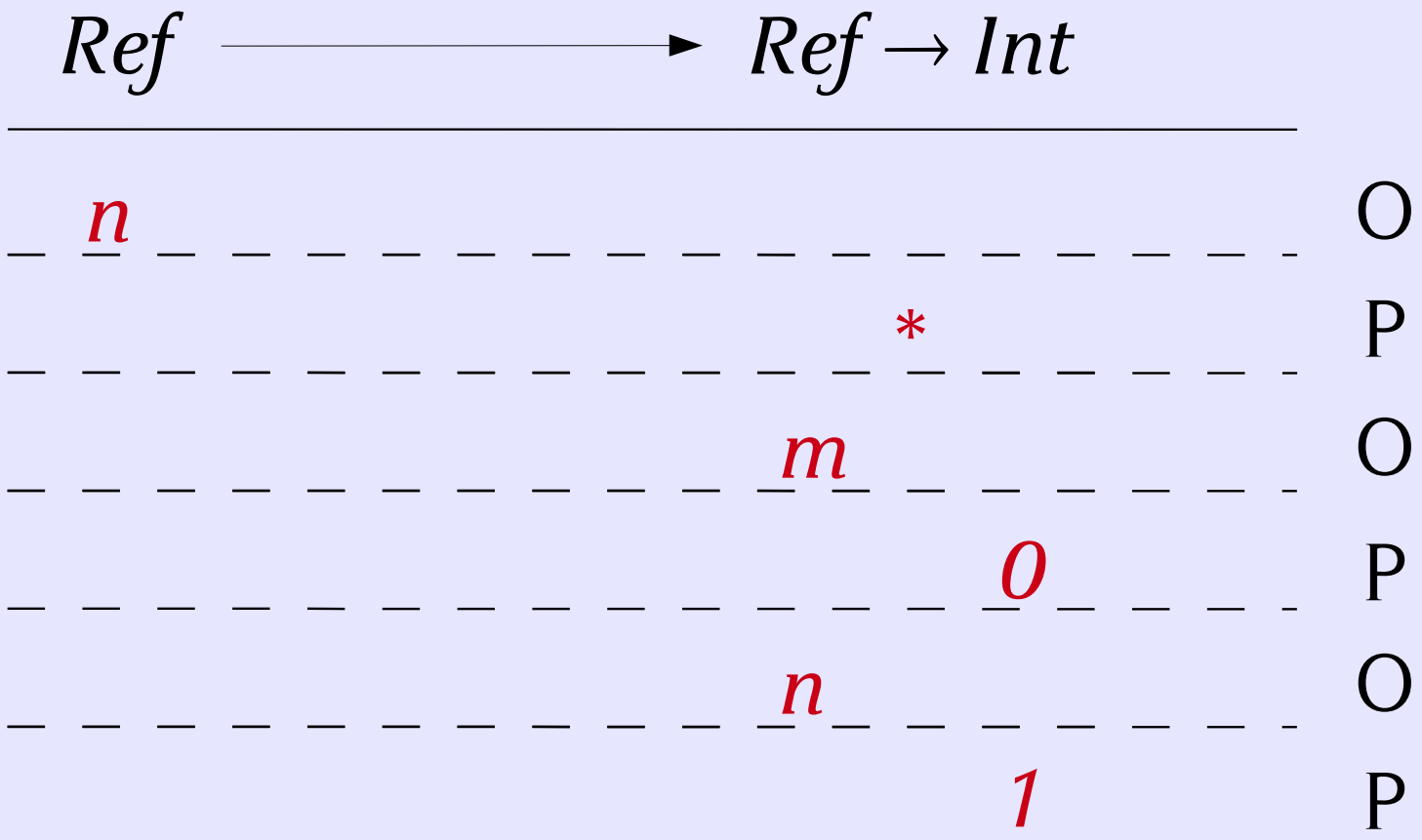

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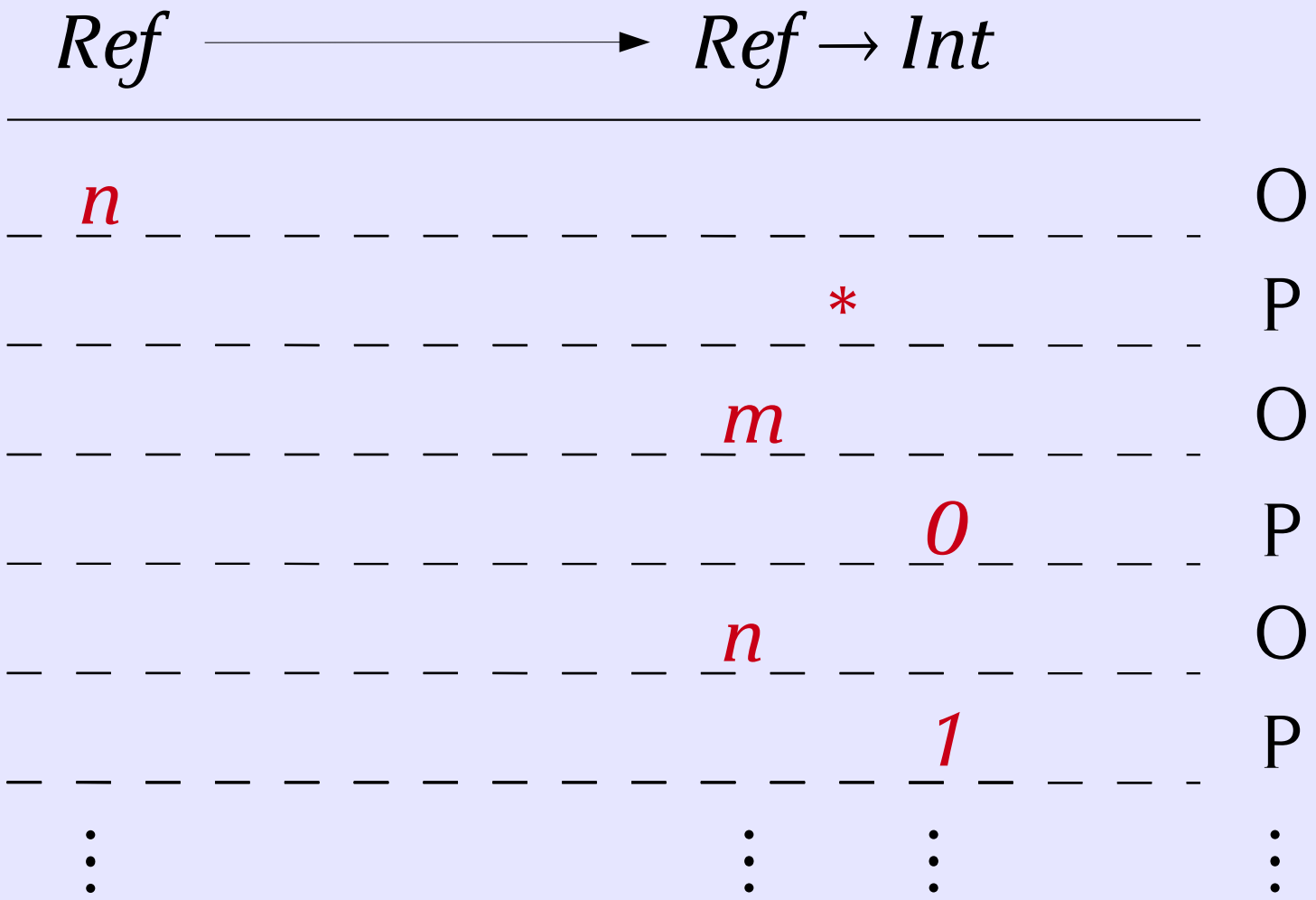

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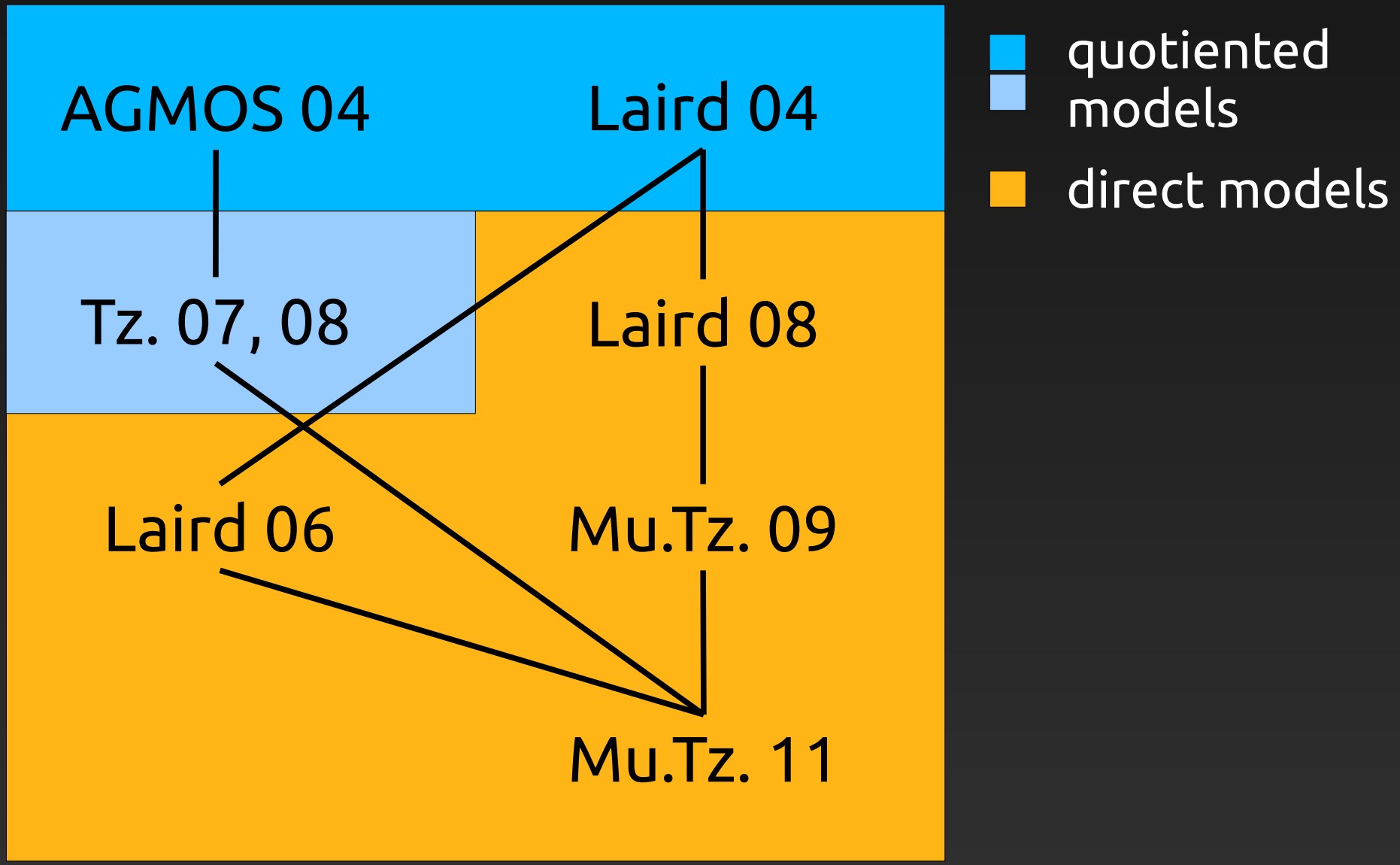

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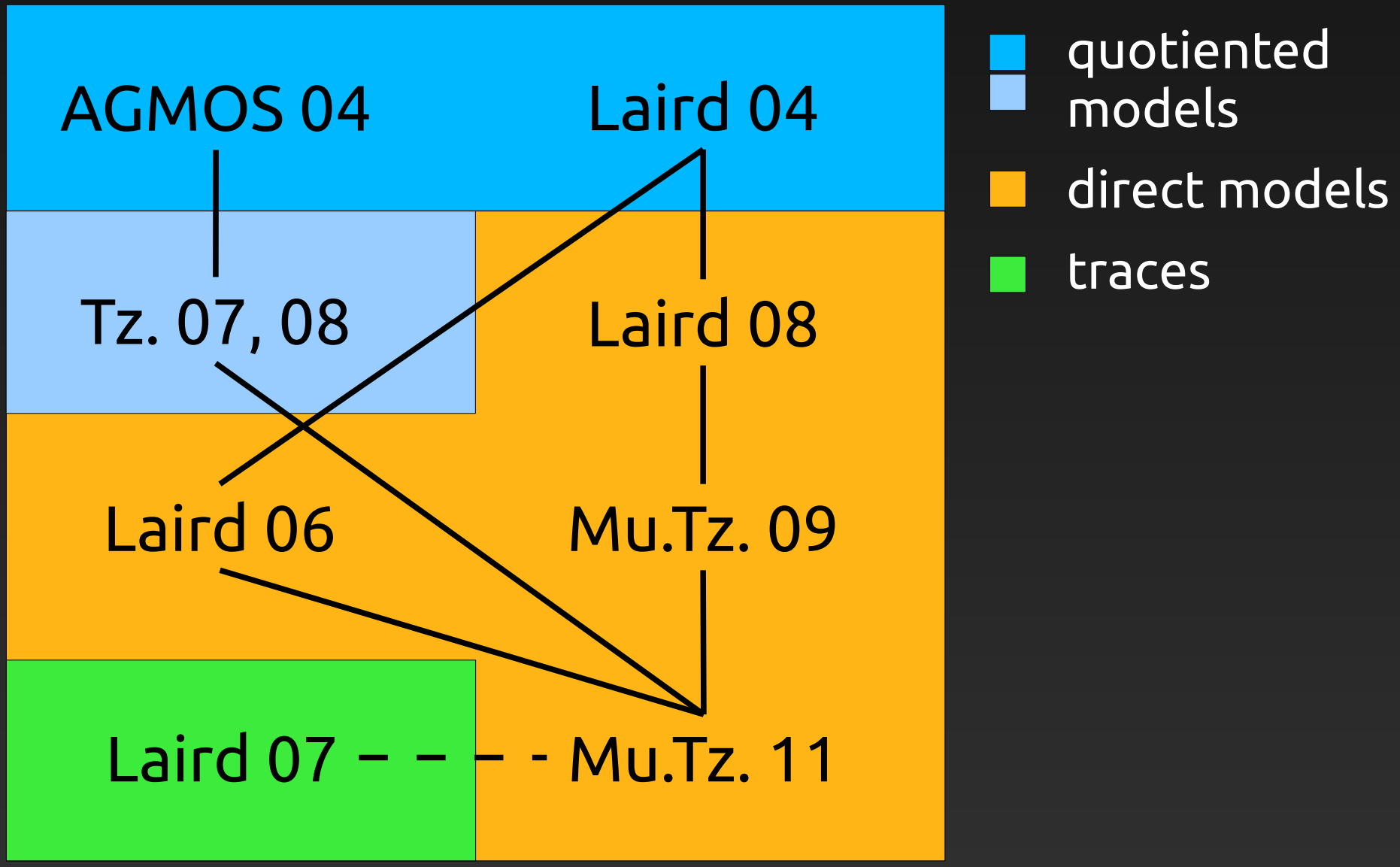

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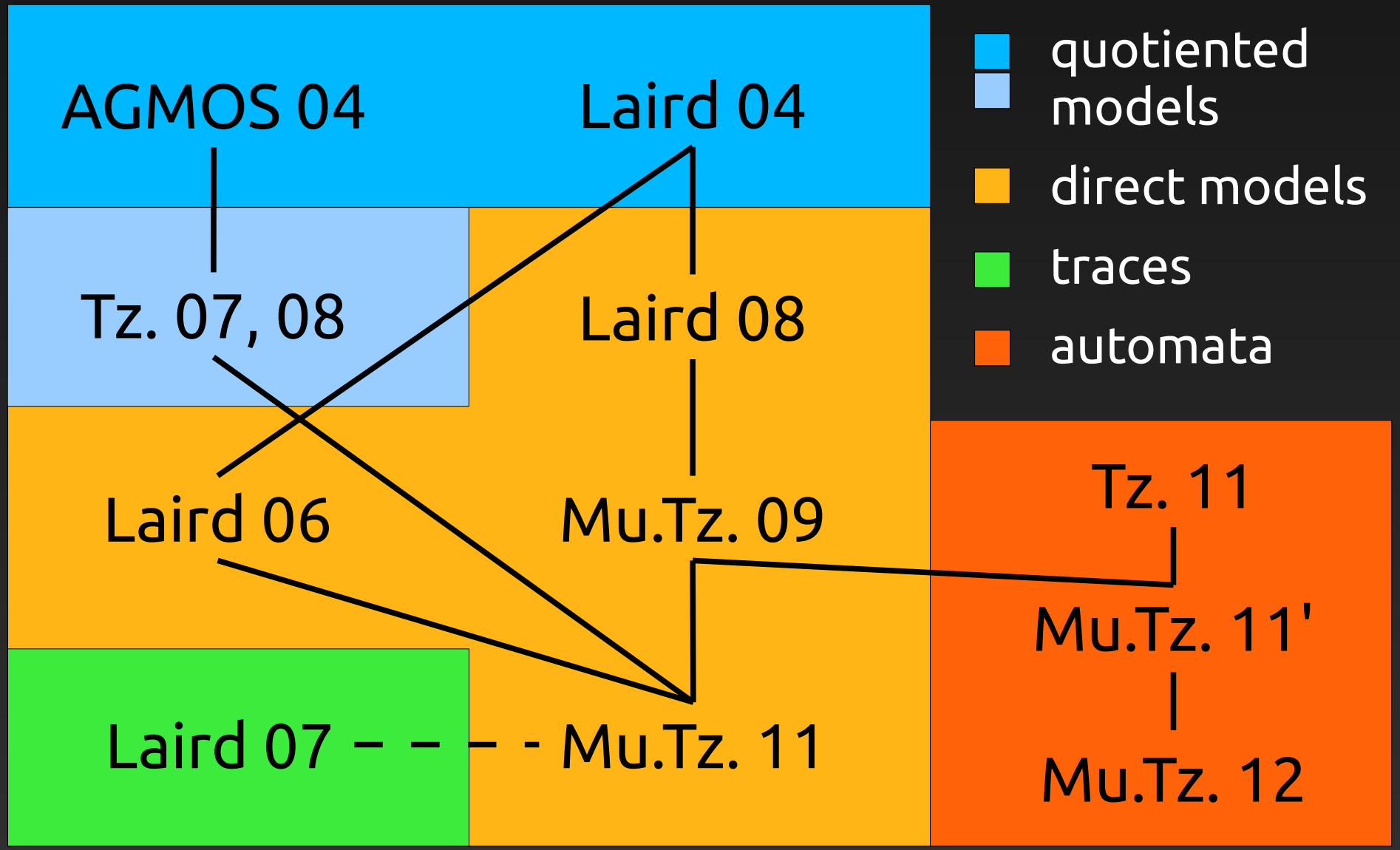

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- AGMOS 04, Laird 04: nu-calculus
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$$S = \bigotimes_A (\mathbf{N}_A \Rightarrow [\![A]\!])$$

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$$M \cong N \iff [\![M]\!] \cong [\![N]\!]$$

Characteristics:

- moves involving names
- moves-with-state (a set/list of names)
- “functional” behaviour + monads for effects

# Direct models

- Laird 06: higher-order channels
  Laird 08: pointers
- Mu.Tz. 09: integer references (Reduced ML)
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Characteristics:

- moves involving names/ moves-with-store
- name-availability conditions/ “direct” effects

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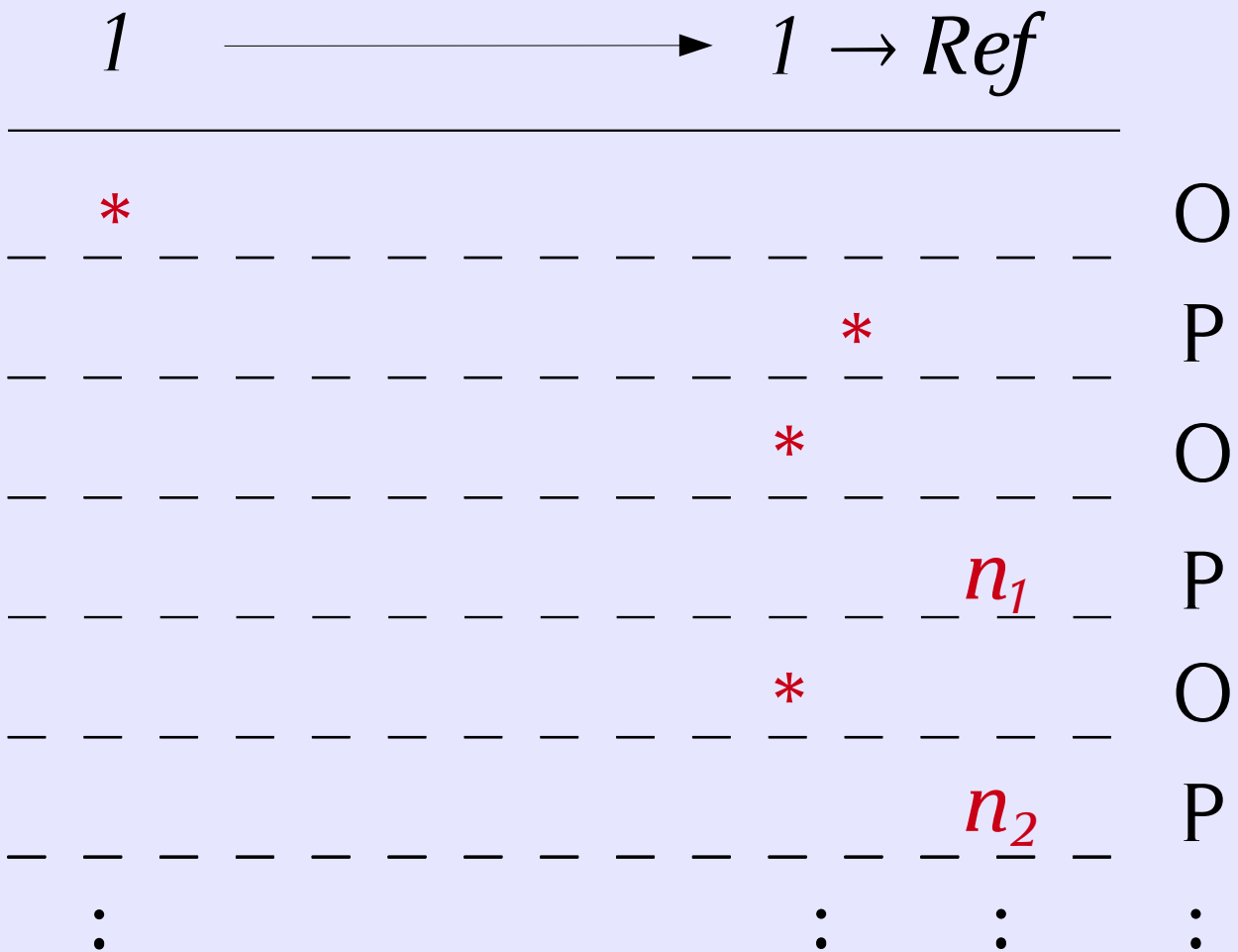

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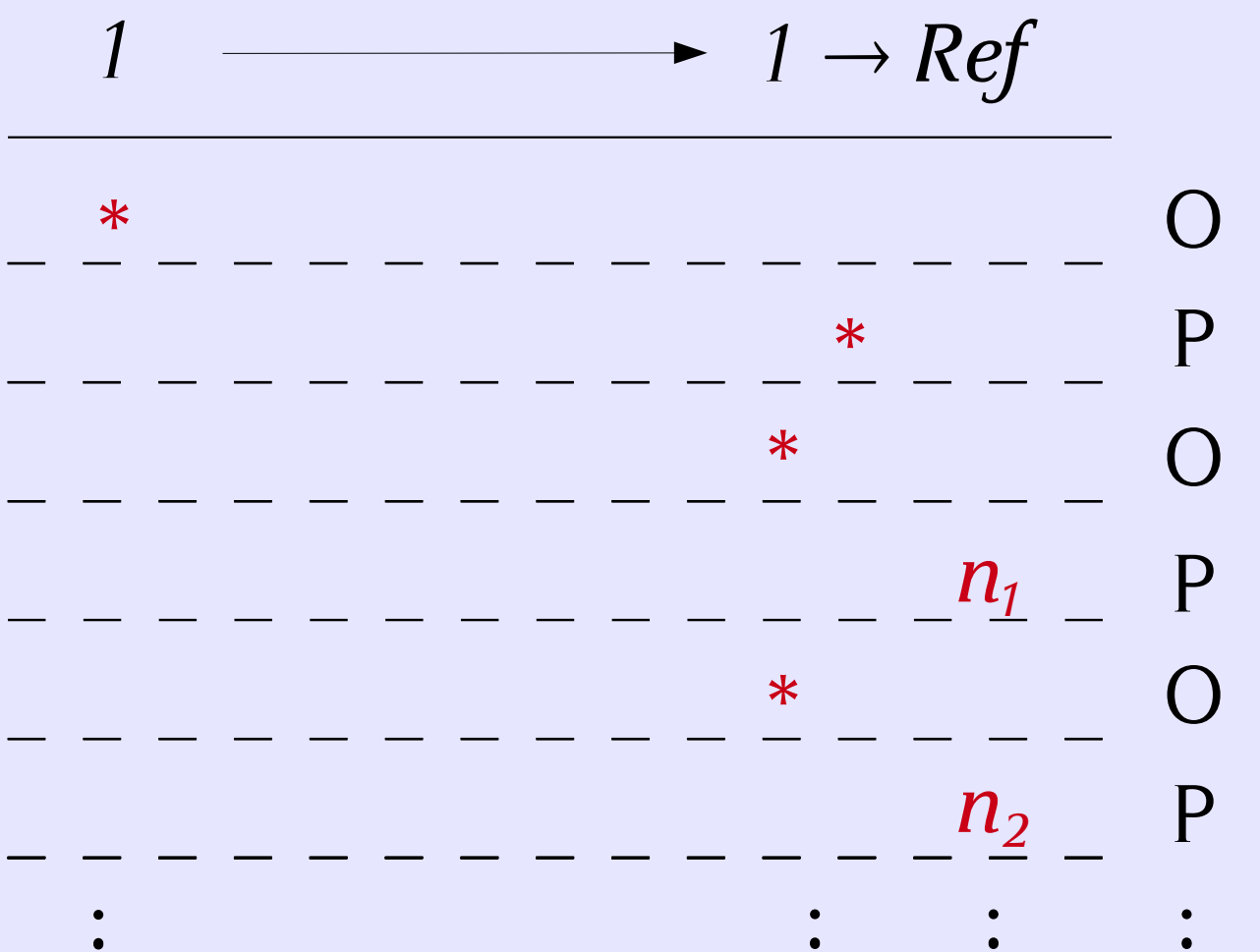

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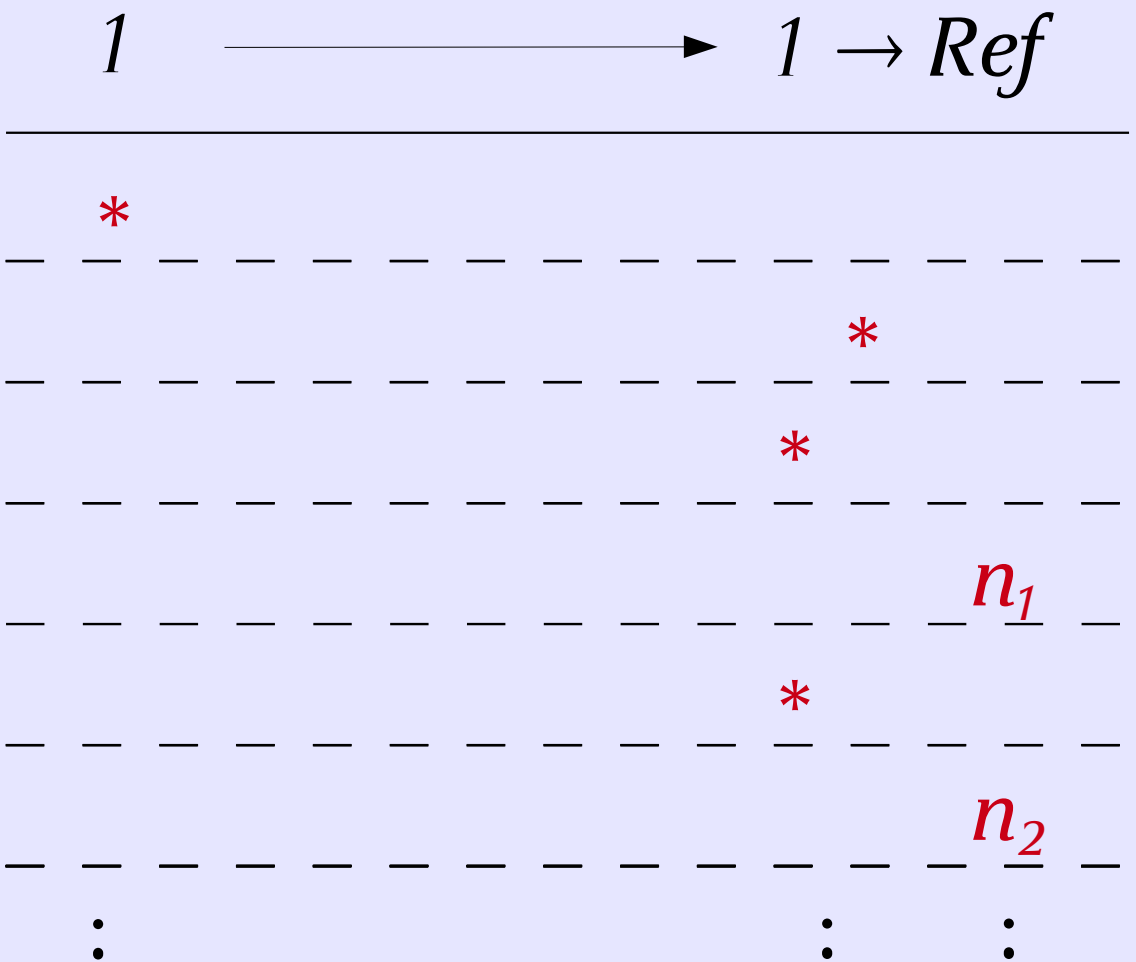


Programs

Game Semantics

Automata

# Fresh-register automata

$\texttt{λz.ref()} \longmapsto \{ * * * n_1 * n_2 * n_3 \ldots \mid n_i\text{'s distinct} \}$

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Automata with names

- Infinite alphabet $\mathcal{N}$
- Freshness recognition

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Finite-state machines with registers

- *Kaminski & Francez (1994)*

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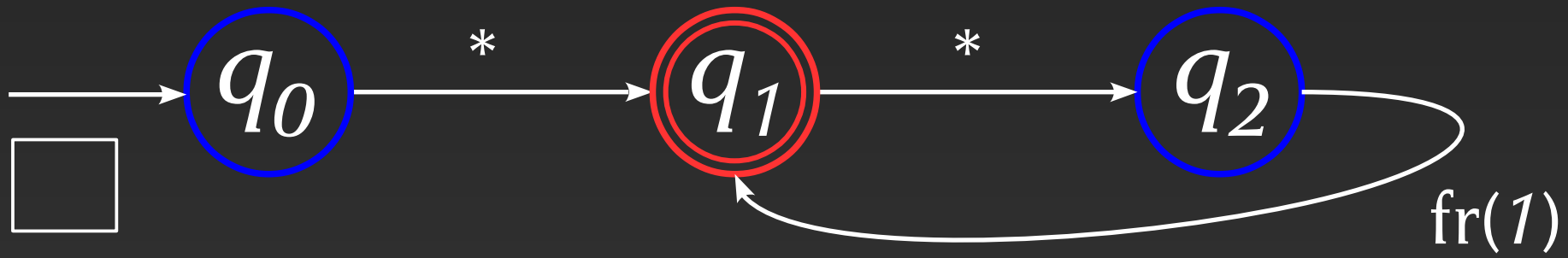

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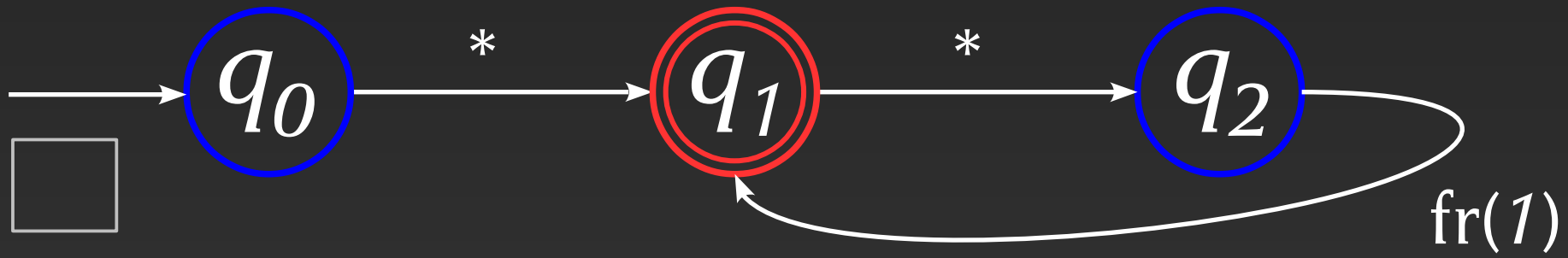

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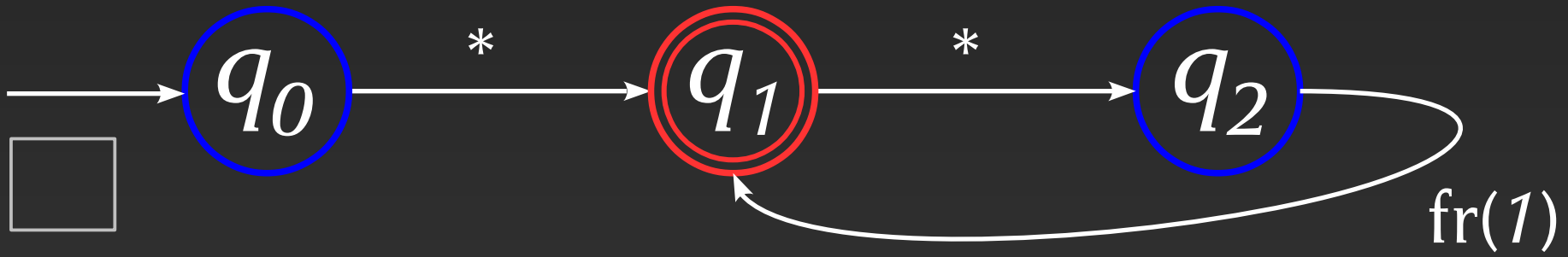

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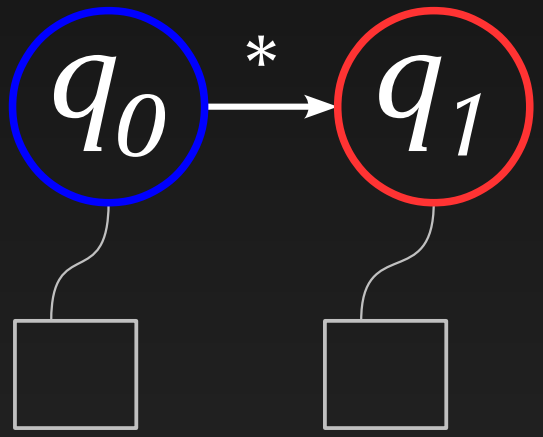


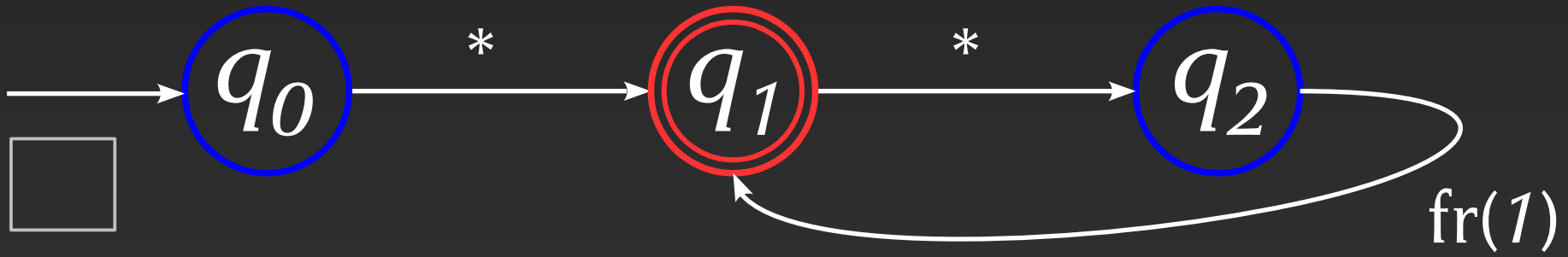

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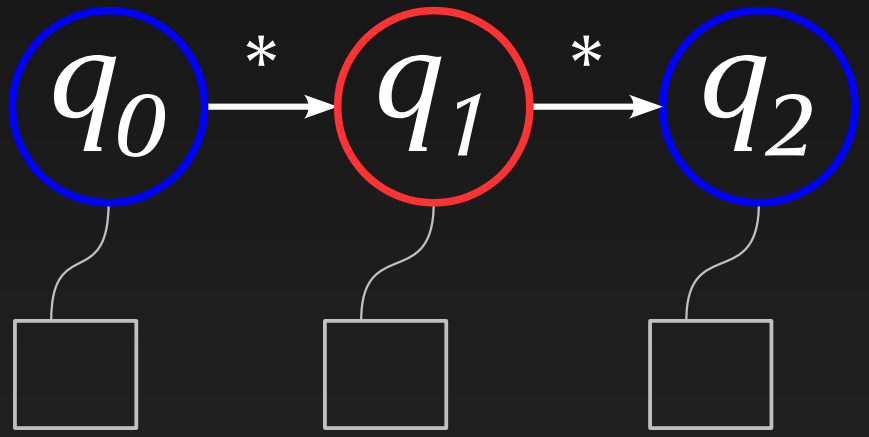


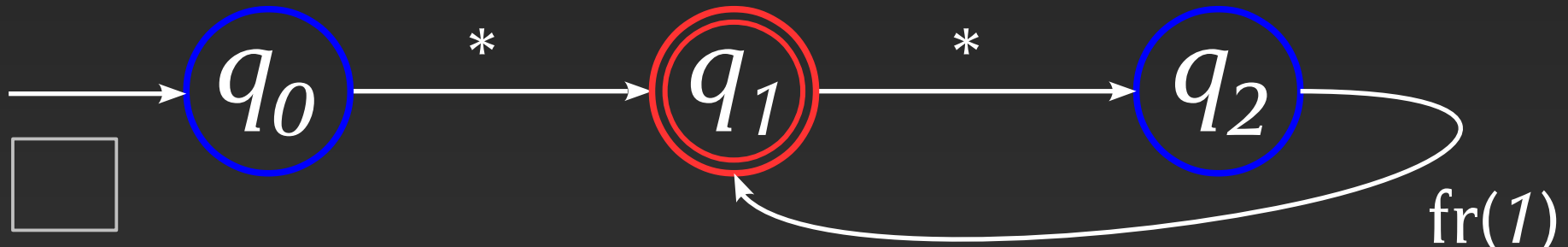

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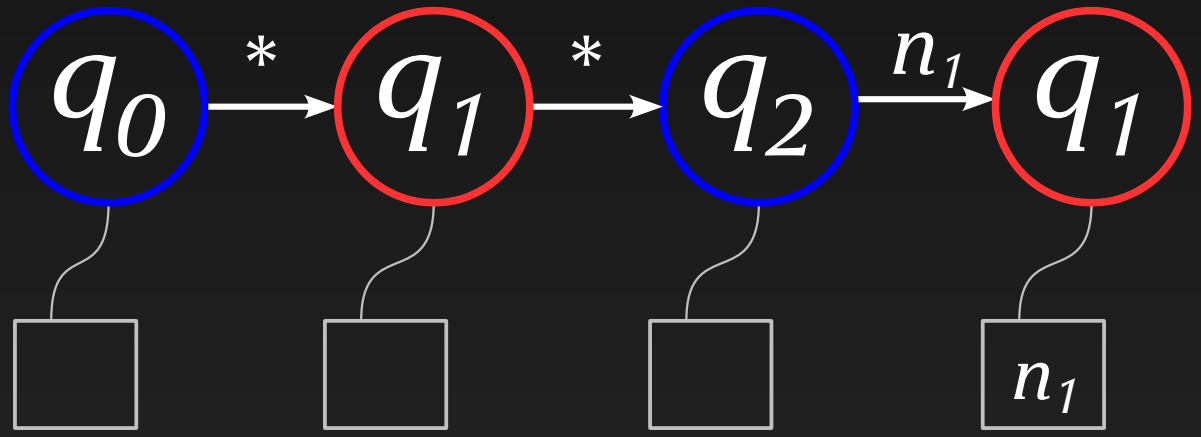
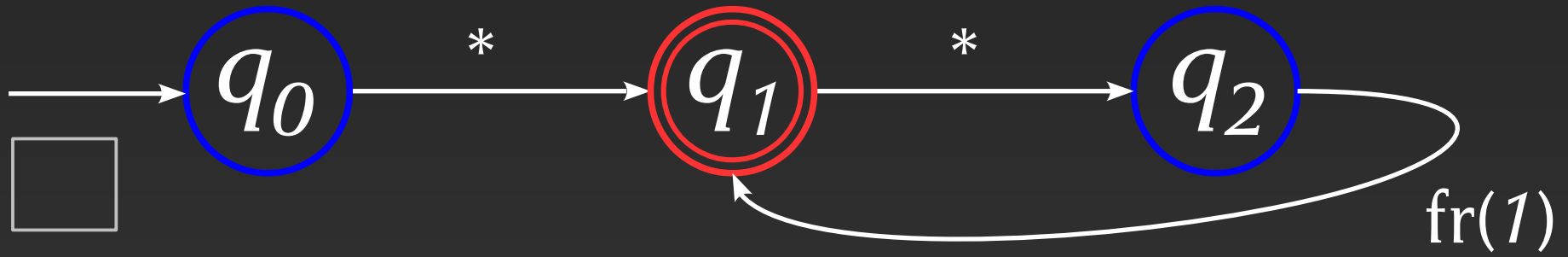
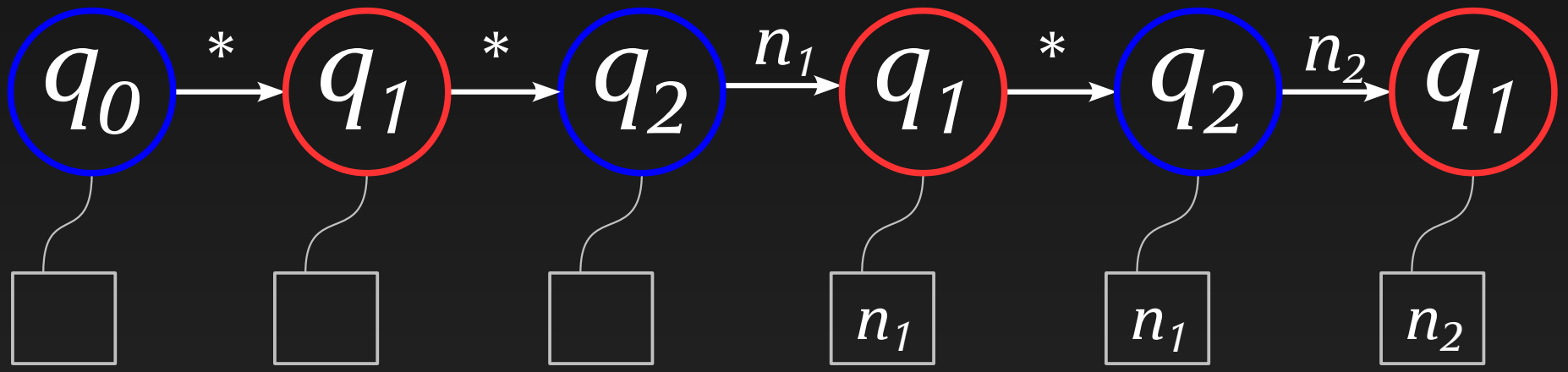
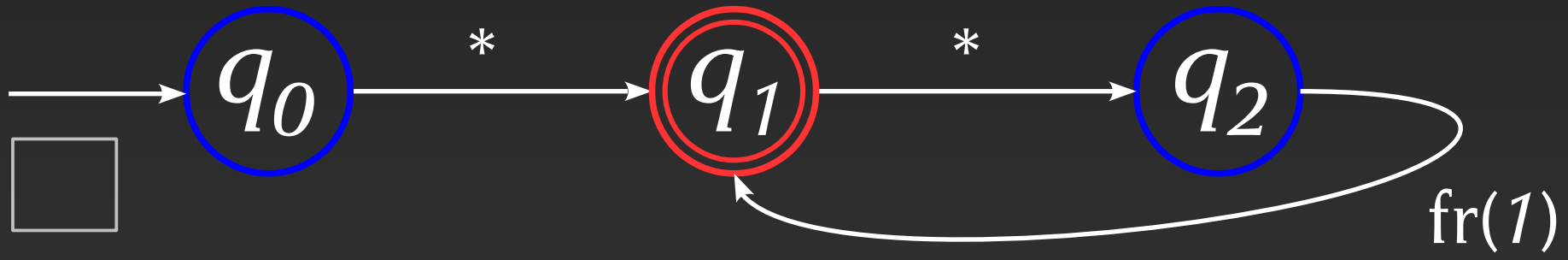


# Fresh-register automata



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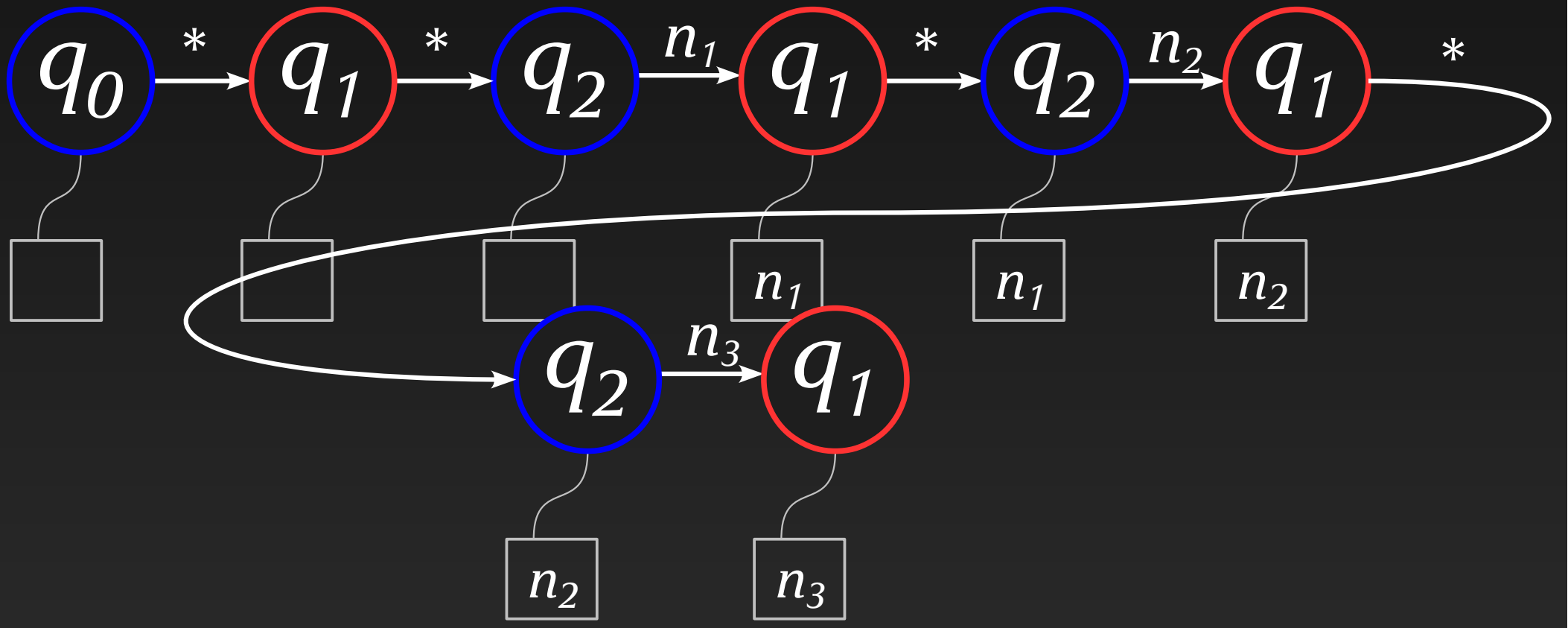


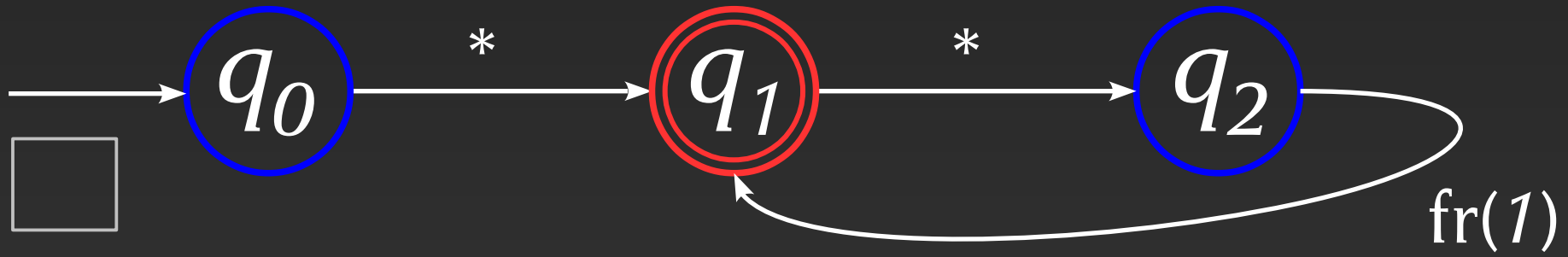

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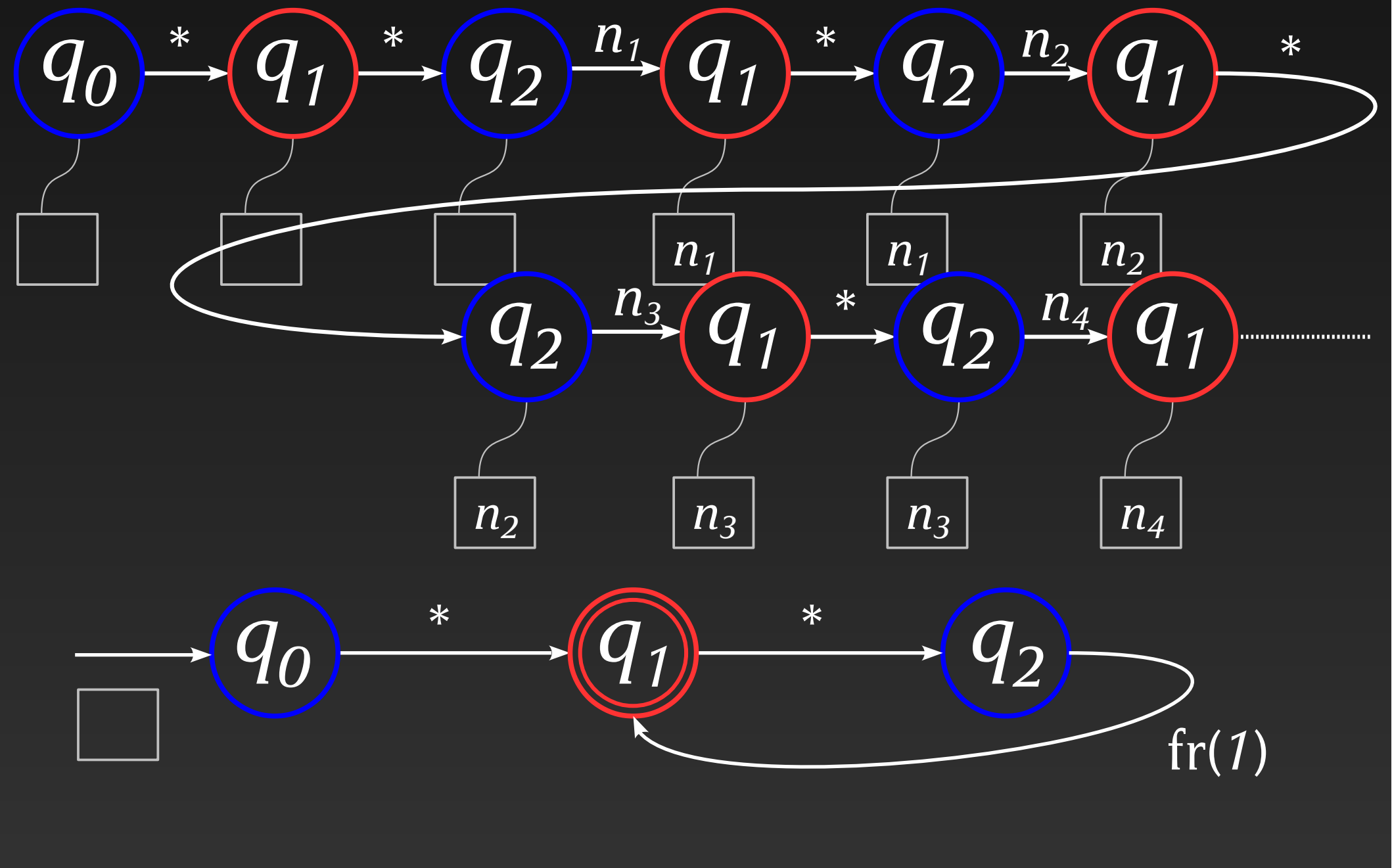
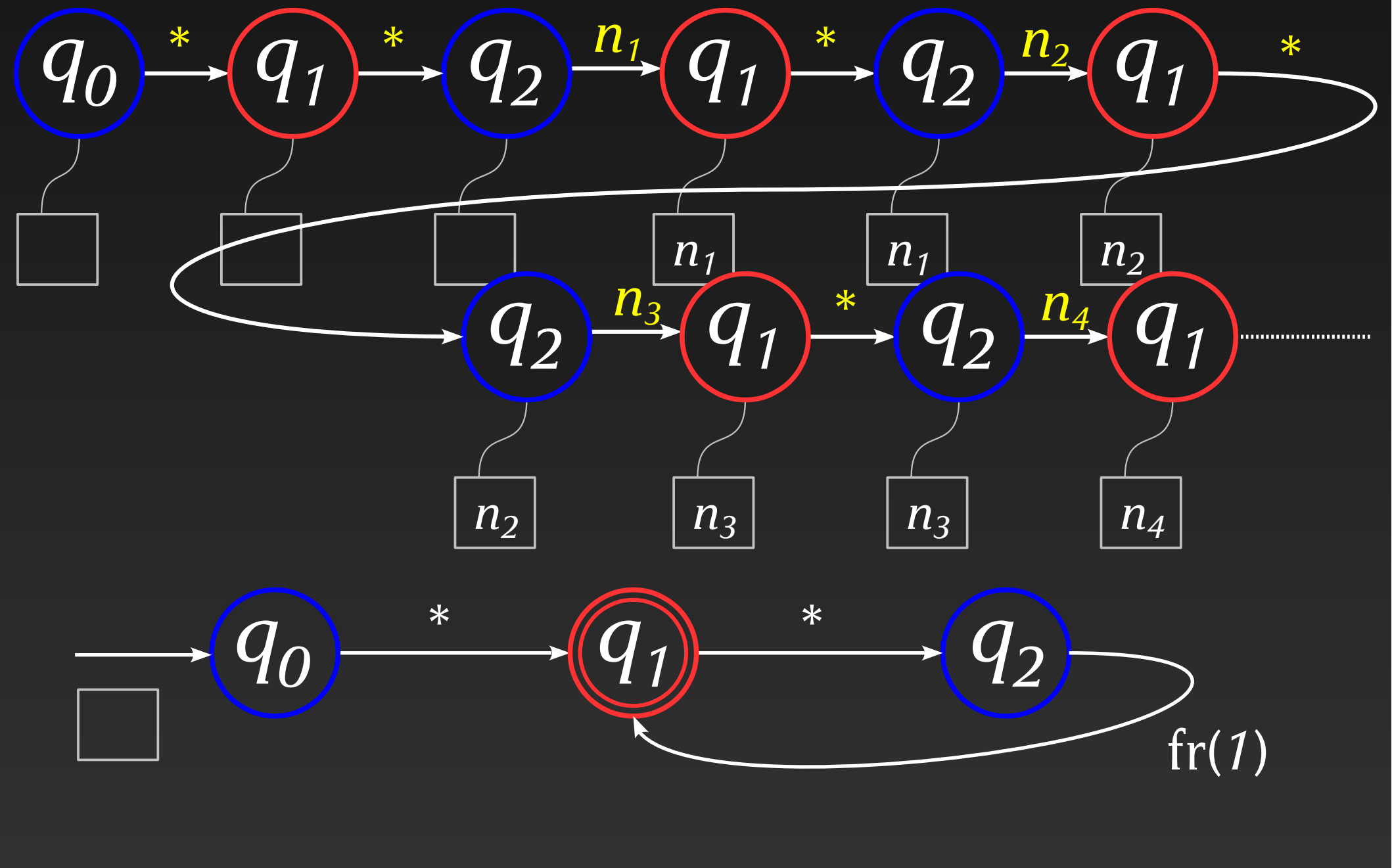
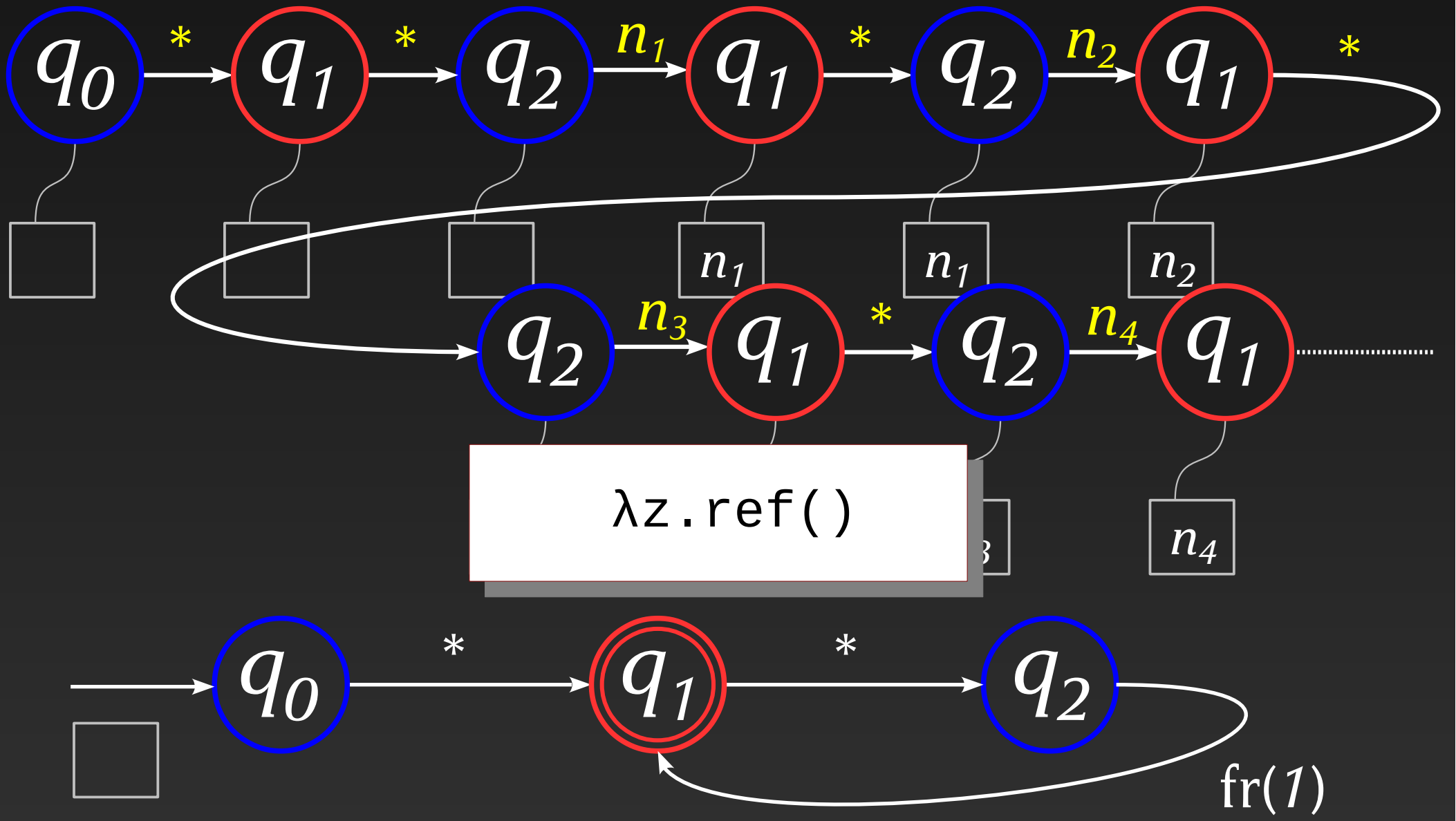


# Fresh-register automata



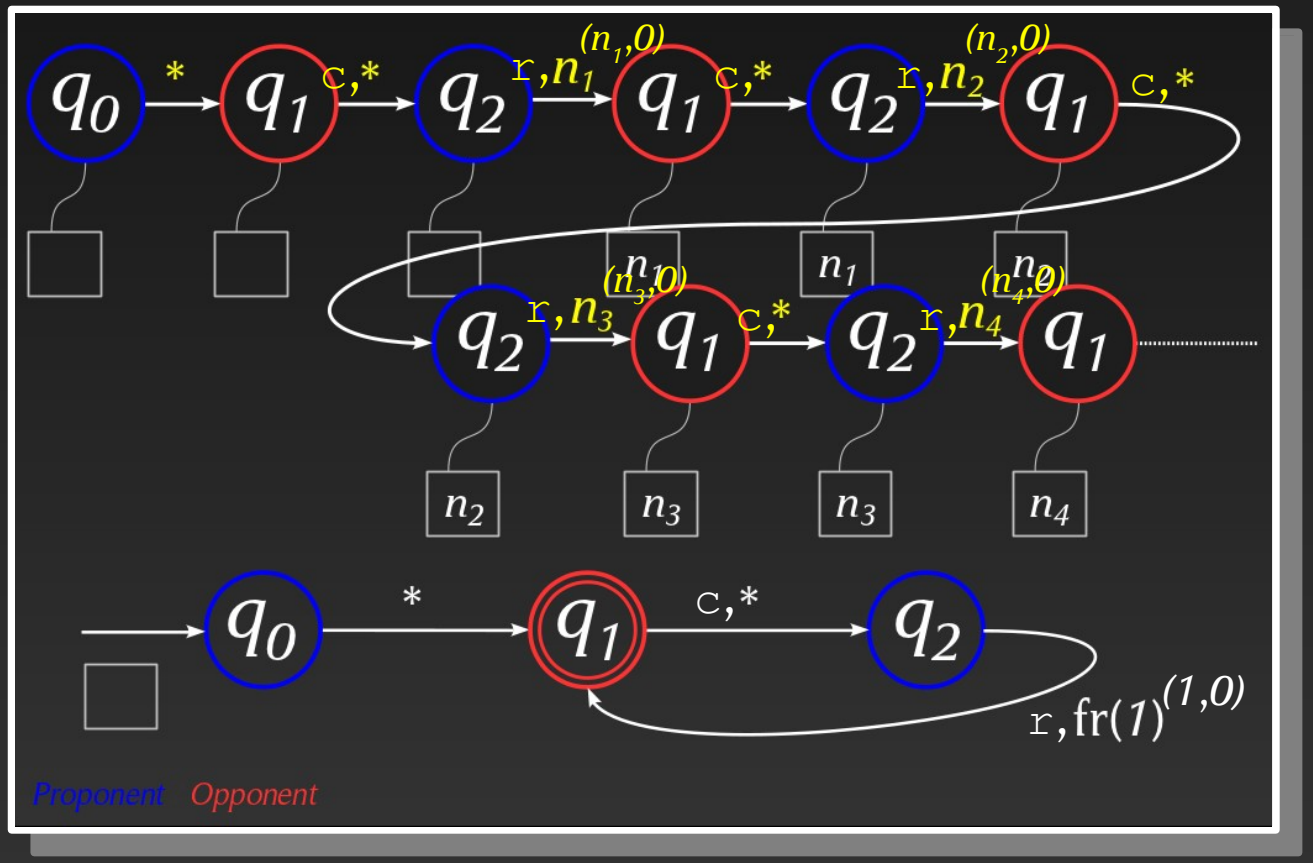
# Fresh-register automata




*Proponent* *Opponent*

# Automata

- Tz.11: Fresh-Register Automata
- Mu.Tz. 11': Algorithmic games for RedML*



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$$M \cong N \iff \mathcal{A}_M \sim \mathcal{A}_N$$

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# Games with names