

the Nominal Games Company

*presents*

**Andrzej Murawski    Steven Ramsay    and    Nikos Tzevelekos**

*in*

**Game Semantic Analysis of Equivalence  
in Interface Middleweight Java**

*an epic story*

MFPS, Nijmegen, June 2015

# what this talk is about

We apply game semantics in program verification  
→ characterising equivalence of Java programs

We work in an open fragment of MJ by use of interfaces

- open programs → need for games
- Java programs → simpler games

Full type-based characterisation:

- queue machine encodings (negative cases)
- pushdown (register) automata (algorithms)

# Interface Middleweight Java (IMJ)

Object calculus based on MJ [Bierman, Parkinson, Pitts]

- Objects, inheritance, casting, **interfaces**

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*Types*       $\theta ::= \text{void} \mid \text{int} \mid \mathcal{I}$

*Interface definitions*

$\Theta ::= \emptyset \mid (f : \theta), \Theta \mid (m : \bar{\theta} \rightarrow \theta), \Theta$

*Interface tables*

$\Delta ::= \emptyset \mid (\mathcal{I} : \Theta), \Delta \mid (\mathcal{I} \langle \mathcal{I} \rangle : \Theta), \Delta$

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interface ident.

field identifier

method identif.

Object calculus based on MJ [Bierman, Parkinson, Pitts]

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# Interface Middleweight Java (IMJ)

## *Terms*

$M ::=$  skip |  $a$  | null |  $x$  |  $i$  |  $M \oplus M$  | if  $M M M$   
| let  $x = M$  in  $M$  |  $M = M$  |  $(\mathcal{I})M$  | while  $M M$   
| new( $x : \mathcal{I}; \mathcal{M}$ ) |  $M.f$  |  $M.f := M$  |  $M.m(\overline{M})$

*Method implementations*      $\mathcal{M} ::= \emptyset$  |  $(m : \lambda \overline{x}. M), \mathcal{M}$

# Interface Middleweight Java (IMJ)

## Terms

$M ::= \text{skip} \mid a \mid \text{null} \mid x \mid i \mid M \oplus M \mid \text{if } M M M$   
 $\mid \text{let } x = M \text{ in } M \mid M = M \mid (\mathcal{I})M \mid \text{while } M M$   
 $\mid \text{new}(x : \mathcal{I}; \mathcal{M}) \mid M.f \mid M.f := M \mid M.m(\overline{M})$

*Method implementations*      $\mathcal{M} ::= \emptyset \mid (m : \lambda \vec{x}. M), \mathcal{M}$

$$\frac{\Delta | \Gamma \vdash M : \text{int} \quad \Delta | \Gamma \vdash M', M'' : \theta}{\Delta | \Gamma \vdash \text{if } M \text{ then } M' \text{ else } M'' : \theta} \quad \frac{\bigwedge_{i=1}^n (\Delta | \Gamma \uplus \{\vec{x}_i : \vec{\theta}_i\} \vdash M_i : \theta_i)}{\Delta | \Gamma \vdash \mathcal{M} : \{m_i : \vec{\theta}_i \rightarrow \theta_i \mid 1 \leq i \leq n\}}$$

$$\frac{\Delta | \Gamma \vdash M, M' : I}{\Delta | \Gamma \vdash M = M' : \text{int}} \quad \frac{\Delta | \Gamma \vdash M : \text{void} \quad \Delta | \Gamma \vdash M' : \theta}{\Delta | \Gamma \vdash M; M' : \theta} \quad \frac{\Delta | \Gamma \vdash M : I \quad \Delta | \Gamma \vdash M' : \theta}{\Delta | \Gamma \vdash M.f := M' : \text{void}} \quad \Delta(I).f = \theta$$

$$\frac{\Delta | \Gamma \vdash M : I}{\Delta | \Gamma \vdash M.f : \theta} \quad \Delta(I).f = \theta \quad \frac{\Delta | \Gamma \vdash M : I'}{\Delta | \Gamma \vdash (I)M : I} \quad \Delta \vdash I \leq I' \quad \forall I' \leq I \quad \frac{\Delta | \Gamma, x : I \vdash \mathcal{M} : \Theta}{\Delta | \Gamma \vdash \text{new}\{x : I; \mathcal{M}\} : I} \quad \Delta(I) \upharpoonright \text{Meths} = \Theta$$

$$\frac{\Delta | \Gamma \vdash M : I \quad \bigwedge_{i=1}^n (\Delta | \Gamma \vdash M_i : \theta_i)}{\Delta | \Gamma \vdash M.m(M_1, \dots, M_n) : \theta} \quad \Delta(I).m = \vec{\theta} \rightarrow \theta \quad \frac{\Delta | \Gamma \vdash M : \theta' \quad \Delta | \Gamma, x : \theta' \vdash M' : \theta}{\Delta | \Gamma \vdash \text{let } x = M \text{ in } M' : \theta}$$



# IMJ: operational semantics

$$S, M \longrightarrow S', M'$$

$S$  stores object names  
+ their types and values

Obj : set of obj. names

$$S, \text{let } x = v \text{ in } M \longrightarrow S, M[v/x]$$

$$S, a = a' \longrightarrow S, 0/1 \quad a, a' \in \text{Obj}$$

$$S, (\mathcal{I})a \longrightarrow S, a \quad \text{if } S(a) = \mathcal{I}' \leq \mathcal{I}$$

$$S, \text{new}(x:\mathcal{I}; M) \longrightarrow S \uplus \{(a, \mathcal{I}, (V_{\mathcal{I}}, M[a/x]))\}, a$$

$V_{\mathcal{I}}$ : default field values

# Program equivalence – decidability

$$M \cong M'$$

*same observable behaviour in every context*

for all closing contexts  $C$ :

$$(C[M], \emptyset) \rightarrow^* (\text{skip}, S) \iff (C[M'], \emptyset) \rightarrow^* (\text{skip}, S')$$

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- we consider a finite-types restriction  $\text{IMJ}_{fin}$
- termination is undecidable already in  $\text{IMJ}_{fin}$  termination → non-equivalence
- we use game semantics to **type-characterise** the fragment of  $\text{IMJ}_{fin}$  that is **decidable for equivalence**

# Termination

$M$  terminates if:  
 $(M, \phi) \rightarrow^* (\text{skip}, \mathcal{S})$

Two distinct sources of undecidability:

I. recursive methods

$\text{new}(x : \mathcal{I}; \text{main}: \lambda y. \{ \dots \mathbf{x.main}(\dots) \dots \} )$

II. objects with methods can be stored

let  $a = \text{new}(x : \mathcal{I}; \text{main}: \lambda y. \{ \dots \} )$  in  
let  $b = \text{new}(x' : \mathcal{I}'; )$  in  $\mathbf{b.var} := a$

**Theorem:** Termination is undecidable for  $\text{IMJ}_{fin}$  terms that may feature I or II.

Conversely, given an  $\text{IMJ}_{fin}$  term  $M$  that does not feature I or II, we can always decide whether it terminates.

# Games for program equivalence

We focus on:  $\text{IMJ}_{fin} - \{\text{I,II}\}$

and employ the fully abstract game semantics for IMJ:

$$M \cong M' \Leftrightarrow \llbracket M \rrbracket = \llbracket M' \rrbracket$$

- for negative results, given a queue machine  $Q$ , devise terms  $M, M'$ :

$$Q \uparrow \Leftrightarrow \llbracket M \rrbracket = \llbracket M' \rrbracket$$

- for decidability, given terms  $M, M'$ , devise automata  $A, A'$ :

$$\llbracket M \rrbracket = \llbracket M' \rrbracket \Leftrightarrow A \sim A'$$

# Nominal game semantics

- Computation is a 2-player game between:
  - *Opponent* (the environment), aka  $O$
  - *Proponent* (the program), aka  $P$
- Programs = *strategies* for  $P$
- *Categories* of games
- models cast in **nominal sets**
- games played using **moves-with-stores**
- special conditions for name **privacy** and **propagation**

[ Gabbay & Pitts '02 ]

# Arena games

free variables

program

output type

$$x_1:\theta_1, \dots, x_n:\theta_n \vdash M:\theta$$

input types

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input types

$$[[M]] : [[\theta_1, \dots, \theta_n]] \longrightarrow [[\theta]]$$

strategy

arenas



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arenas

moves

$$[[\text{void}]] = \{ * \}$$

$$[[\text{int}]] = \{ 0, 1, -1, \dots \}$$

$$[[\mathcal{I}]] = \{ a, b, \dots \}$$

$a, b, \dots \in \mathcal{N}_{\mathcal{I}}$

$\mathcal{N}_{\mathcal{I}}$  a set of *names*

# Arenas of moves

$$[[M]] : [[\theta_1, \dots, \theta_n]] \longrightarrow [[\theta]]$$

That's all!

arenas

moves

$$[[\text{void}]] = \{ * \}$$

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# Games for IMJ programs

- use names and moves-with-store:

$$\Sigma = \{ (a, ), (b, (f_1, 4), (f_2, c)), \dots \}$$

- not really HO programs: no functions, only methods
  - *there is no need for “pointers” between moves*

effectively, moves of the form  $m^\Sigma$  where  $m$  is either:

- an arena move
- an object method call/return

# Examples

$x : \text{Var}, f : \text{Fun} \vdash f.\text{val} (x.\text{val}) + 1 : \text{int}$

$\text{Var} : \{ \text{val} : \text{int} \} \quad \text{Fun} : \{ \text{val} : \text{int} \rightarrow \text{int} \}$

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$\mathcal{O} : (\mathbf{x}, \mathbf{f})^{(x.\text{val} = 4)}$

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*O* :  $\text{ret } f.\text{val}(50)^{(x.\text{val} = 73)}$



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$O : (x, f)^{(x.\text{val} = 4)}$

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$O : \text{ret } f.\text{val}(50)^{(x.\text{val} = 73)}$

$P : 51^{(x.\text{val} = 73)}$

$\llbracket x : \text{Var}, f : \text{Fun} \vdash f.\text{val} (x.\text{val}) + 1 : \text{int} \rrbracket$

$= \{ (x, f)^{(x.\text{val} = i)} \text{call } f.\text{val}(i)^{(x.\text{val} = i)} \text{ret } f.\text{val}(j)^{(x.\text{val} = i')} (j+1)^{(x.\text{val} = i')} \}$

# IMJ example: game semantics

$M_1$ : let  $u = \text{new}(\text{Var}_{\text{Emp}})$  in  
 $\text{new}(M_1)$ : Cell

$M_1$ : get:  $\lambda(). u.\text{val}$ ,  
 set:  $\lambda y. u.\text{val} := y$

$\Delta =$  Empty:  $\emptyset$ ,  
 Cell: (get: void  $\rightarrow$  Empty,  
 set: Empty  $\rightarrow$  void),  
 $\text{Var}_{\text{Emp}}$ : (val: Empty),  
 $\text{Var}_{\text{Int}}$ : (val: int)

$\llbracket M_1 \rrbracket =$   $\begin{matrix} \color{red}{O} & \color{blue}{P} & & \color{gray}{O} & & \color{gray}{P} \\ * & n^{\Sigma_0} & (\text{call } n.\text{get}()^{\Sigma_0} \text{ ret } n.\text{get}(\text{nil})^{\Sigma_0})^* & & & \\ & & \text{call } n.\text{set}(n_1)^{\Sigma_1} \text{ ret } n.\text{set}()^{\Sigma_1} & & & \\ & & (\text{call } n.\text{get}()^{\Sigma_1} \text{ ret } n.\text{get}(n_1)^{\Sigma_1})^* & & & \\ & & \text{call } n.\text{set}(n_2)^{\Sigma_2} \text{ ret } n.\text{set}()^{\Sigma_2} \dots & & & \end{matrix}$

$\Sigma_i = \{ (n : \text{Cell}, ) \} \cup \{ (n_j : \text{Empty}, ) \mid 1 \leq j \leq i \}$

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**O P O P**

$\llbracket M_1 \rrbracket = * n^{\Sigma_0} (\text{call } n.\text{get}()^{\Sigma_0} \text{ret } n.\text{get}(\text{nul})^{\Sigma_0} )^*$

call  $n.\text{set}(n_1)^{\Sigma_1}$  ret  $n.\text{set}()^{\Sigma_1}$

(call  $n.\text{get}()^{\Sigma_1}$  ret  $n.\text{get}(n_1)^{\Sigma_1} )^*$

call  $n.\text{set}(n_2)^{\Sigma_2}$  ret  $n.\text{set}()^{\Sigma_2} \dots$

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# IMJ example: game semantics

$M_2 :$ let $b = \text{new}(\text{Var}_{\text{Int}})$ in let $u_1 = \text{new}(\text{Var}_{\text{Emp}})$ in let $u_2 = \text{new}(\text{Var}_{\text{Emp}})$ in $\text{new}(M_2) : \text{Cell}$	$M_2 :$ get: $\lambda().$ if $b.\text{val}$ then $b.\text{val} := 0; u_1.\text{val}$ else $b.\text{val} := 1; u_2.\text{val},$ set: $\lambda y. u_1.\text{val} := y;$ $u_2.\text{val} := y$
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$$\begin{aligned}
 \llbracket M_1 \rrbracket = & \quad \color{red}{O} \quad \color{blue}{P} \quad \quad \quad \color{red}{O} \quad \quad \quad \color{blue}{P} \\
 & * n^{\Sigma_0} \left( \text{call } n.\text{get}()^{\Sigma_0} \text{ ret } n.\text{get}(\text{nul})^{\Sigma_0} \right)^* \\
 & \quad \text{call } n.\text{set}(n_1)^{\Sigma_1} \text{ ret } n.\text{set}()^{\Sigma_1} \\
 & \quad \left( \text{call } n.\text{get}()^{\Sigma_1} \text{ ret } n.\text{get}(n_1)^{\Sigma_1} \right)^* \\
 & \quad \text{call } n.\text{set}(n_2)^{\Sigma_2} \text{ ret } n.\text{set}()^{\Sigma_2} \dots = \llbracket M_2 \rrbracket
 \end{aligned}$$

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# Undecidability

One source of undecidability:

$P$  passes objects of type  $\mathcal{I}$ , and  $\mathcal{I}$  contains methods

$\mathcal{I} : (\text{step: void} \rightarrow \text{void})$   
 $N : (\text{val: int})$

we can encode  
computations of  
queue machines!



step



state

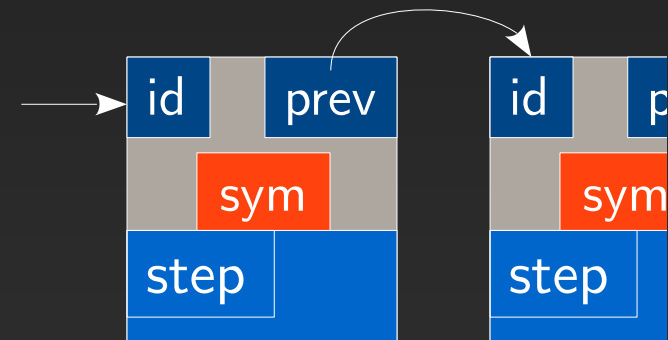
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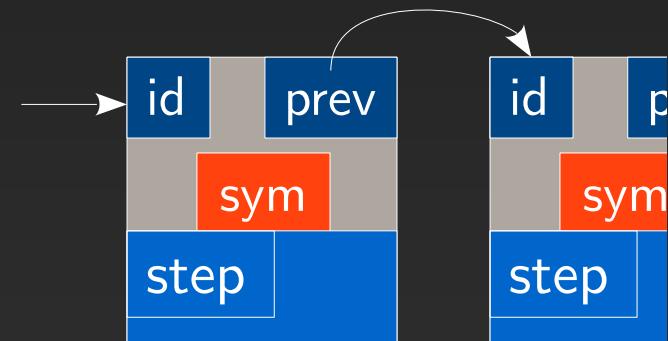
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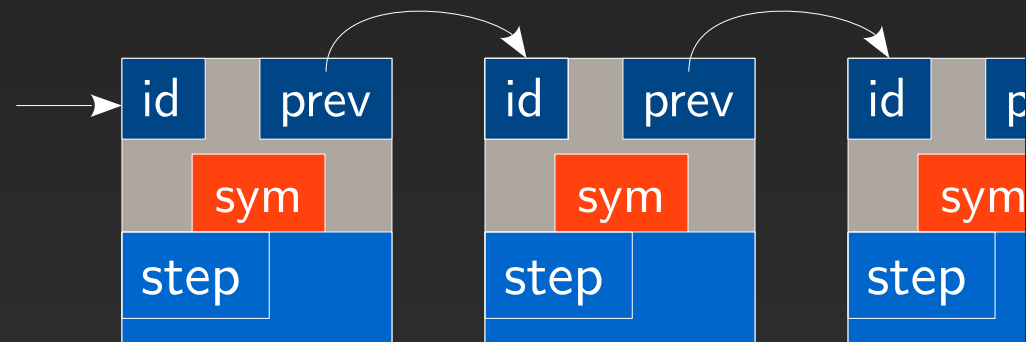
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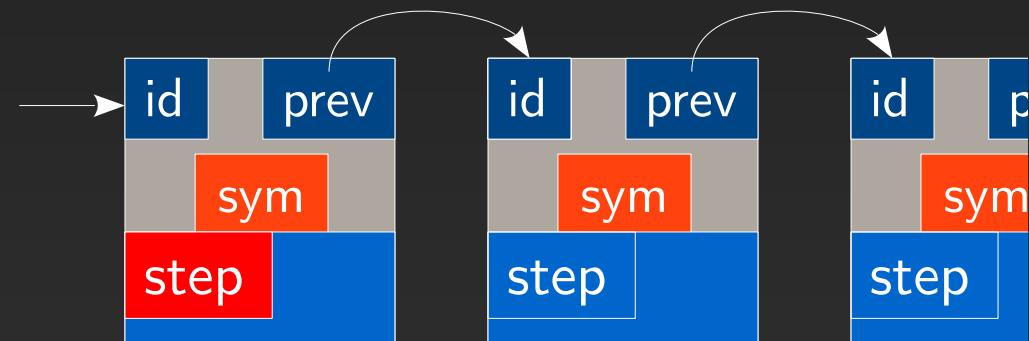
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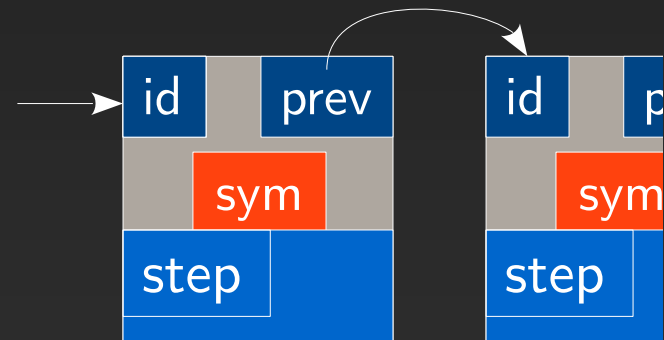
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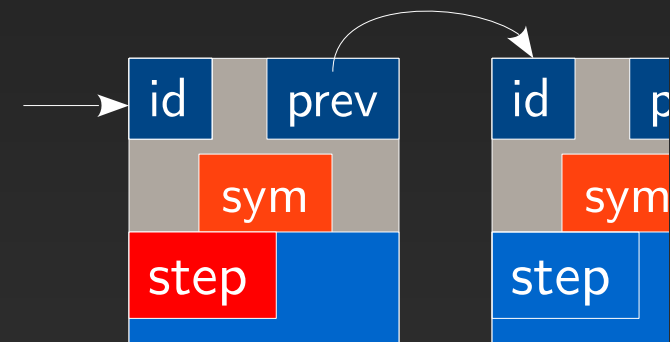
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 $N : (\text{val}: \text{int})$

we can encode  
computations of  
queue machines!



# Undecidability

One source of undecidability:

$P$  passes objects of type  $\mathcal{I}$ , and  $\mathcal{I}$  contains methods

$\mathcal{I} : (\text{step}: \text{void} \rightarrow \text{void})$   
 $N : (\text{val}: \text{int})$

we can encode  
computations of  
queue machines!

step

state

id p  
sym  
step



# Undecidability

$$x : L \vdash M : R$$

Three cases for undecidability:

- $L = (\dots, m : \mathcal{I} \rightarrow \theta)$
- $R = (\dots, m : \theta \rightarrow \mathcal{I})$
- $R = (\dots, m : \mathcal{I}' \rightarrow \theta)$  where  $\mathcal{I}' = (\dots, m : \mathcal{I} \rightarrow \theta)$

and  $\mathcal{I}$  contains methods (i.e. is higher-order)

# Decidable types

$$x : L \vdash M : R$$
$$G ::= \text{void} \mid \text{int} \mid (f : G)$$
$$L ::= \text{void} \mid \text{int} \mid (f : G, m : G \rightarrow L)$$
$$R ::= \text{void} \mid \text{int} \mid (f : G, m : L \rightarrow G)$$

# Games for program equivalence

We focus on:  $\text{IMJ}_{fin} - \{\text{I,II}\}$

and employ the fully abstract game semantics for IMJ:

$$M \cong M' \Leftrightarrow \llbracket M \rrbracket = \llbracket M' \rrbracket$$

- for negative results, given a queue machine  $Q$ , devise terms  $M, M'$ :

$$Q \uparrow \Leftrightarrow \llbracket M \rrbracket = \llbracket M' \rrbracket$$

- for decidability, given terms  $M, M'$ , devise automata  $A, A'$ :

$$\llbracket M \rrbracket = \llbracket M' \rrbracket \Leftrightarrow A \sim A'$$

# From games to automata

$$\begin{array}{c} M \cong M' \\ \Leftrightarrow \\ \llbracket M \rrbracket = \llbracket M' \rrbracket \end{array}$$

$M$  Programs



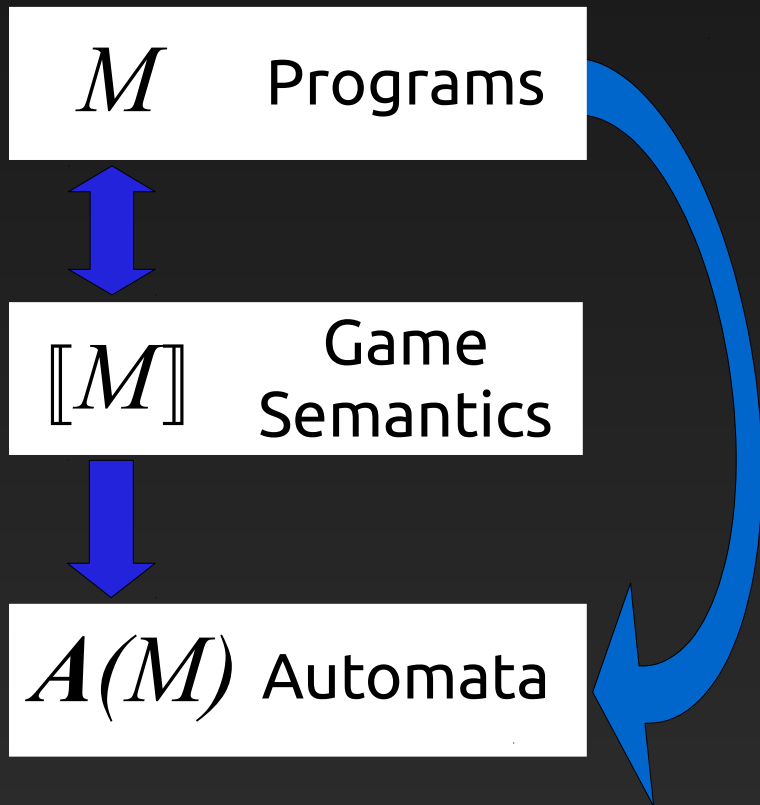
$\llbracket M \rrbracket$  Game Semantics



$A(M)$  Automata



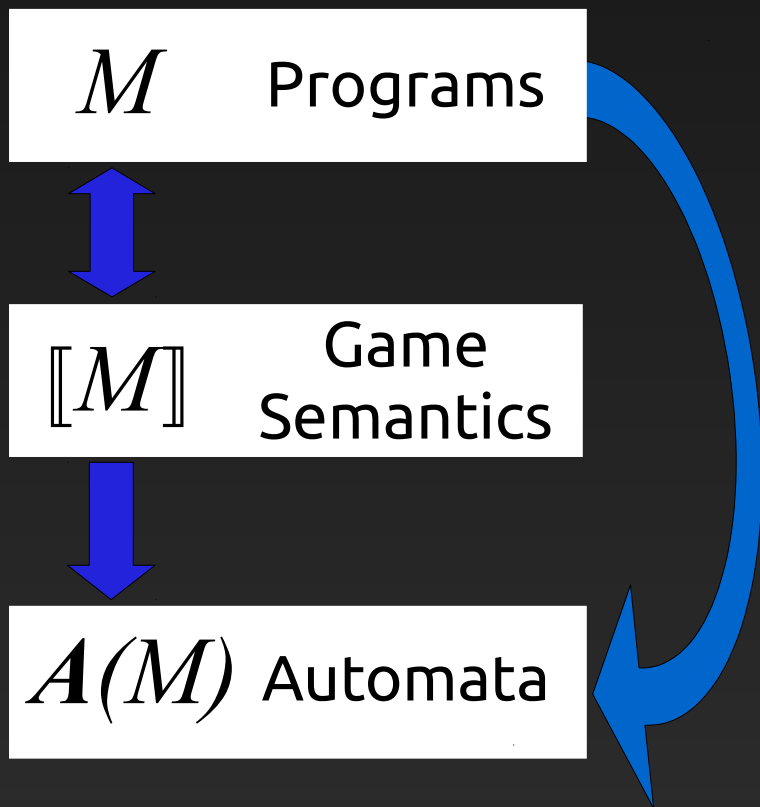
# From games to automata



Translation done in two steps:

- terms reduced to **canonical forms**
- canonical terms are **compositionally** transformed into automata

# From games to automata



Translation done in two steps:

- terms reduced to **canonical forms**
- canonical terms are **compositionally** transformed into automata

We work with pushdown automata:


- over **infinite alphabets**
- **visibly** pushdown & **deterministic**

$$M \cong M' \Leftrightarrow \llbracket M \rrbracket = \llbracket M' \rrbracket \Leftrightarrow A \sim A' \Leftrightarrow A \otimes A' = 0$$

# Coneqct

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 sjr coneqct

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## Wiki

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### coneqct / Home

## Coneqct: a contextual equivalence checking tool for Interface Middleweight Java

[Home](#) | [Downloads](#) | [Syntax](#) | [Examples](#)

### Requirements

The checker runs on the .NET platform ( $\geq 4.5$ ), and hence requires a recent implementation of the .NET Common Language Infrastructure (CLI) to be installed on your system.

- On Windows we recommend Microsoft's ".NET Framework", the latest stable version is 4.5.2: <https://www.microsoft.com/en-us/download/details.aspx?id=42643>.
- On Linux or Mac we recommend Xamarin's "Mono": <http://www.mono-project.com/download/>

### Installation

- Download the latest assemblies from [downloads](#). All the required assemblies are packaged together in a zip file named "coneqct-XXX.zip" where "XXX" denotes the revision number.
- Unzip to any convenient location, this creates a new directory "coneqct-XXX" in which resides the executable "coneqct.exe".
- To verify that all is well, on the command line navigate to the directory "coneqct-XXX" and run the command:
  - ".\coneqct.exe" on Windows or,
  - "mono ./coneqct.exe" on Linux or Mac. If the installation is working correctly, the usage message will be printed out to the terminal.

### Usage

On Windows, to check the equivalence of two IMJ terms defined in the file "terms.inp", run:

```
> .\coneqct.exe \path\to\terms.inp
```

On Linux or Mac you should prefix this command by "mono" (and use appropriate slashes):

```
> mono ./coneqct.exe /path/to/terms.inp
```

A number of example inputs are bundled with the installation. For example, after navigating to the root of the directory "coneqct-XXX", to verify the "extended types" equivalence adapted from Benton and Leperchey's "Relational Reasoning in a Nominal Semantics for Storage" on Mac, run:

```
> mono ./coneqct.exe inputs/inp2.imj
```

See [Syntax](#) for a detailed description of the syntax of the input file format.

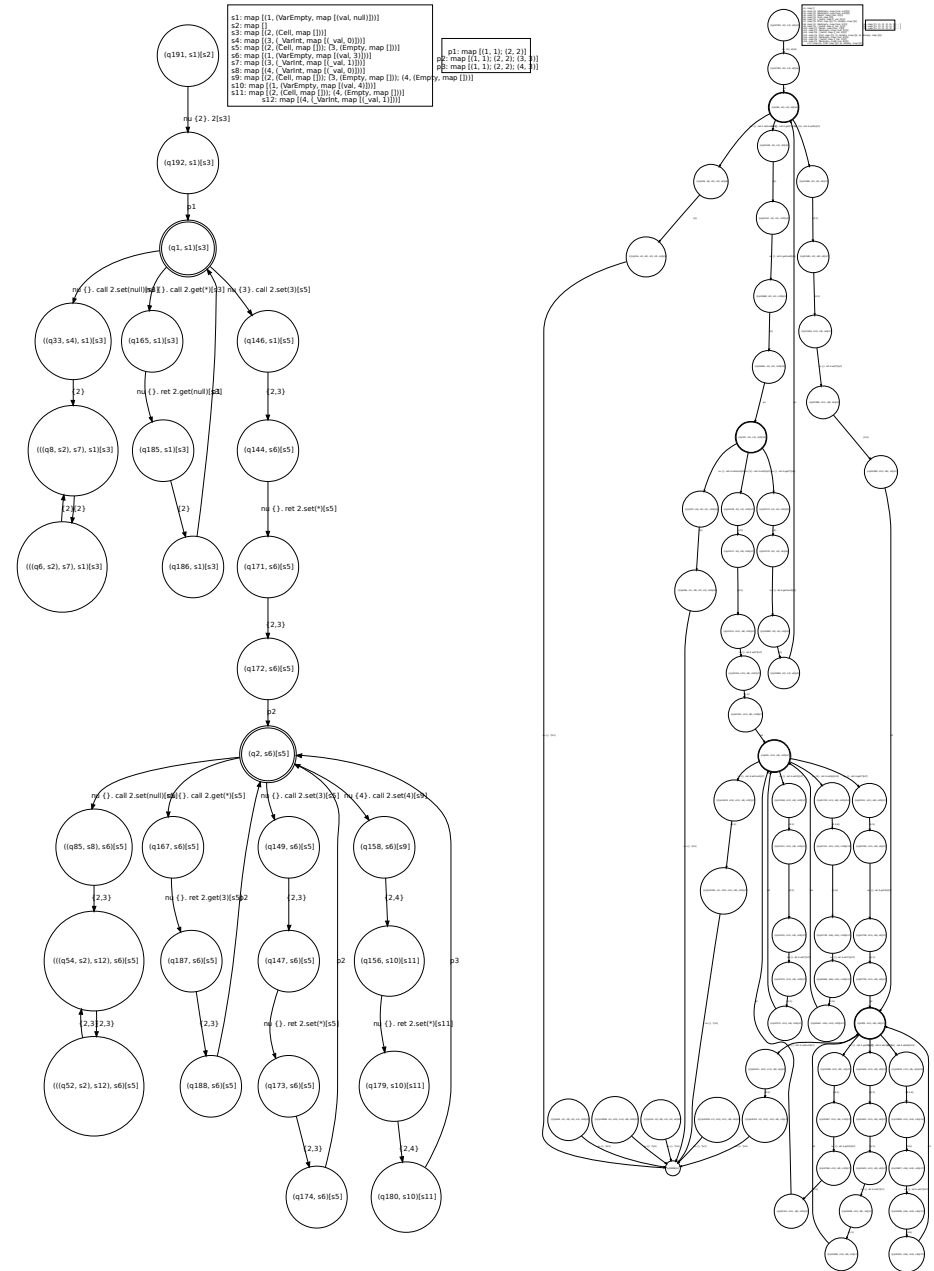
The tool can be configured using some command-line options:

- maxint <int>: set the value of maxint to <int>

# Coneqct

$M_1 \equiv \text{let } v = \text{new } \{ \_ : \text{Var}_{\text{Empty}} \} \text{ in}$   
 $\text{new } \{ \_ : \text{Cell};$   
 $\text{get} : \lambda \_ . v.\text{val},$   
 $\text{set} : \lambda y . \text{if } y = \text{null} \text{ then div else } v.\text{val} := y \}$

$M_2 \equiv \text{let } b = \text{new } \{ \_ : \text{Var}_{\text{int}} \} \text{ in}$   
 $\text{let } v = \text{new } \{ \_ : \text{Var}_{\text{Empty}} \} \text{ in}$   
 $\text{let } w = \text{new } \{ \_ : \text{Var}_{\text{Empty}} \} \text{ in}$   
 $\text{new } \{ \_ : \text{Cell};$   
 $\text{get} : \lambda \_ . \text{if } b.\text{val} = 1 \text{ then } (b.\text{val} := 0; v.\text{val})$   
 $\qquad \qquad \qquad \text{else } (b.\text{val} := 1; w.\text{val}),$   
 $\text{set} : \lambda y . \text{if } y = \text{null} \text{ then div}$   
 $\qquad \qquad \qquad \text{else } v.\text{val} := y; w.\text{val} := y \}$

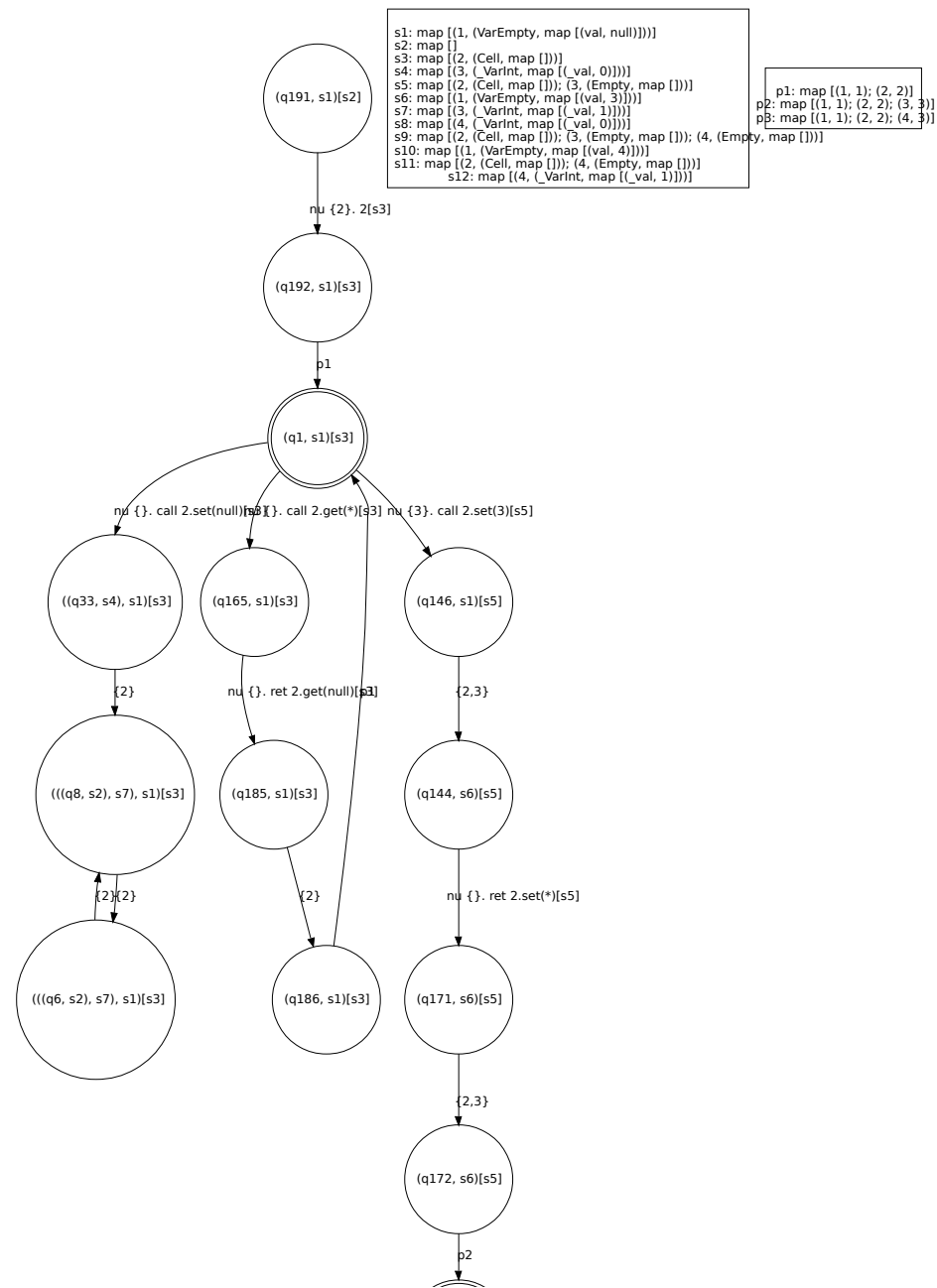




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# Wrapping up

Games for IMJ:

- are simple and **operational**
- used here for **characterising program equivalence**
- **automated** decision via FPDRA's → **Coneqct**

Further on:

- general model checking
- logics (over infinite alphabets)

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*the End*

thanks!