Games with names

Nikos Tzevelekos

Queen Mary, University of London

What this talk is about

Generation of new resources is a pervasive feature in computation (references, objects, channels, etc.)

We call these names

We present game semantics for computation with names

Computation with names

$\vdash$ `λx.ref(0)` : `com` $\to$ `intref`

```
let f = [_] in { f() == f() }
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Computation with names

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$\vdash$ `λx.ref(0)` : `com` $\rightarrow$ `intref`

`let f = [_] in { f() == f() }` $\longmapsto$ `false`

Example: Reduced ML

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+ integer references + CBV

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$$f : \mathsf{intref} \to \mathsf{int} \;\vdash\; \lambda y.\, \mathsf{let}\, x = \mathsf{ref}(0)\, \mathsf{in}\, f(x)$$
$$\cong \quad \mathsf{let}\, x = \mathsf{ref}(0)\, \mathsf{in}\, \lambda y.\, x := 0; f(x) \;\; : \mathsf{com} \to \mathsf{int}$$

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$$\cong \quad \mathsf{let}\ x = \mathsf{ref}(0)\ \mathsf{in}\ f(x);\, x \ : \mathsf{intref}$$

Two ways to model references

Reynolds

- *Idealized Algol (1978)*

References are *pairs:*

```
intref =
 (com → int)x(int → com)
```

$\longmapsto$ $(\mathbf{1} \to \mathbf{Z}) \times (\mathbf{Z} \to \mathbf{1})$

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- *nu-calculus (1993)*

References are *names:*

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⟼ **`N`** (names)

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References are *names:*

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$\longmapsto$ $\mathbf{N}$ (names)

- Notion of *resource (name):*
  - atomic values
  - infinitely many
  - comparable for equality

Bad variables

- Many pairs of ref type are *not* references
  - e.g. `mkvar(λx.3,λx.())`
- In Idealized Algol:
  - no notion of *reference equality test*
  - spurious non-equivalences:

    `x:=0;!x` vs. `x:=0;0`

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```

In call-by-name IA:

- Equivalence *not* affected by `mkvar`
- Approximation *is* affected

Both affected in:

- CBN IA + control
- CBN IA + non-det.
- call-by-value IA

Game Semantics

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- Computation is modelled as a 2-player game between:
  - *Opponent* (the environment)
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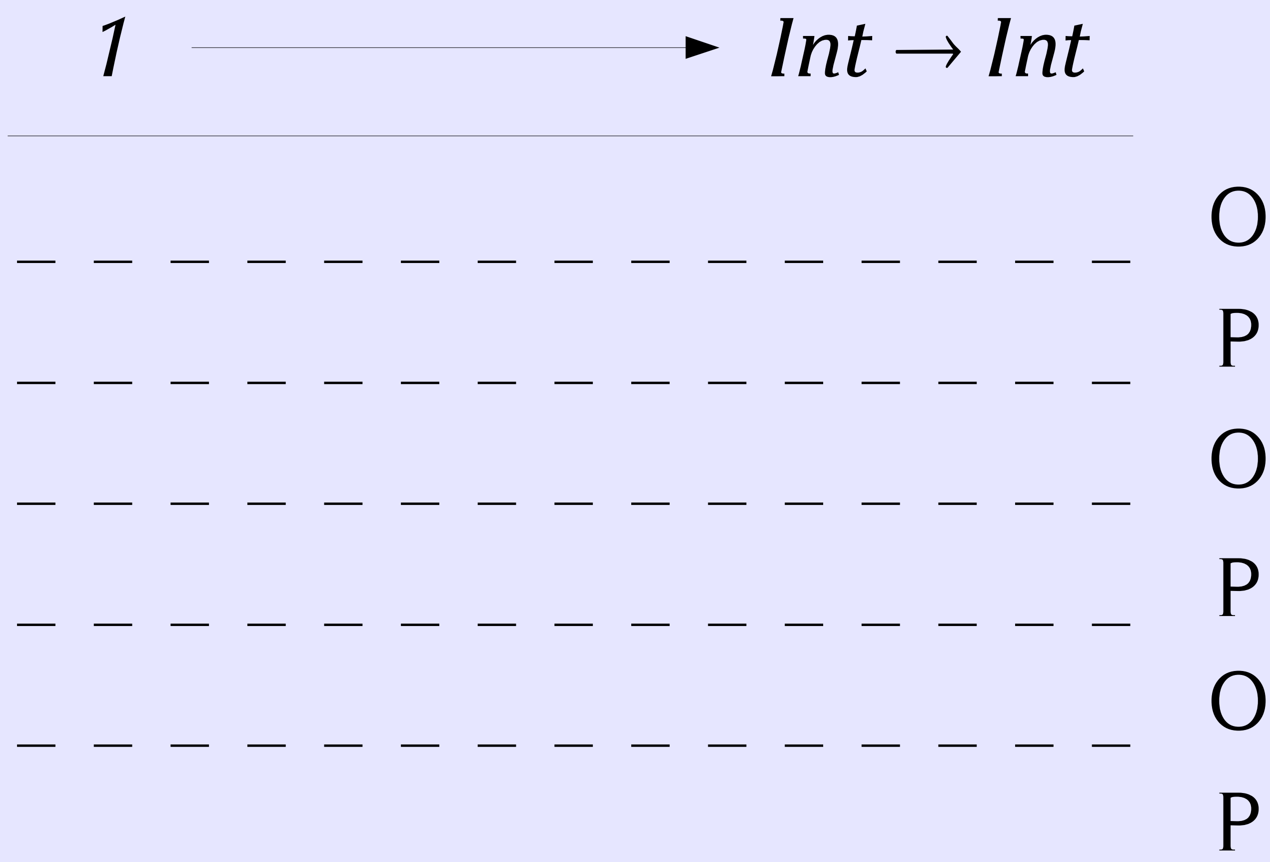
Example

$\vdash$ `λx.x+1 : int → int`



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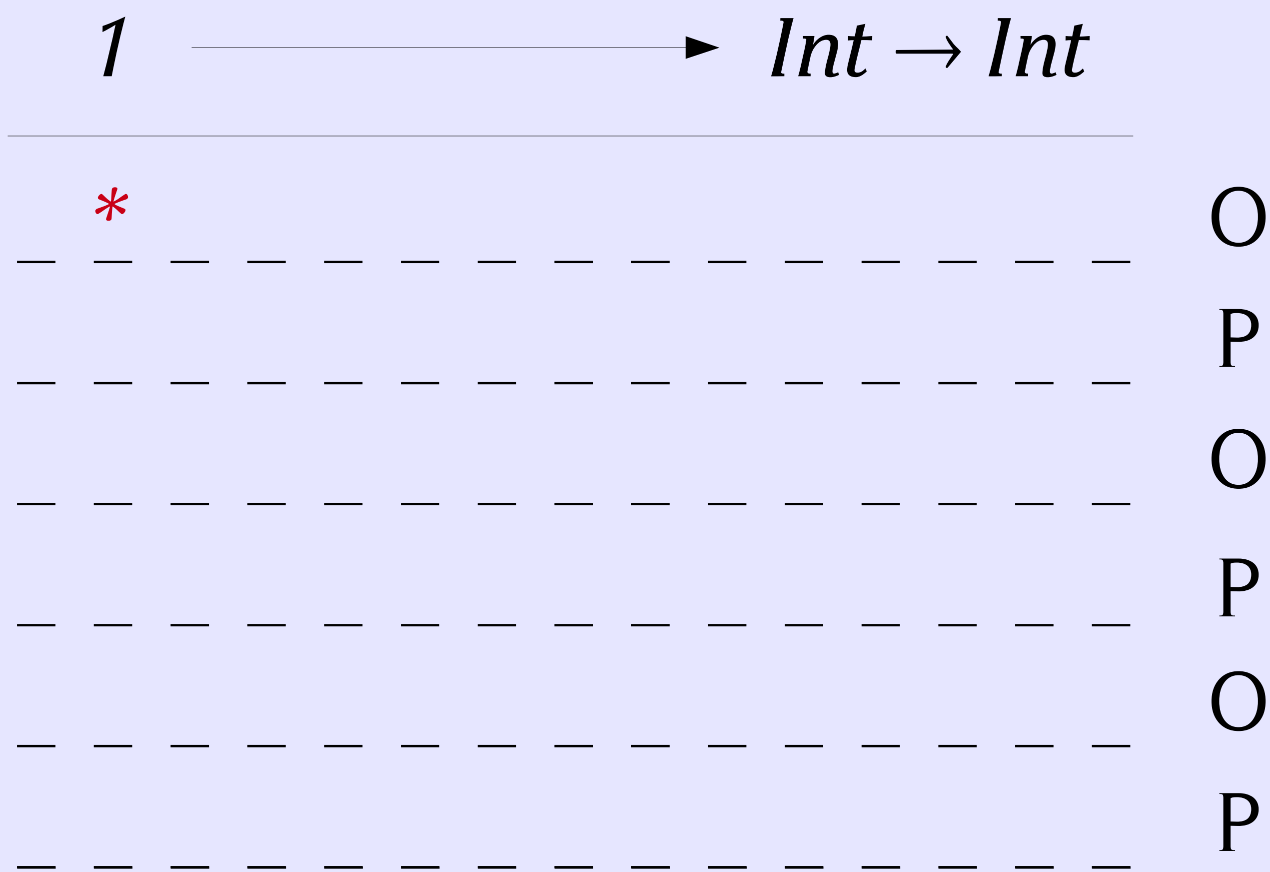

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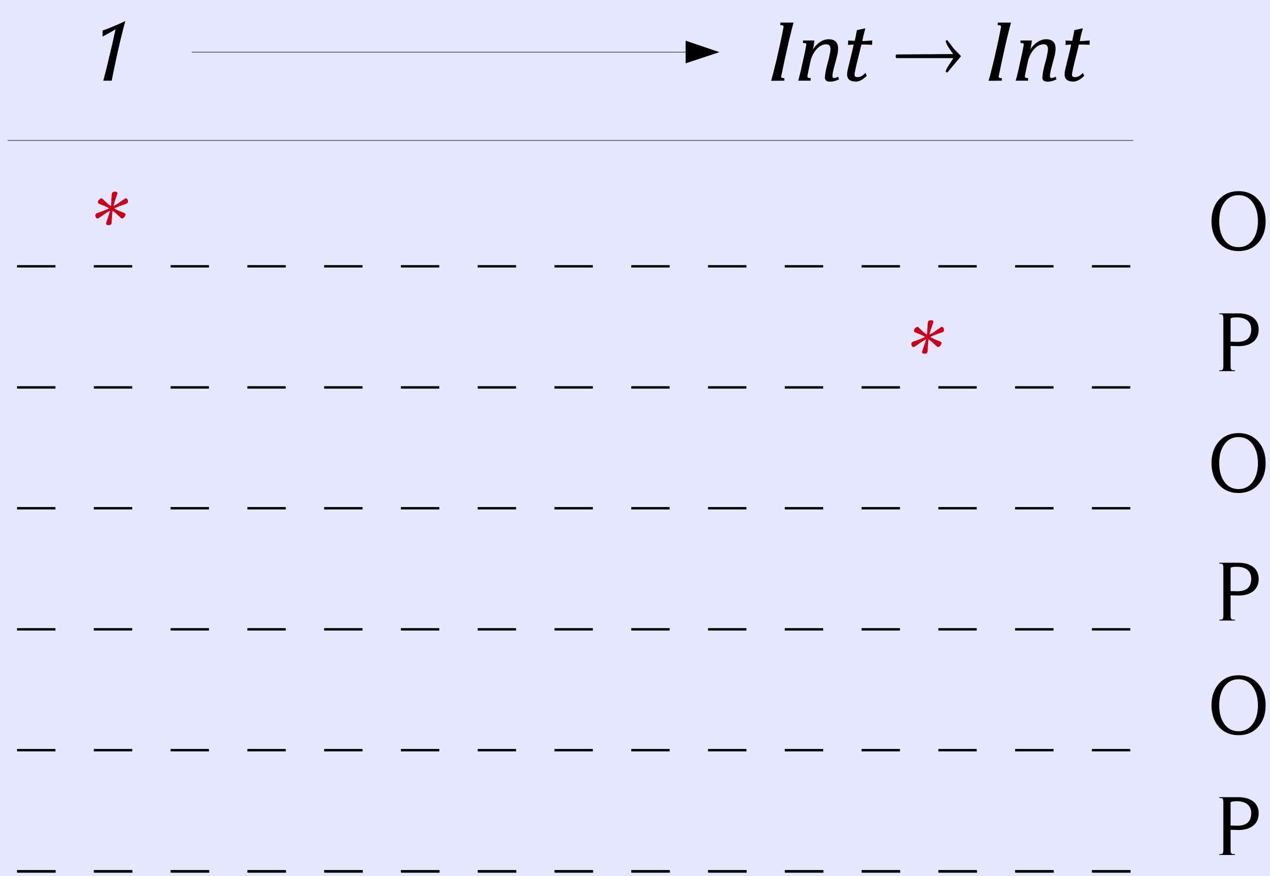

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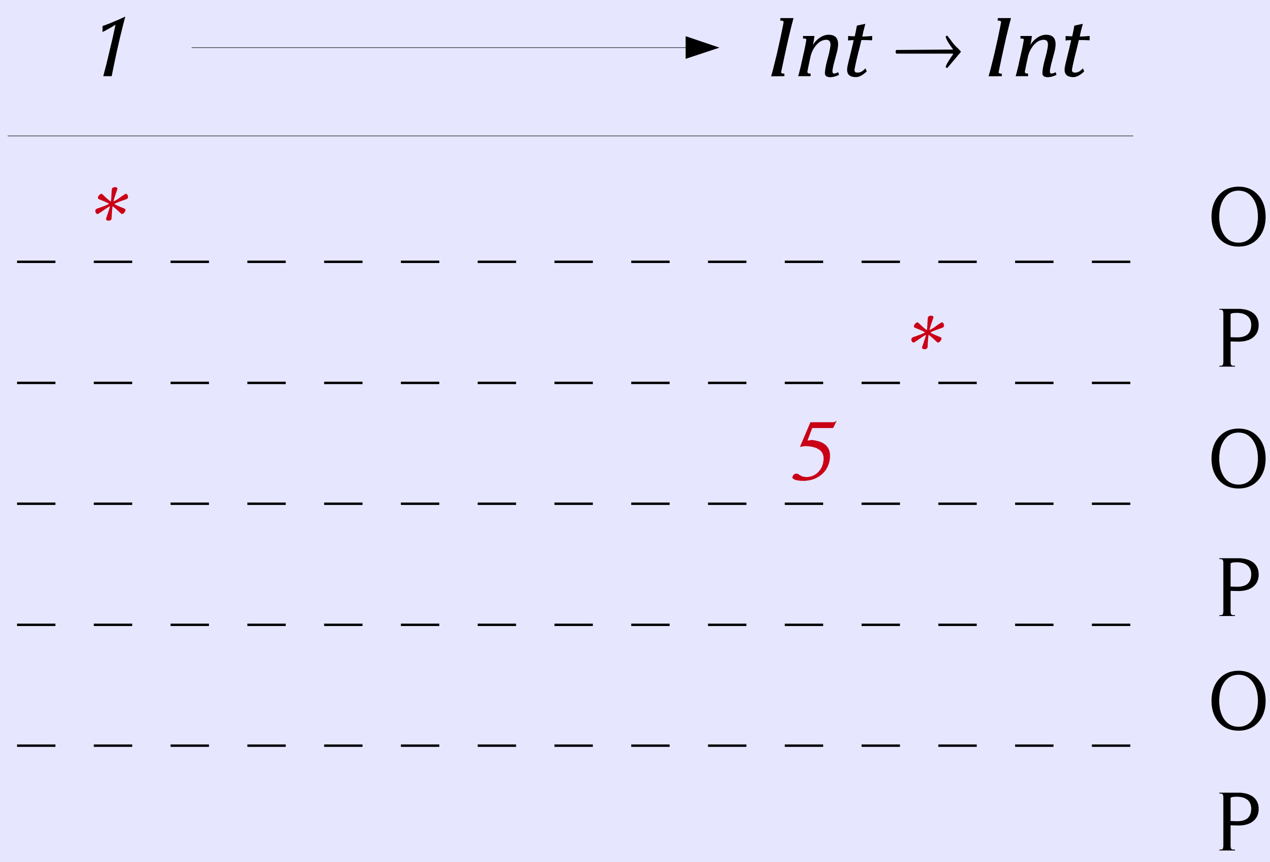

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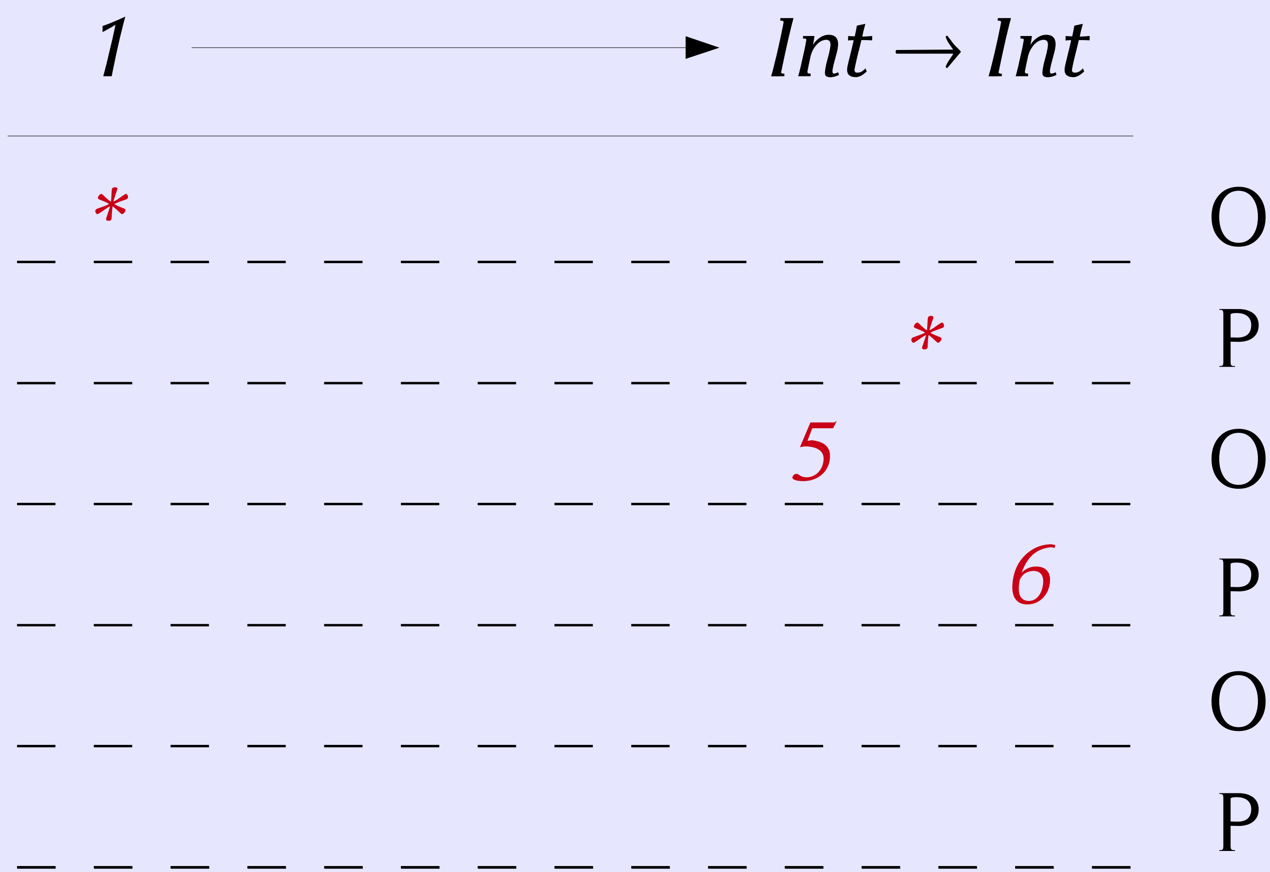

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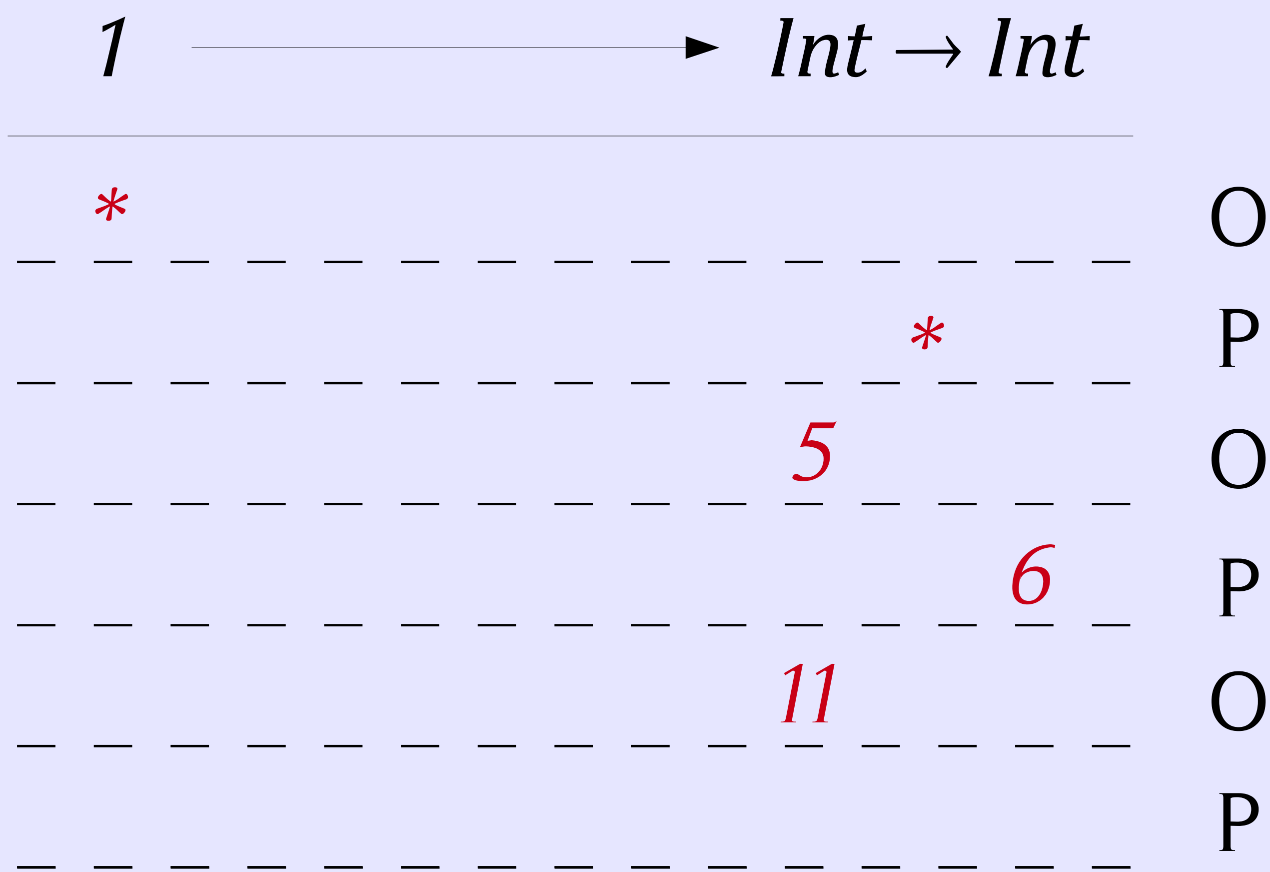

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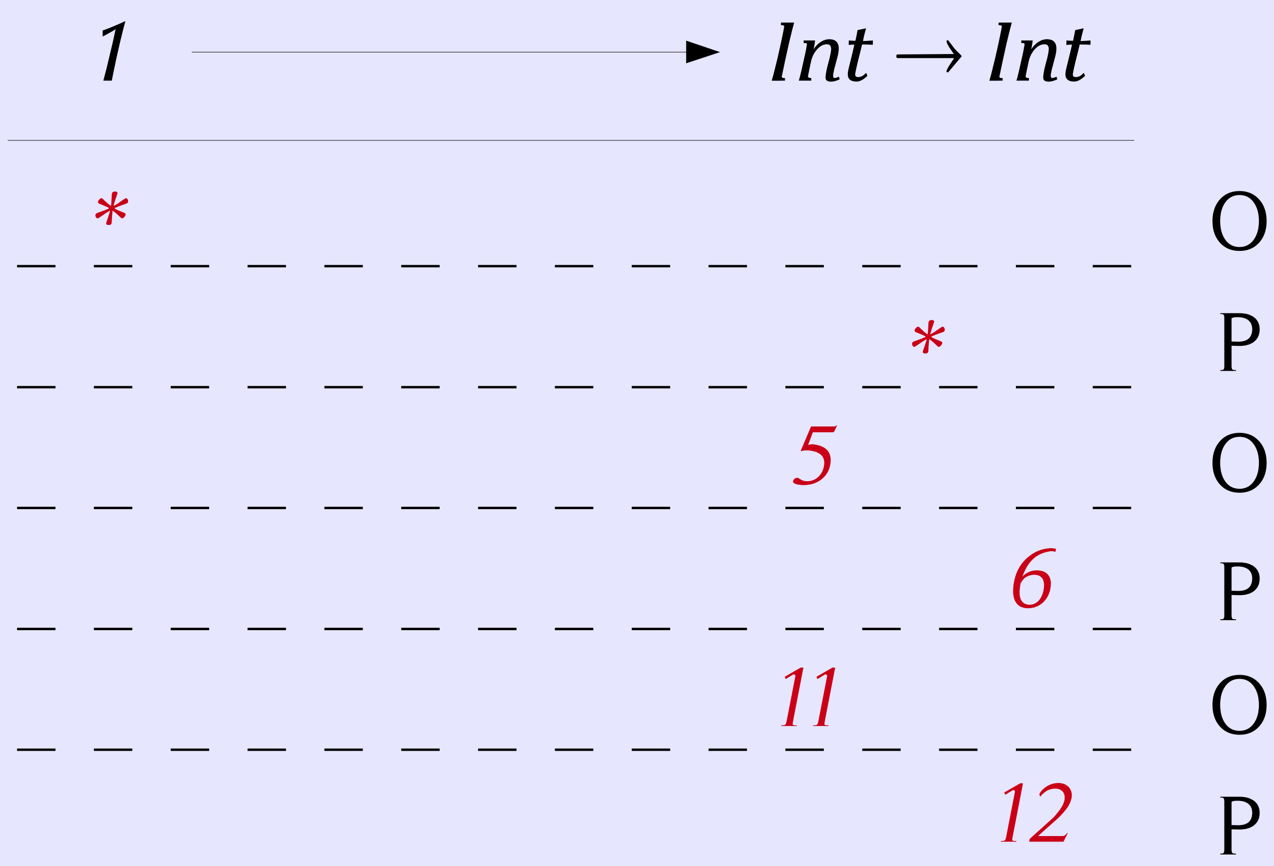

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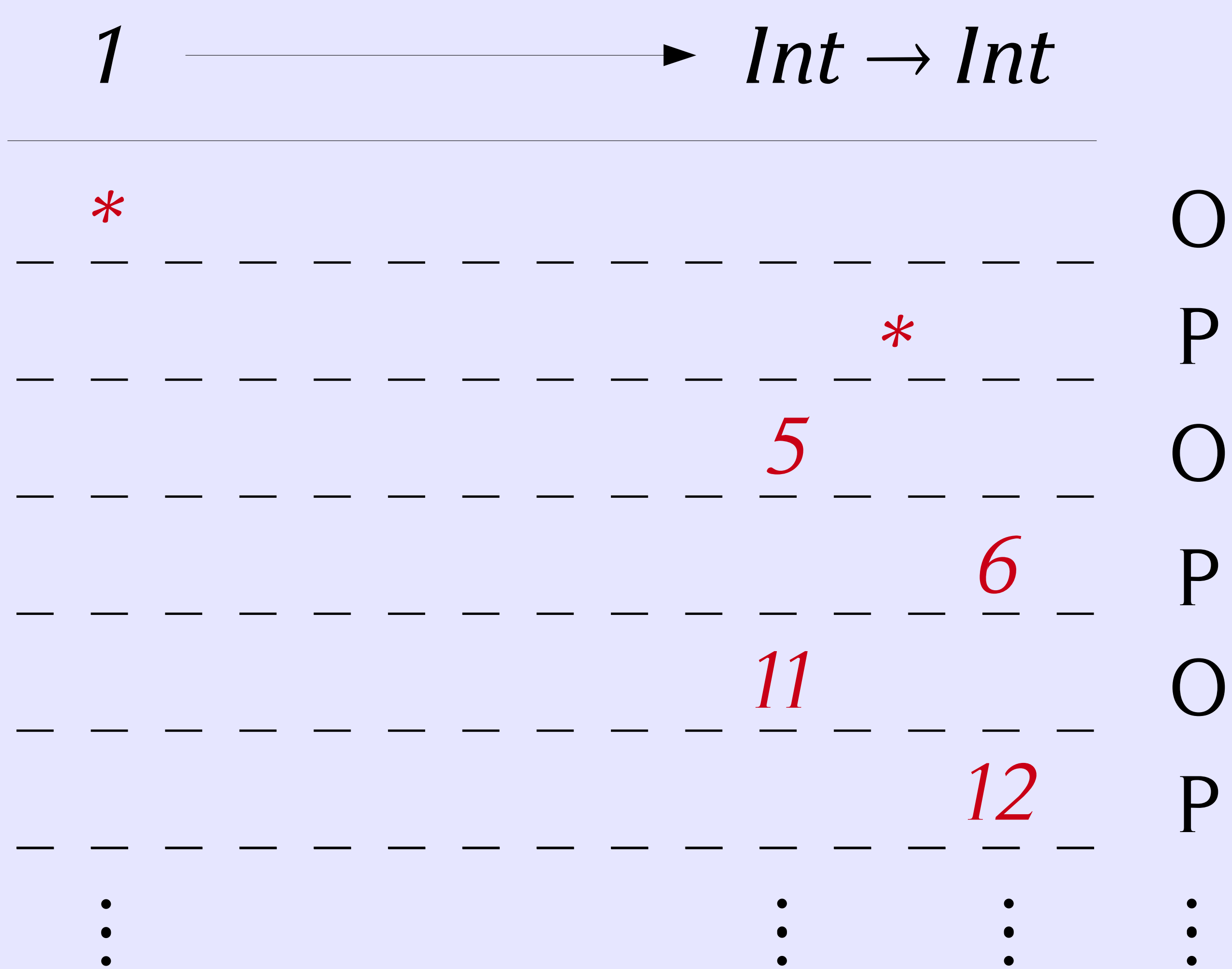

Example

`f : int→int ⊢ λx.f(x)+1 : int→int`

$$Int \to Int \longrightarrow Int \to Int$$

_	_	$*$	_	_	_	_	_	_	_	_	_	_	_	_	O
_	_	_	_	_	_	_	_	_	_	_	_	_	_	_	P
_	_	_	_	_	_	_	_	_	_	_	_	_	_	_	O
_	_	_	_	_	_	_	_	_	_	_	_	_	_	_	P
_	_	_	_	_	_	_	_	_	_	_	_	_	_	_	O
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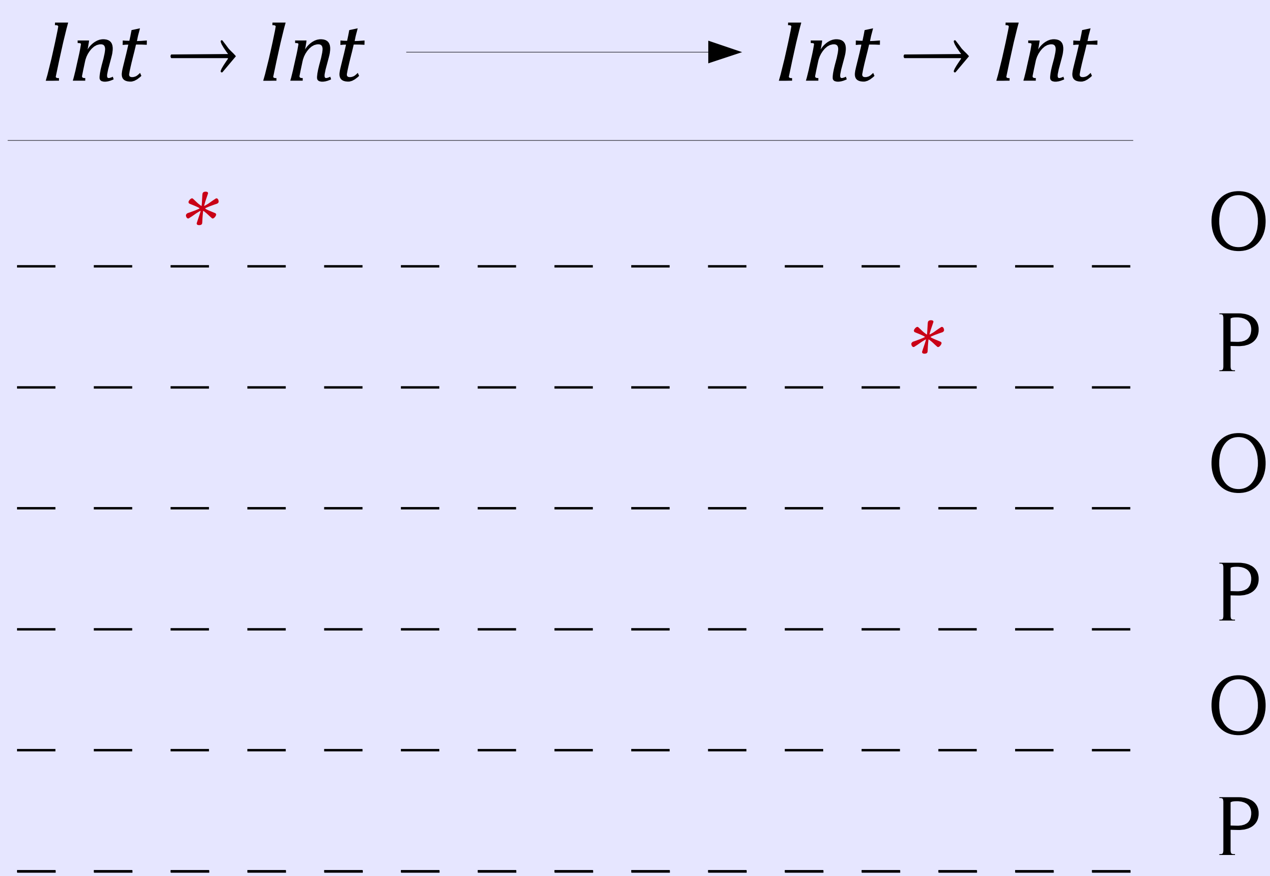

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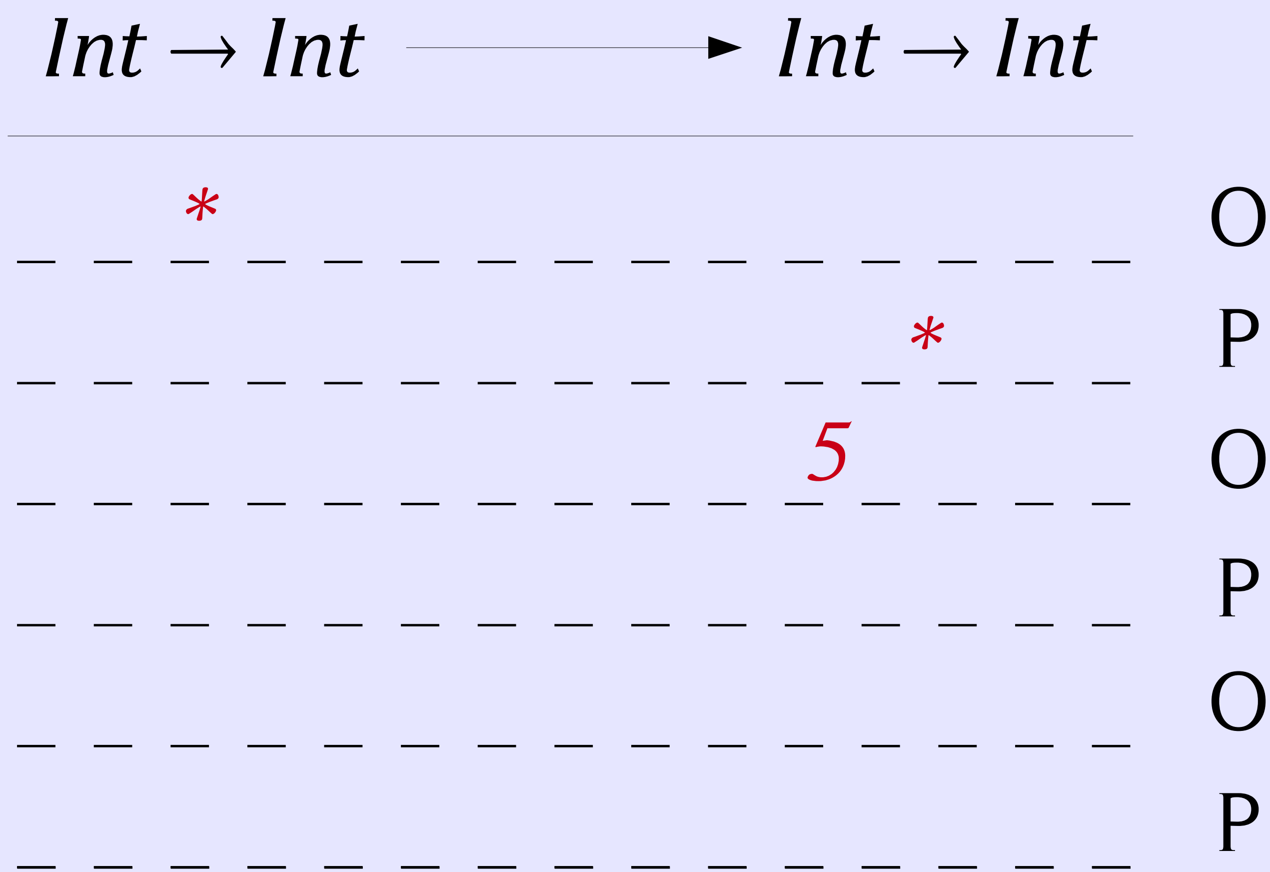
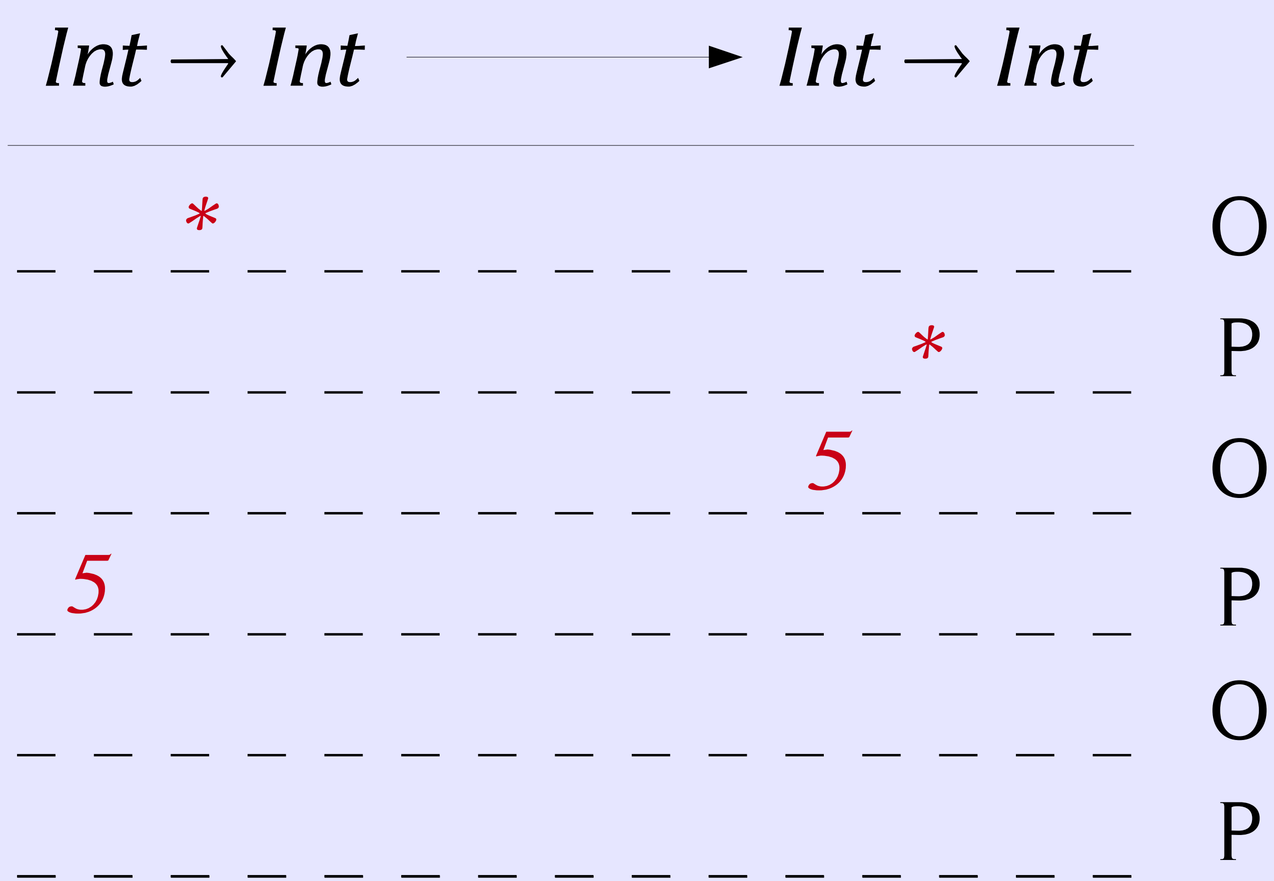


Example

$$\texttt{f : int} \rightarrow \texttt{int} \vdash \lambda\texttt{x.f(x)+1 : int} \rightarrow \texttt{int}$$



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Int	$\rightarrow$	Int	$\longrightarrow$	Int	$\rightarrow$	Int	
	$*$						O
					$*$		P
				5			O
5							P
		14					O
							P

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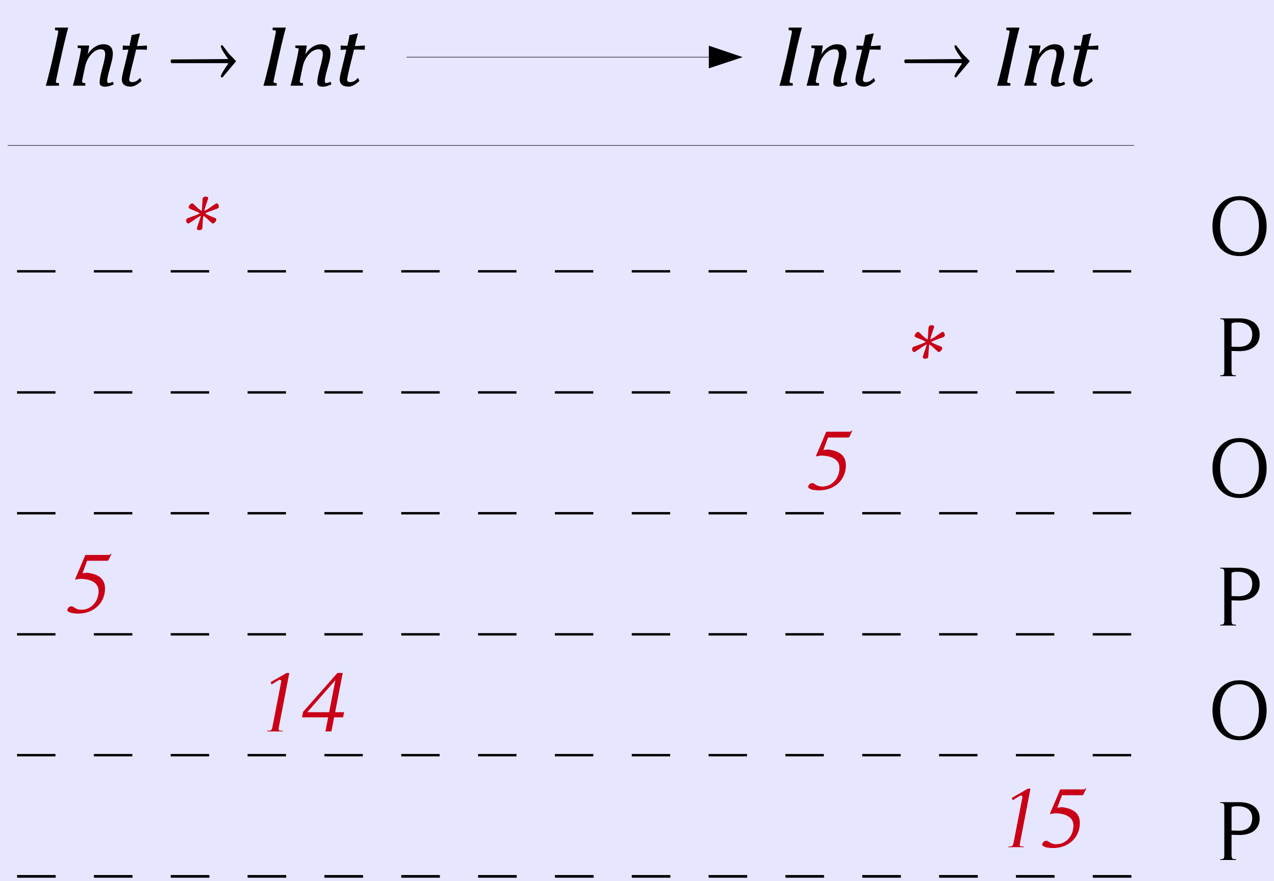

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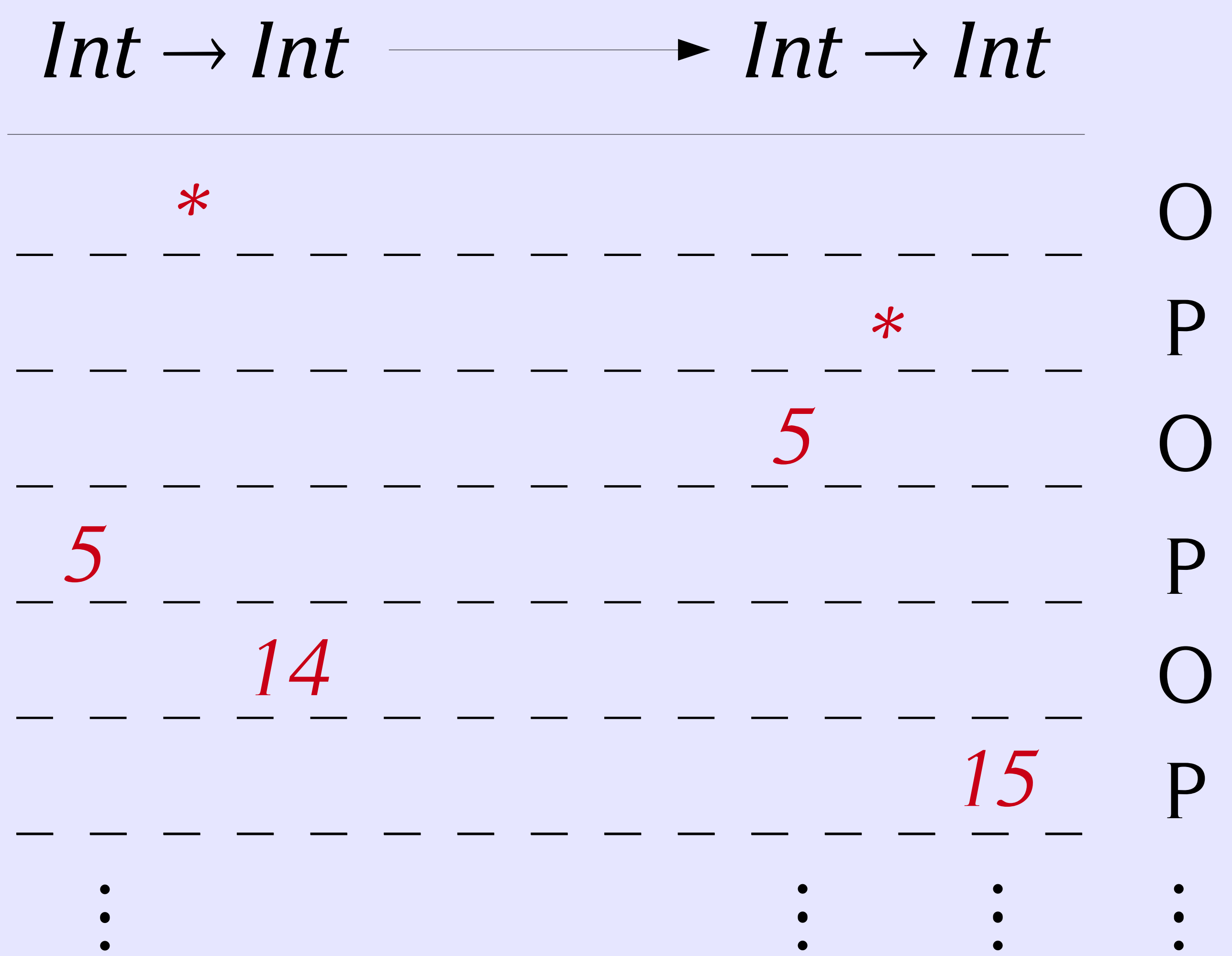

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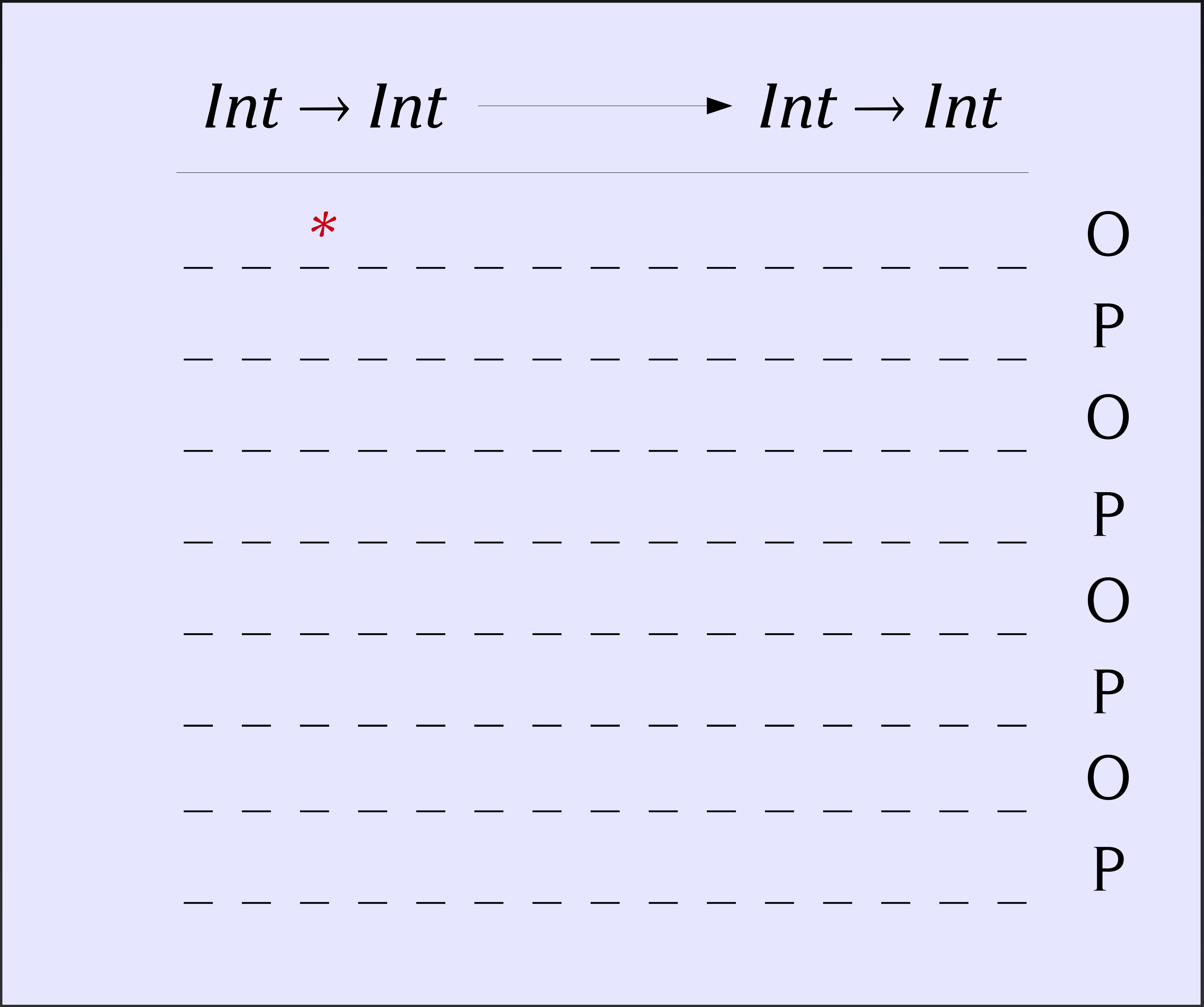

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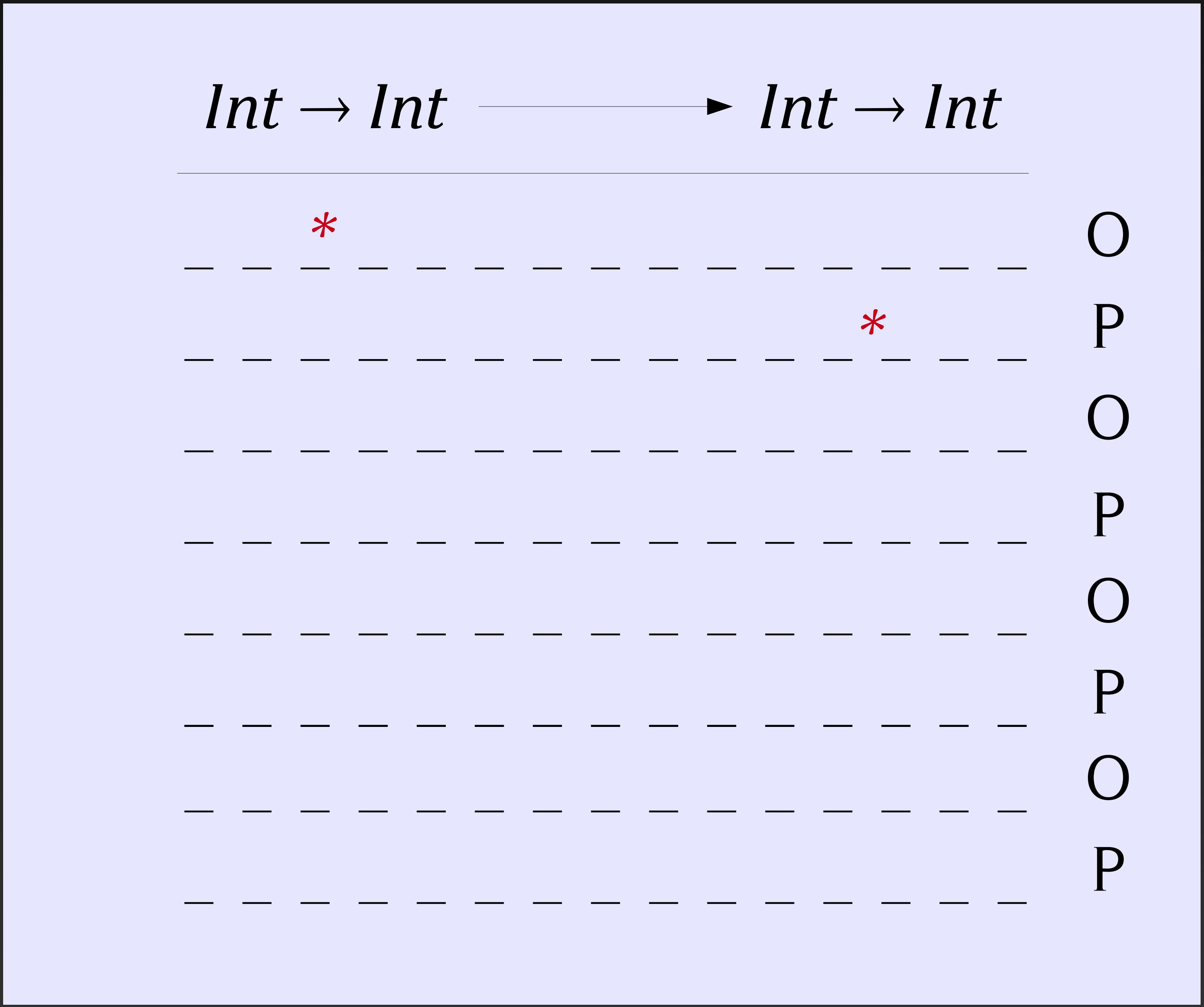

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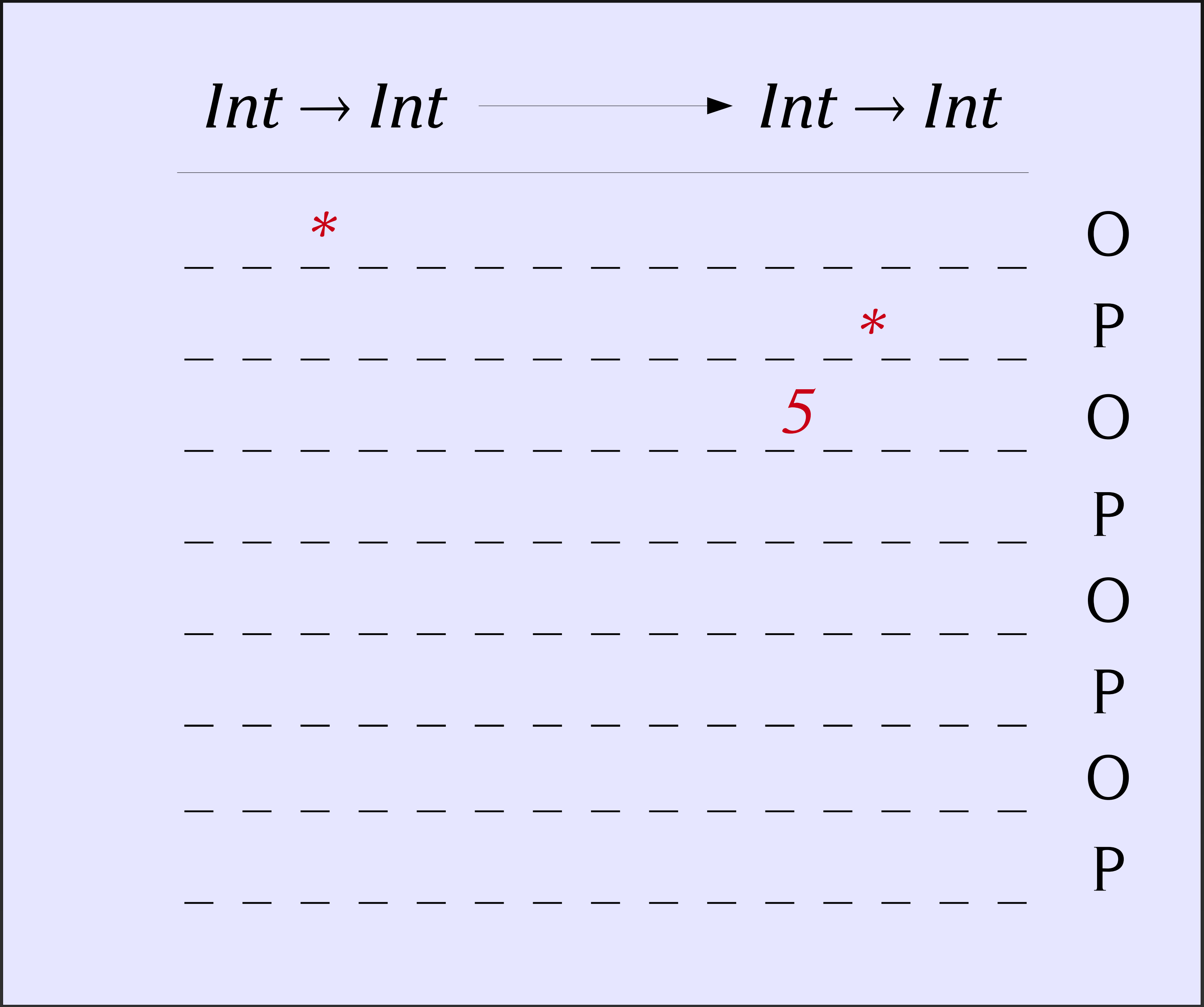

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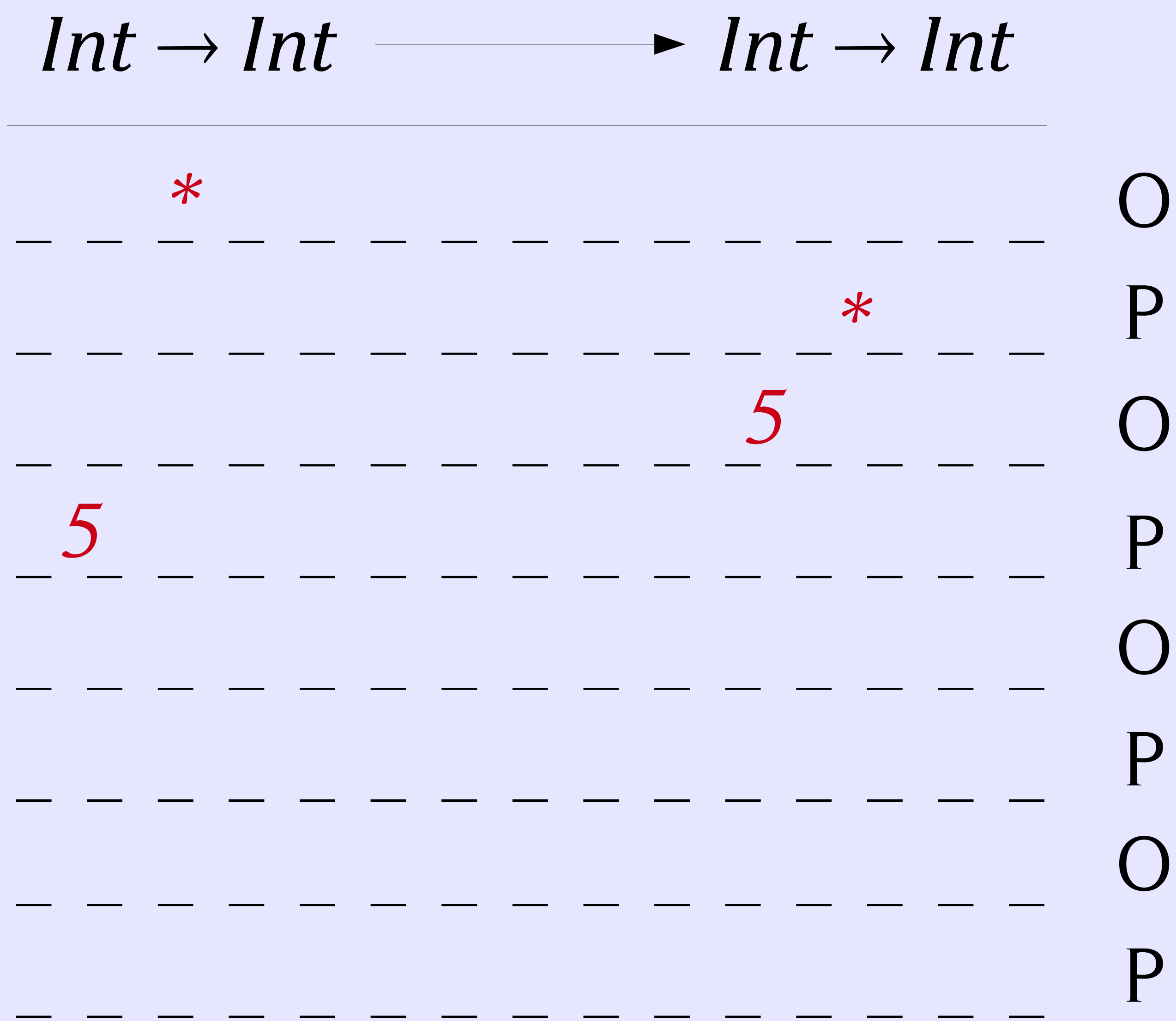

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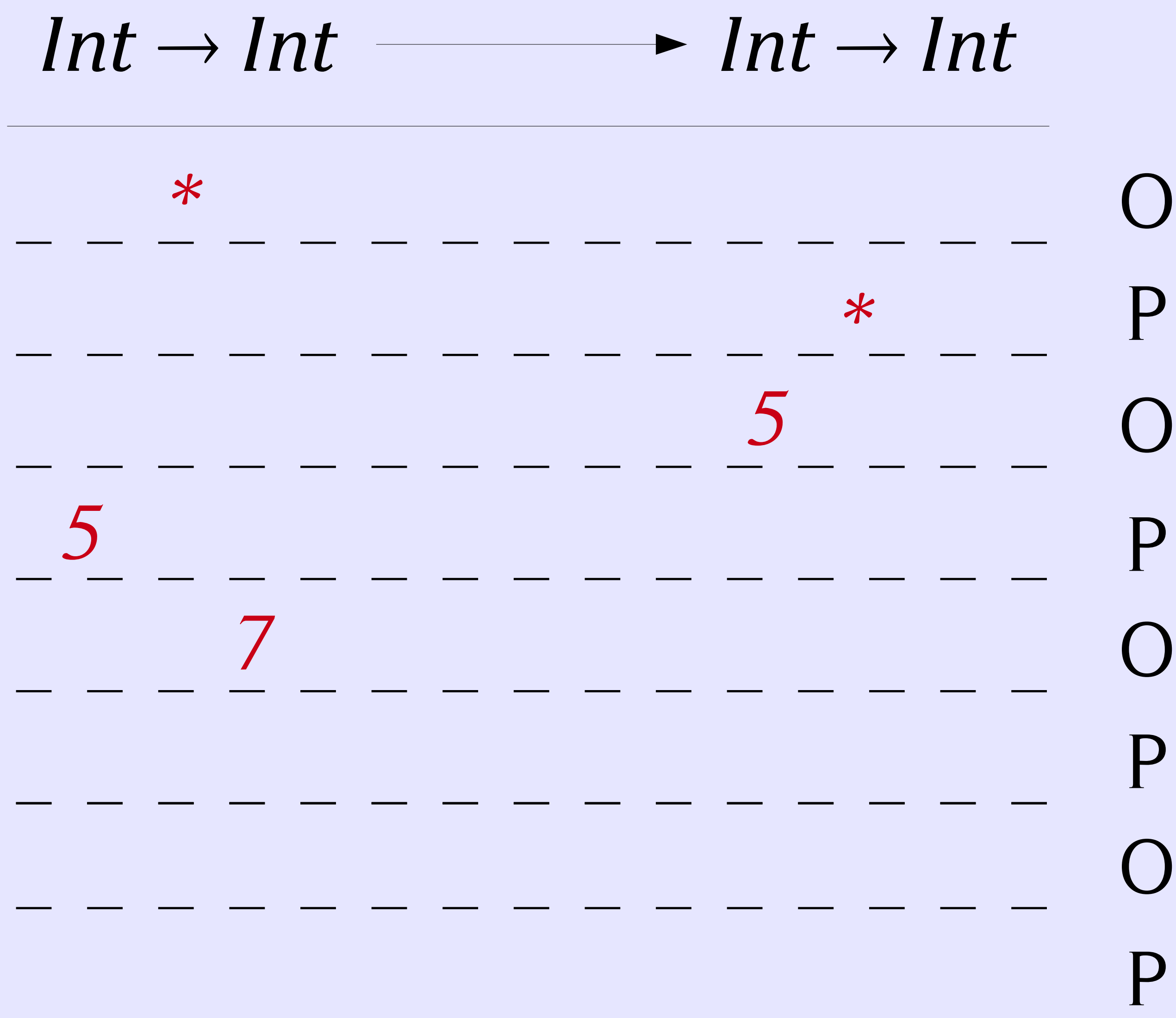

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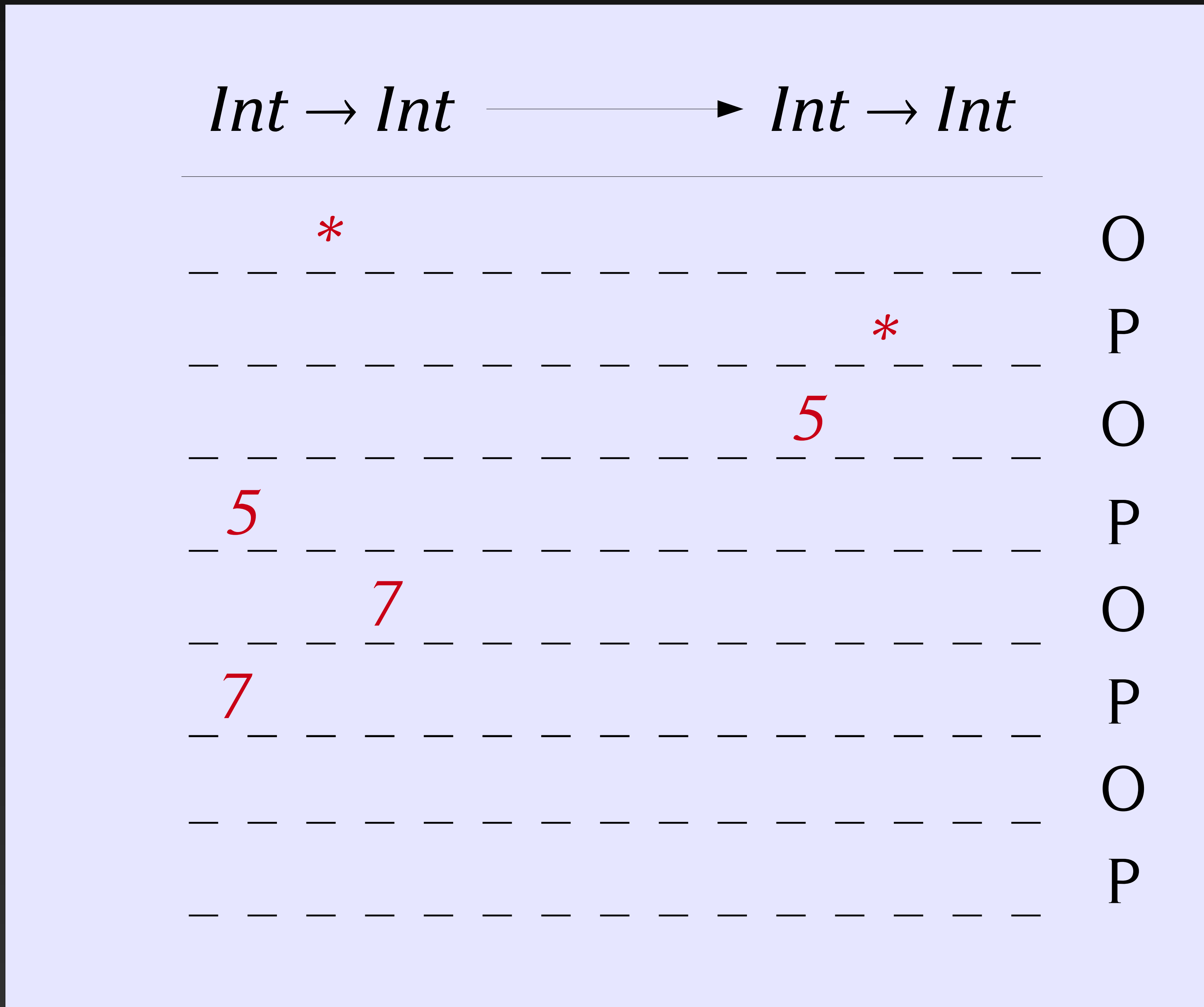

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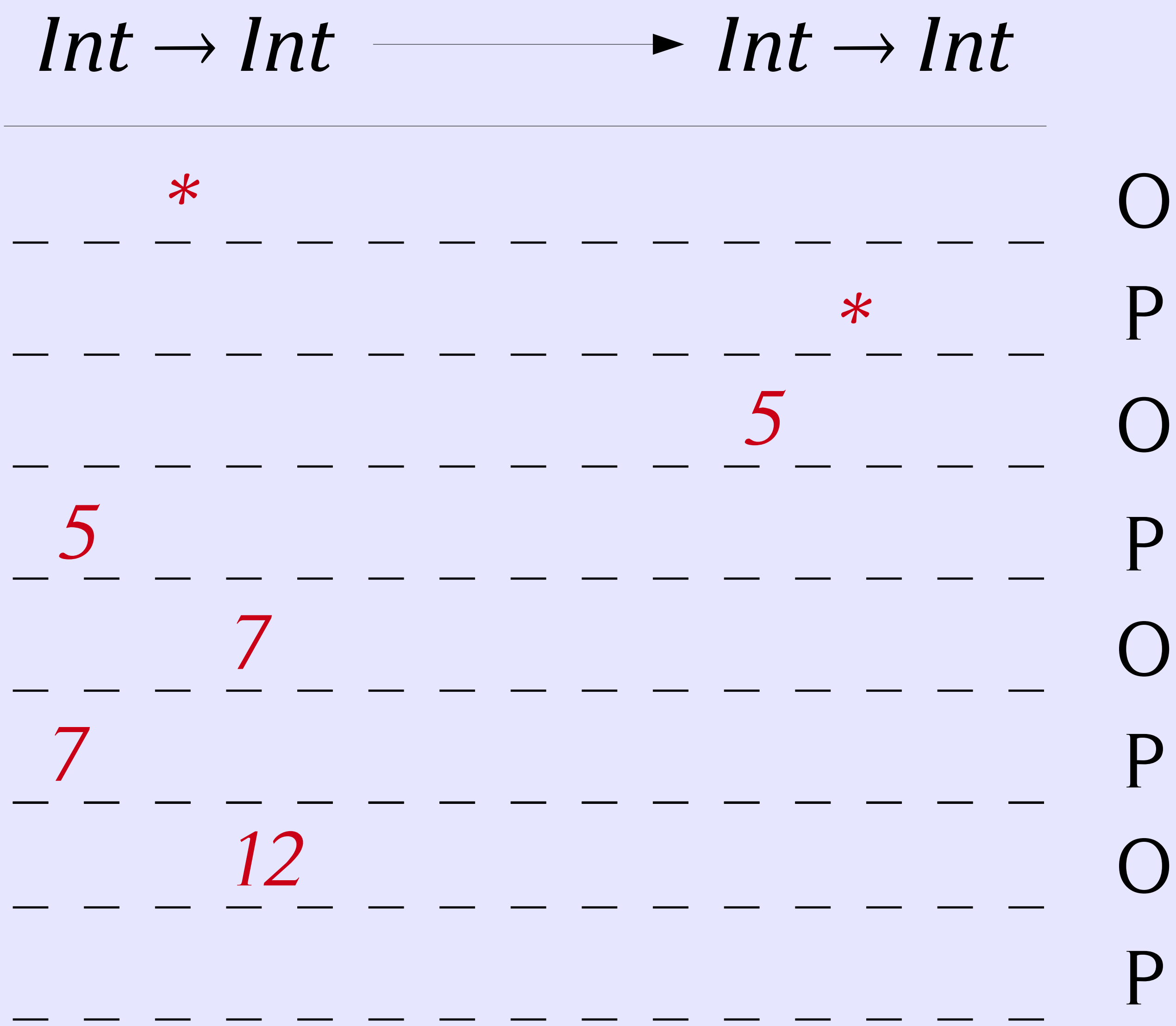

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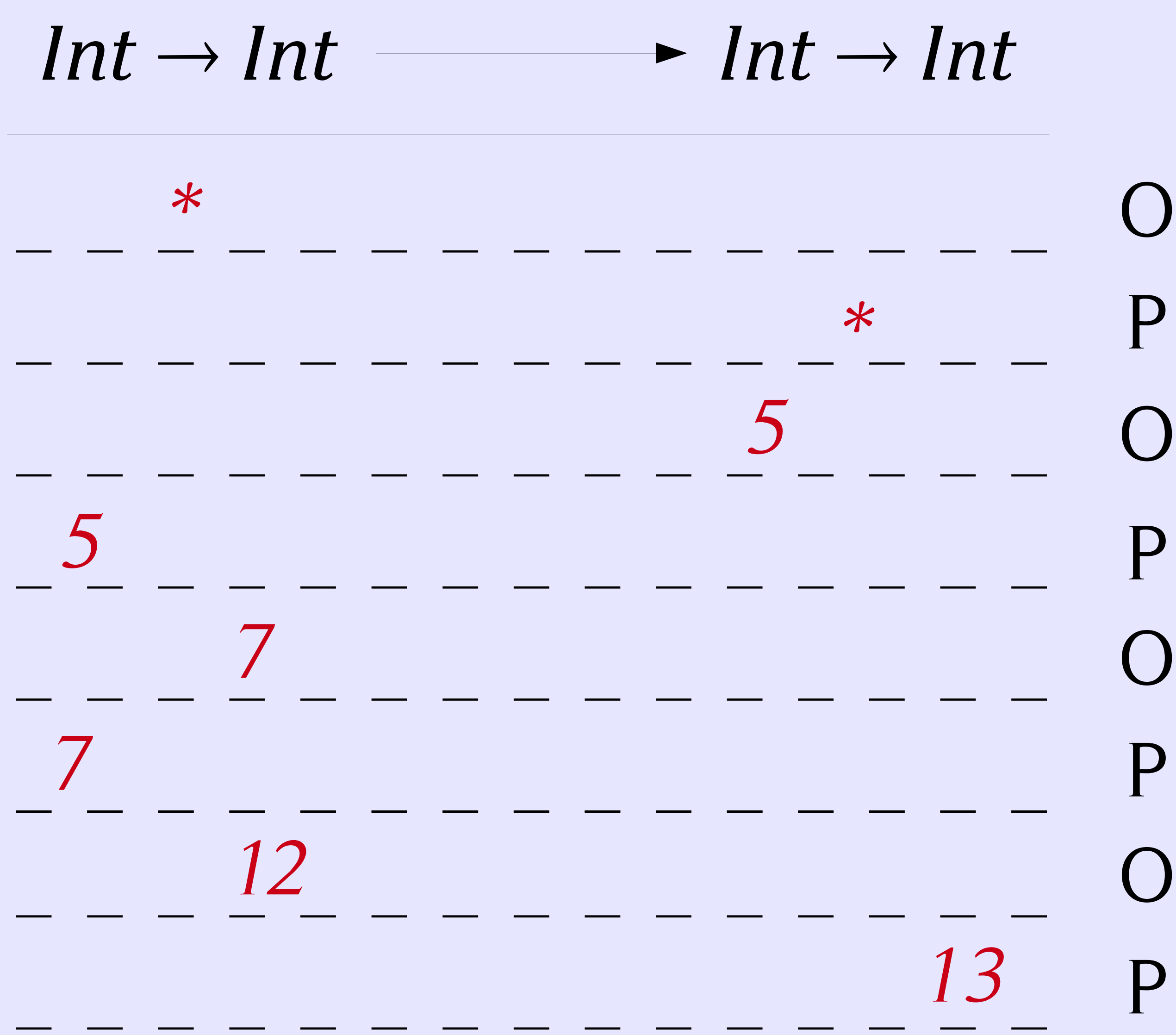

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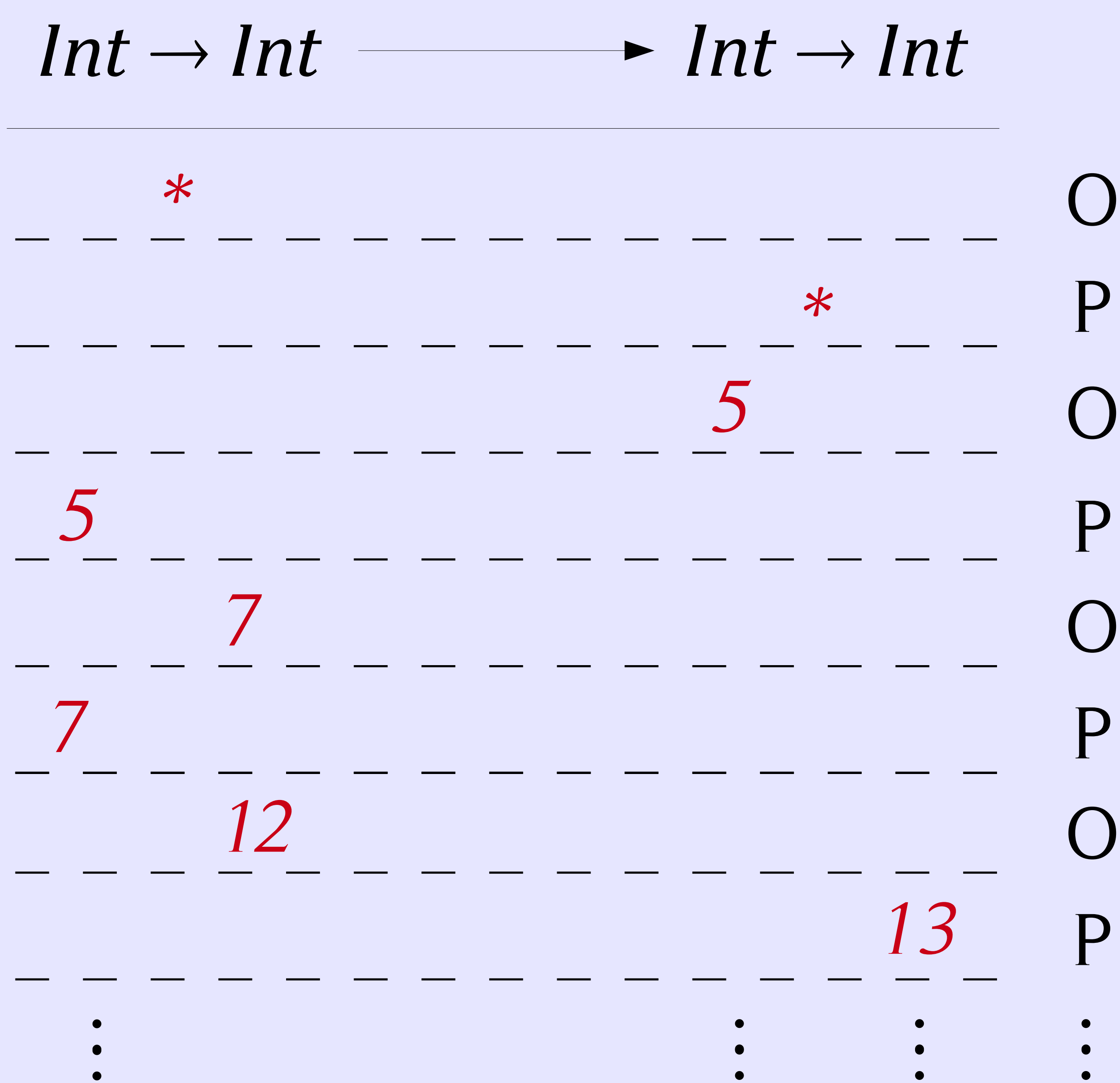
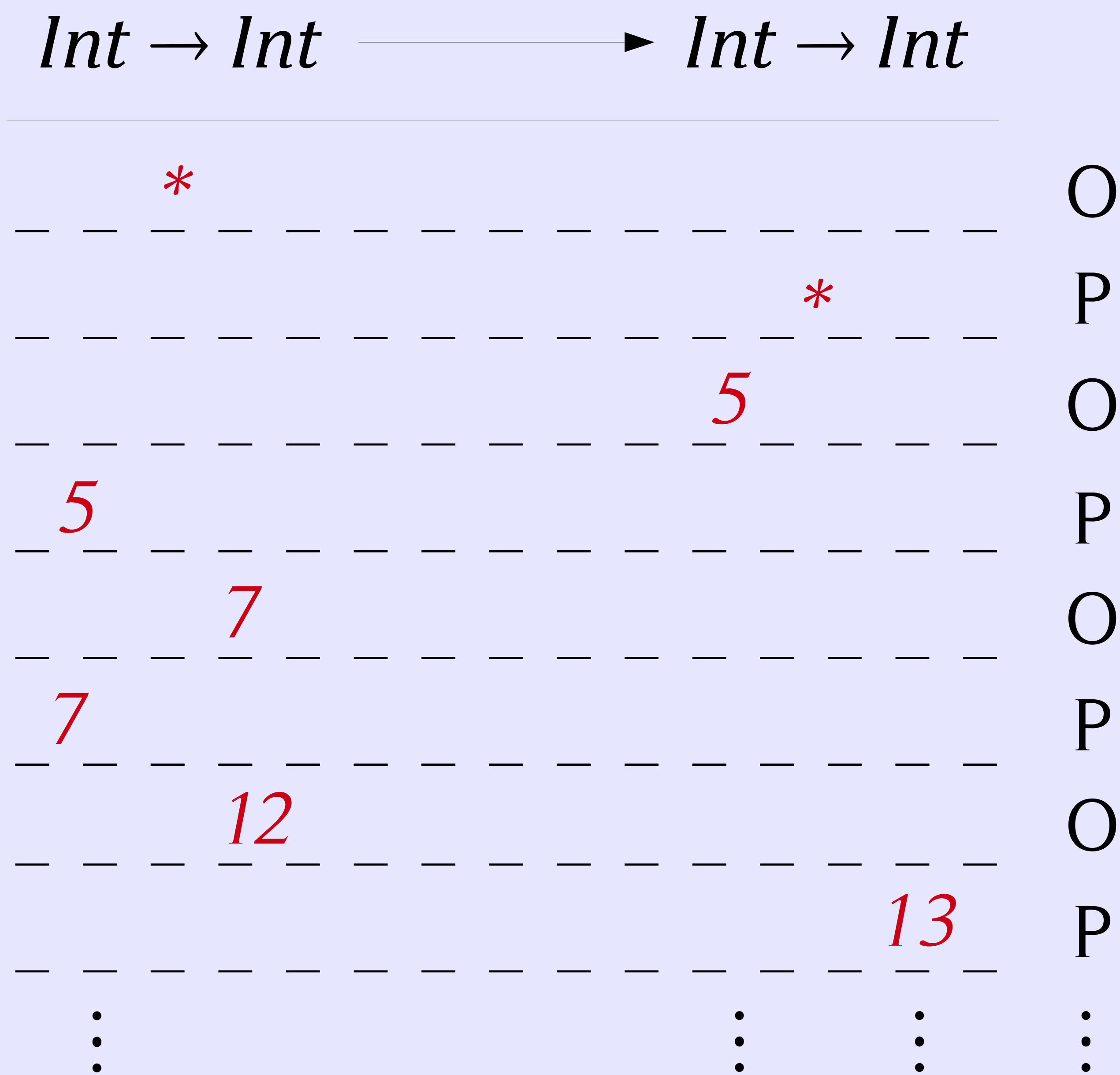


$$f : \mathtt{int} \to \mathtt{int} \vdash \lambda x.f(f(x))+1 : \mathtt{int} \to \mathtt{int}$$



Game Semantics

- Computation is modelled as a 2-player game between:
  - *Opponent* (the environment)
  - *Proponent* (the program)
- Qualitative games
- Programs = *strategies* for Proponent

Composition

$\vdash$ `λx.x+1 : int` $\longrightarrow$ `int`

`f : int` $\longrightarrow$ `int` $\vdash$ `λx.f(x)+1 : int` $\longrightarrow$ `int`

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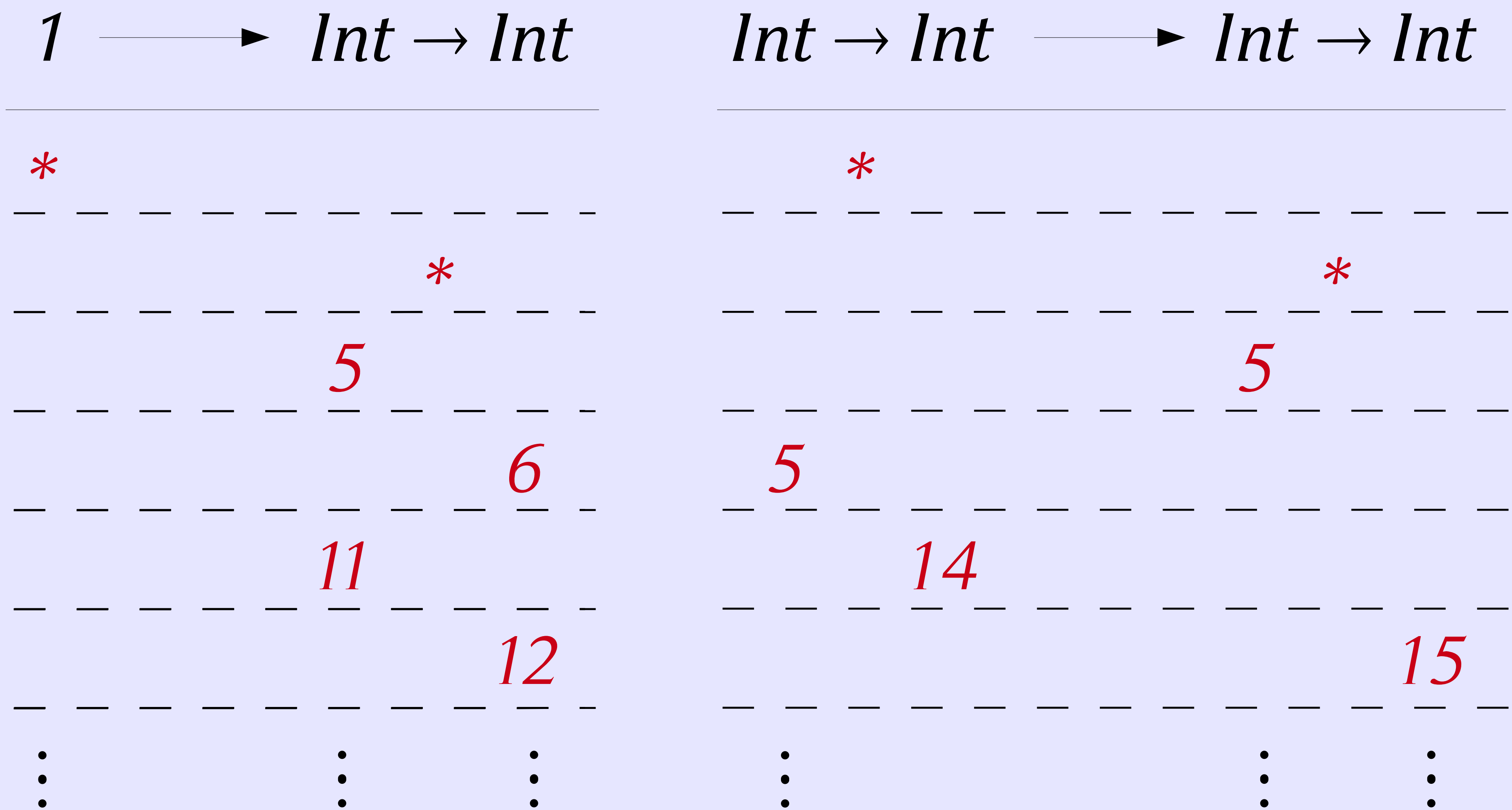

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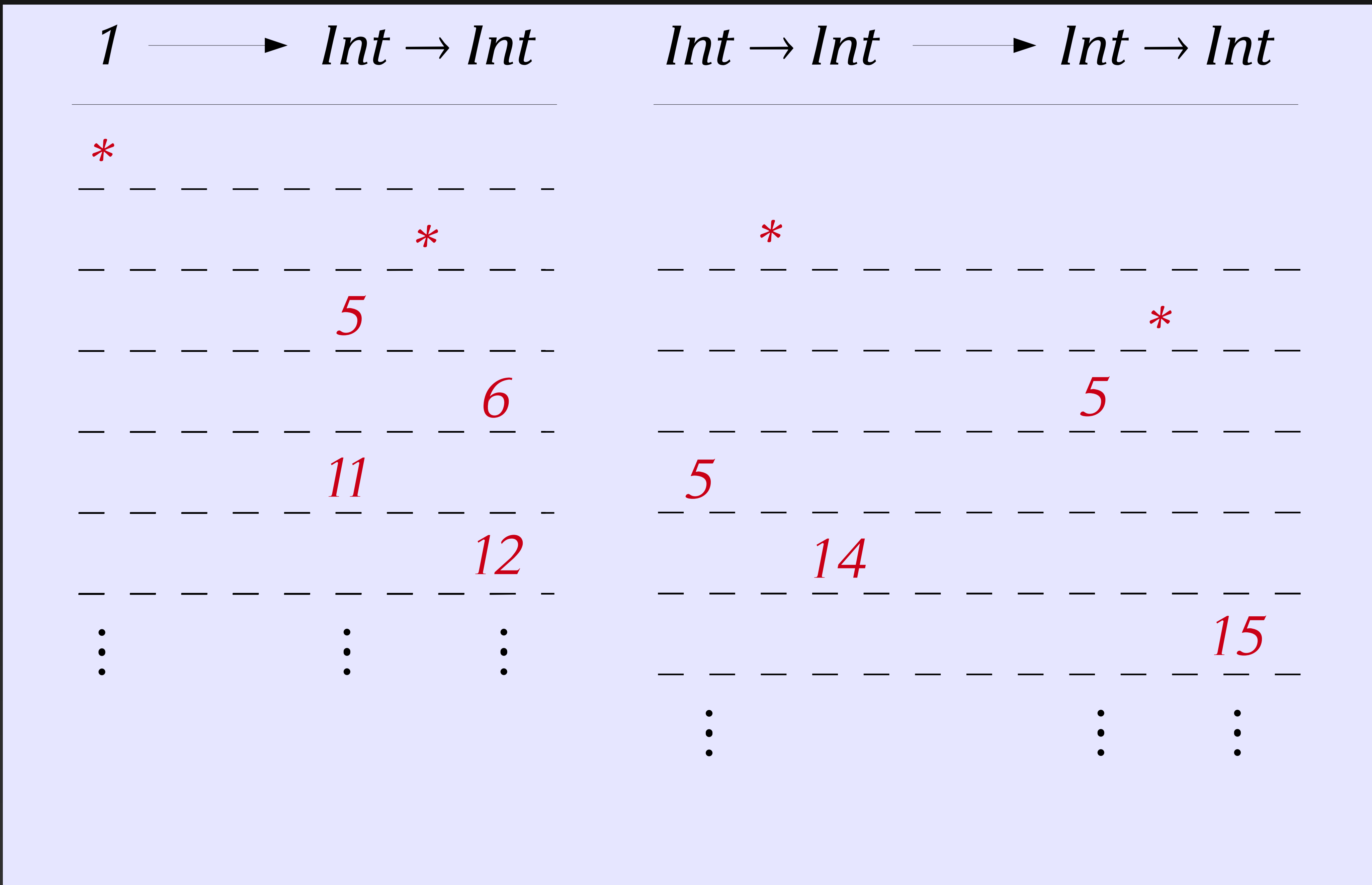

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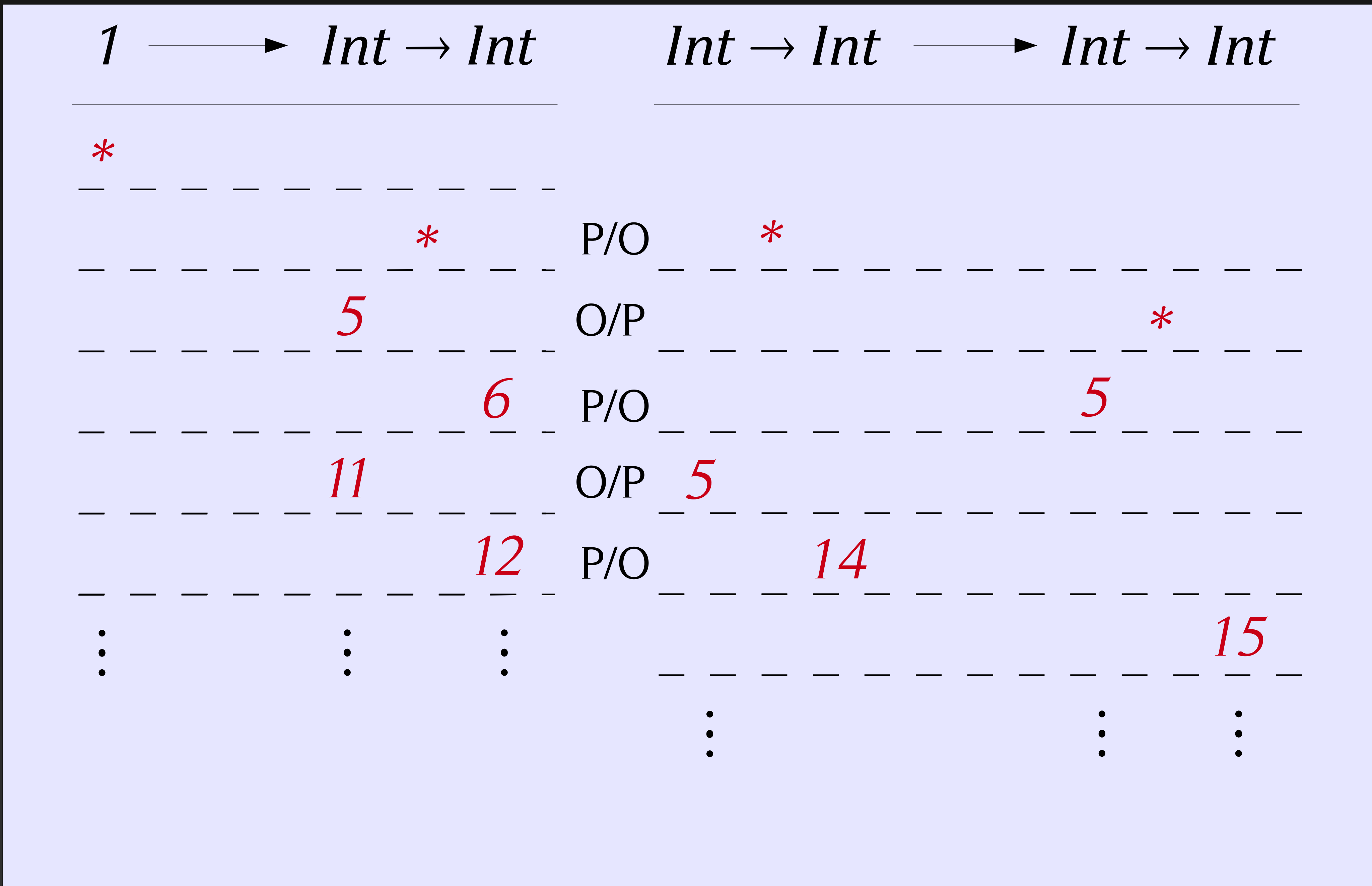

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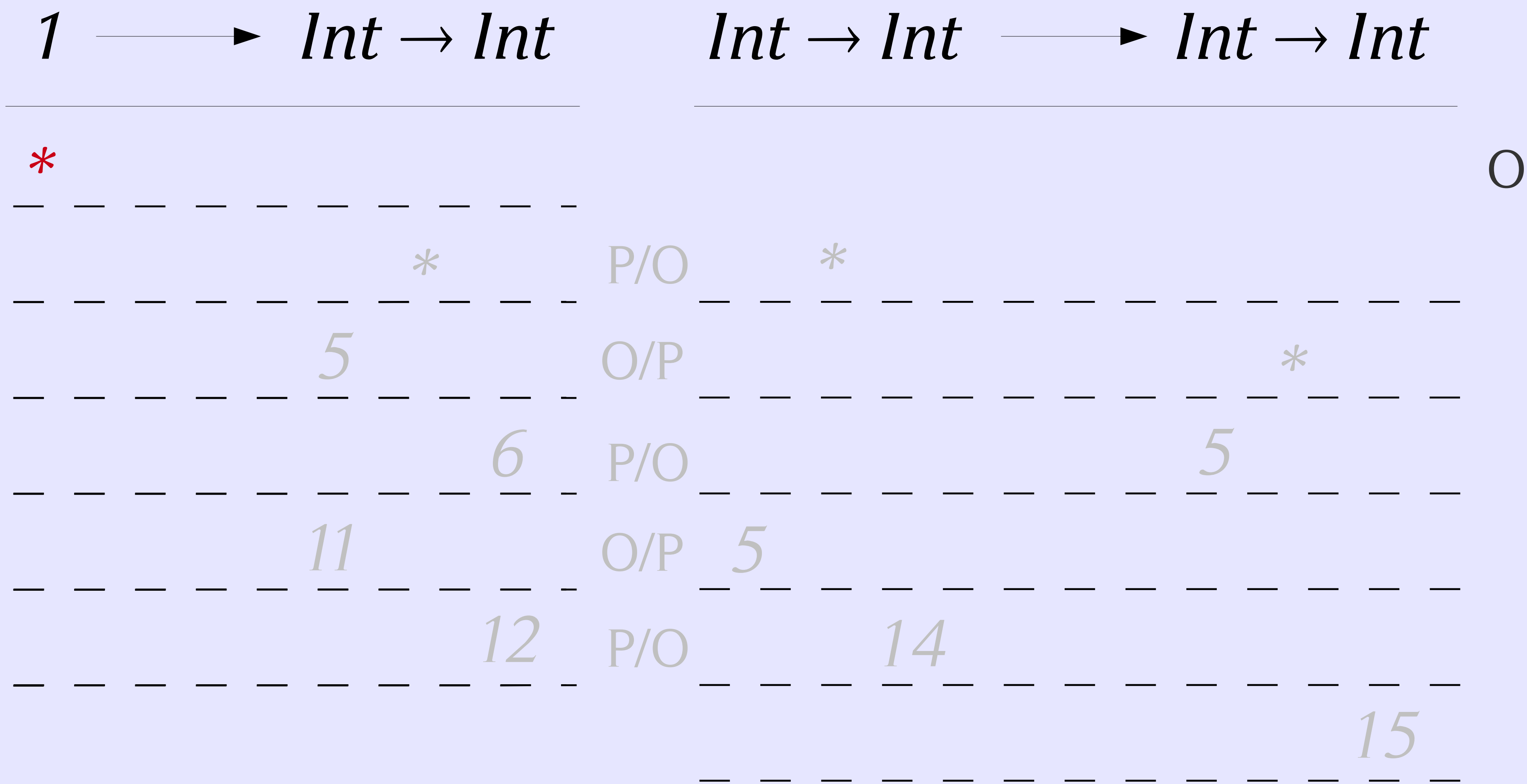

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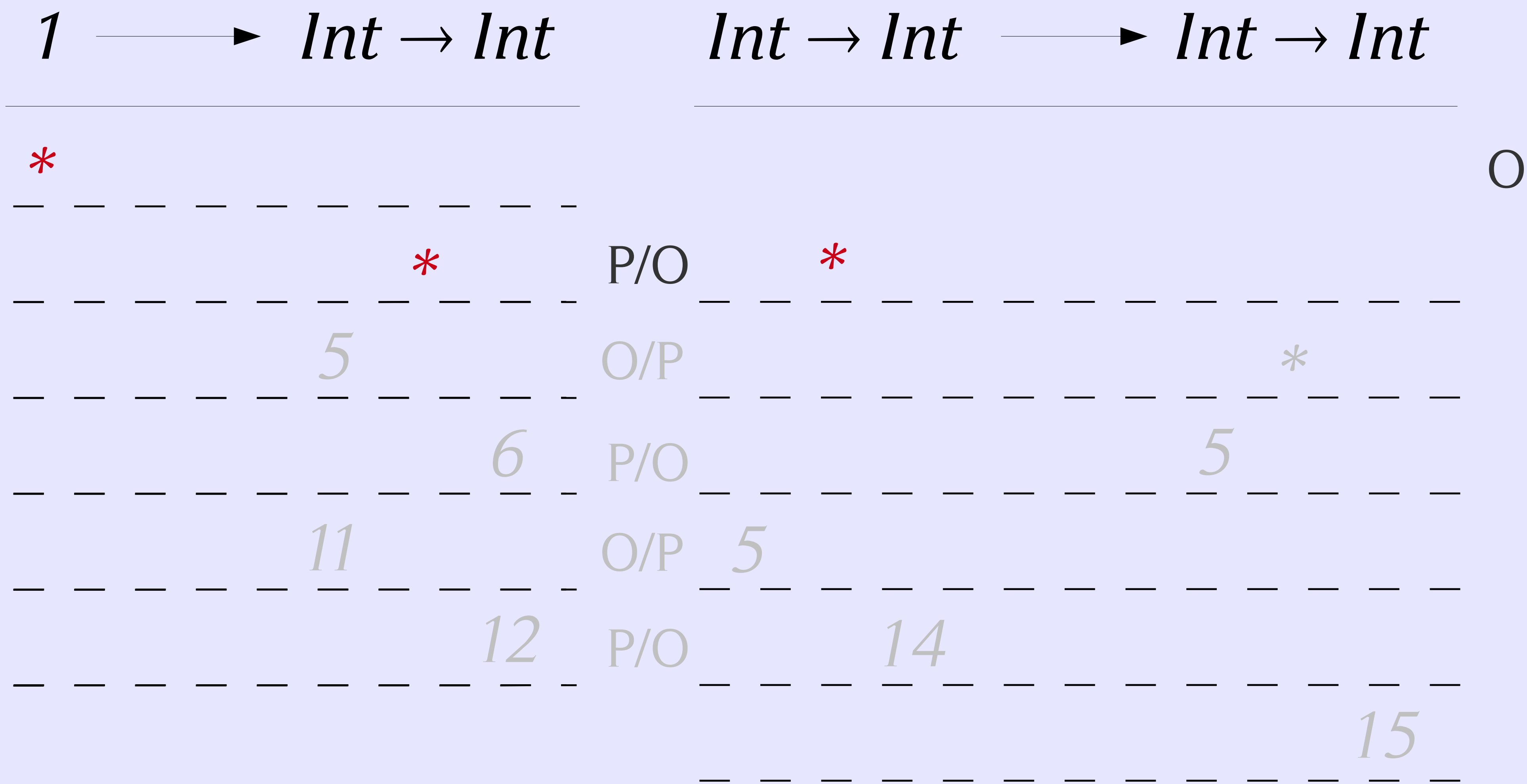

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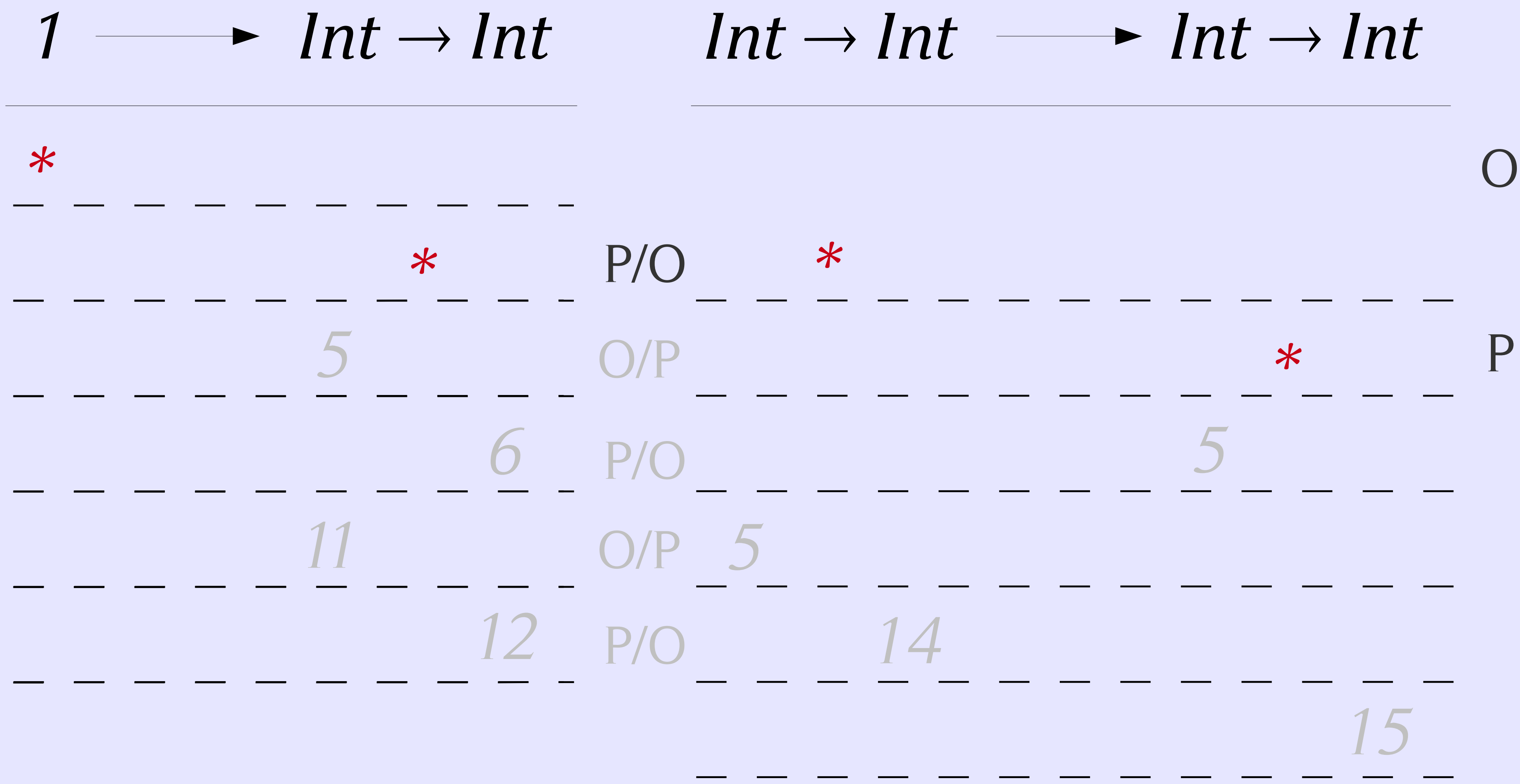

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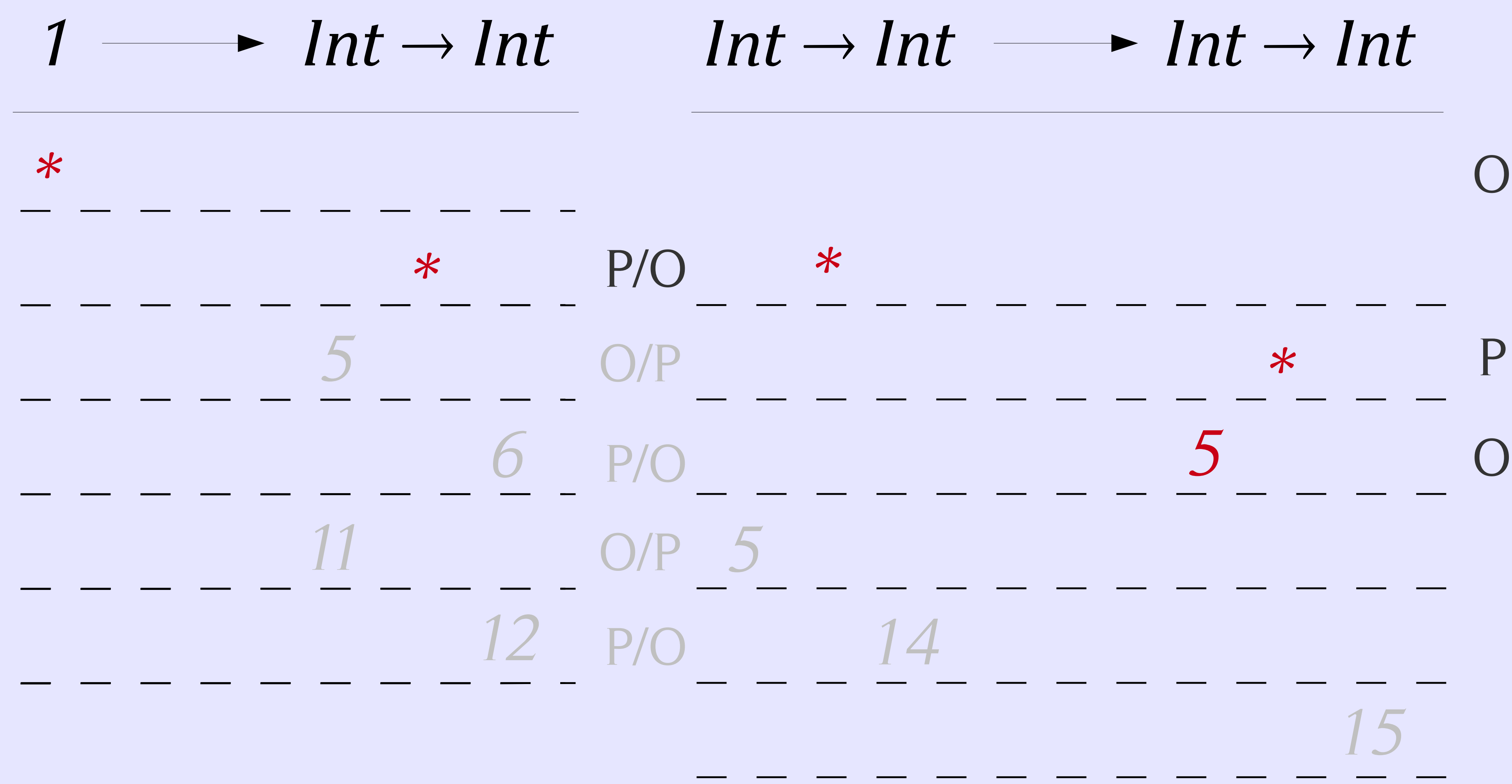

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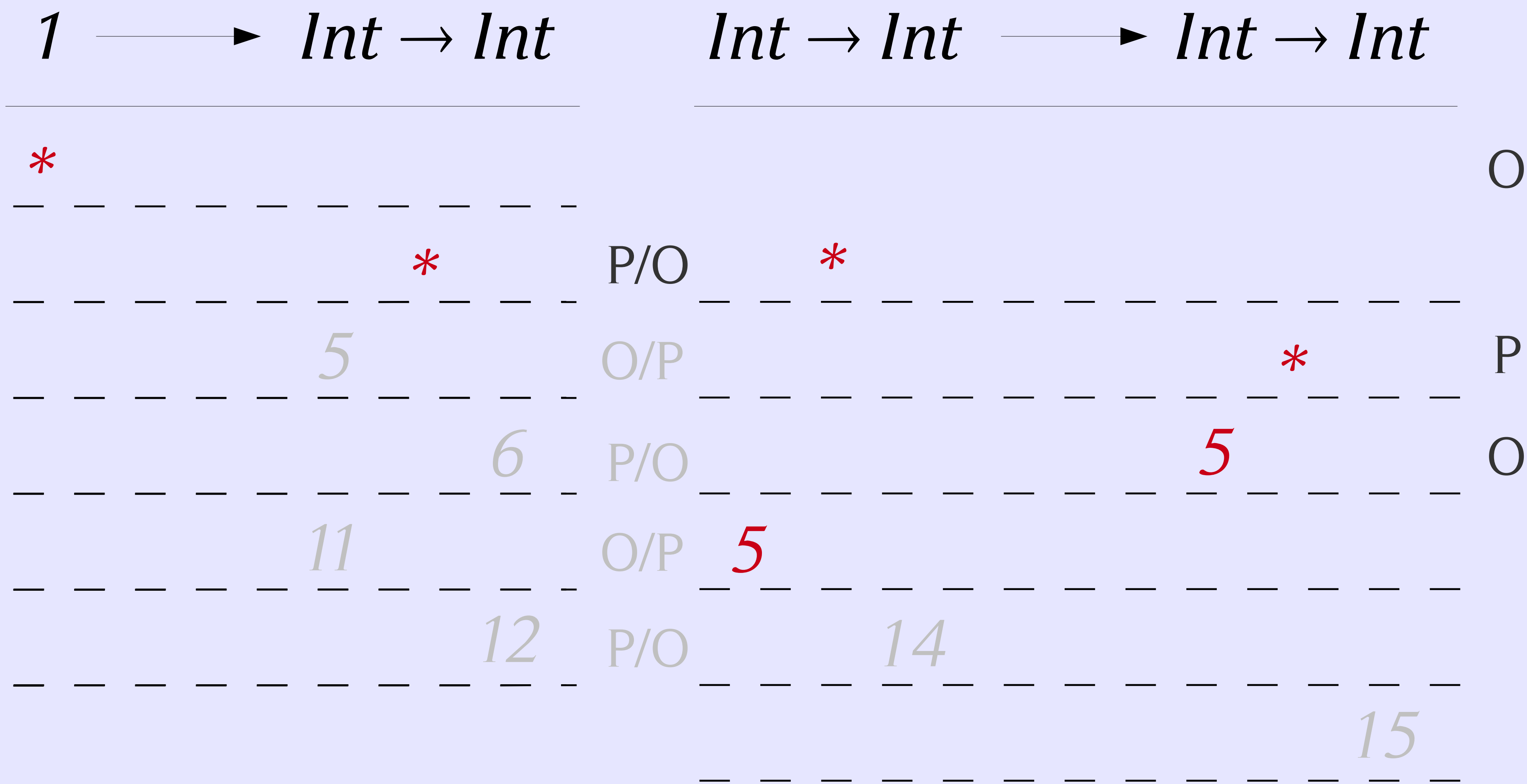

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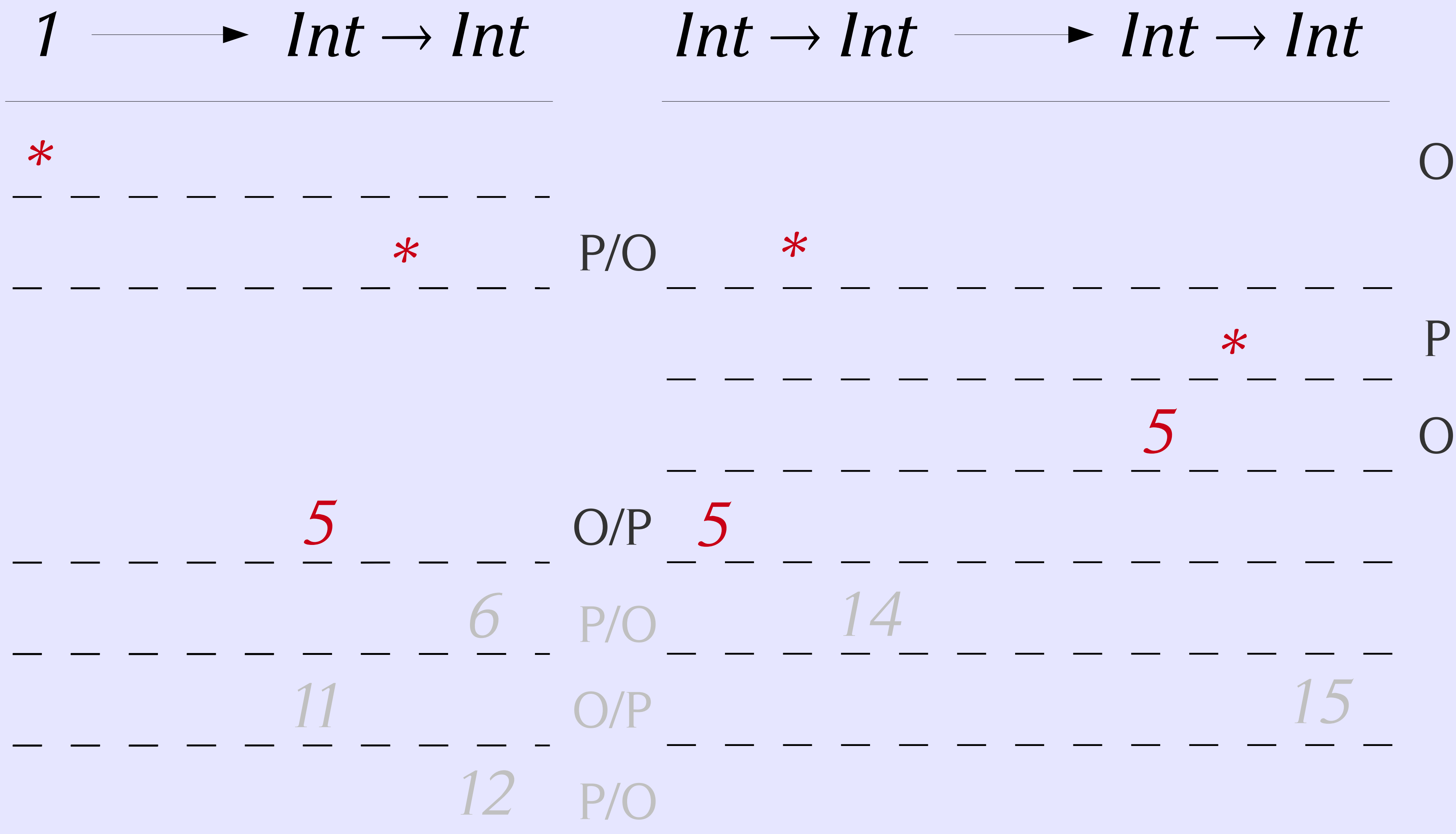

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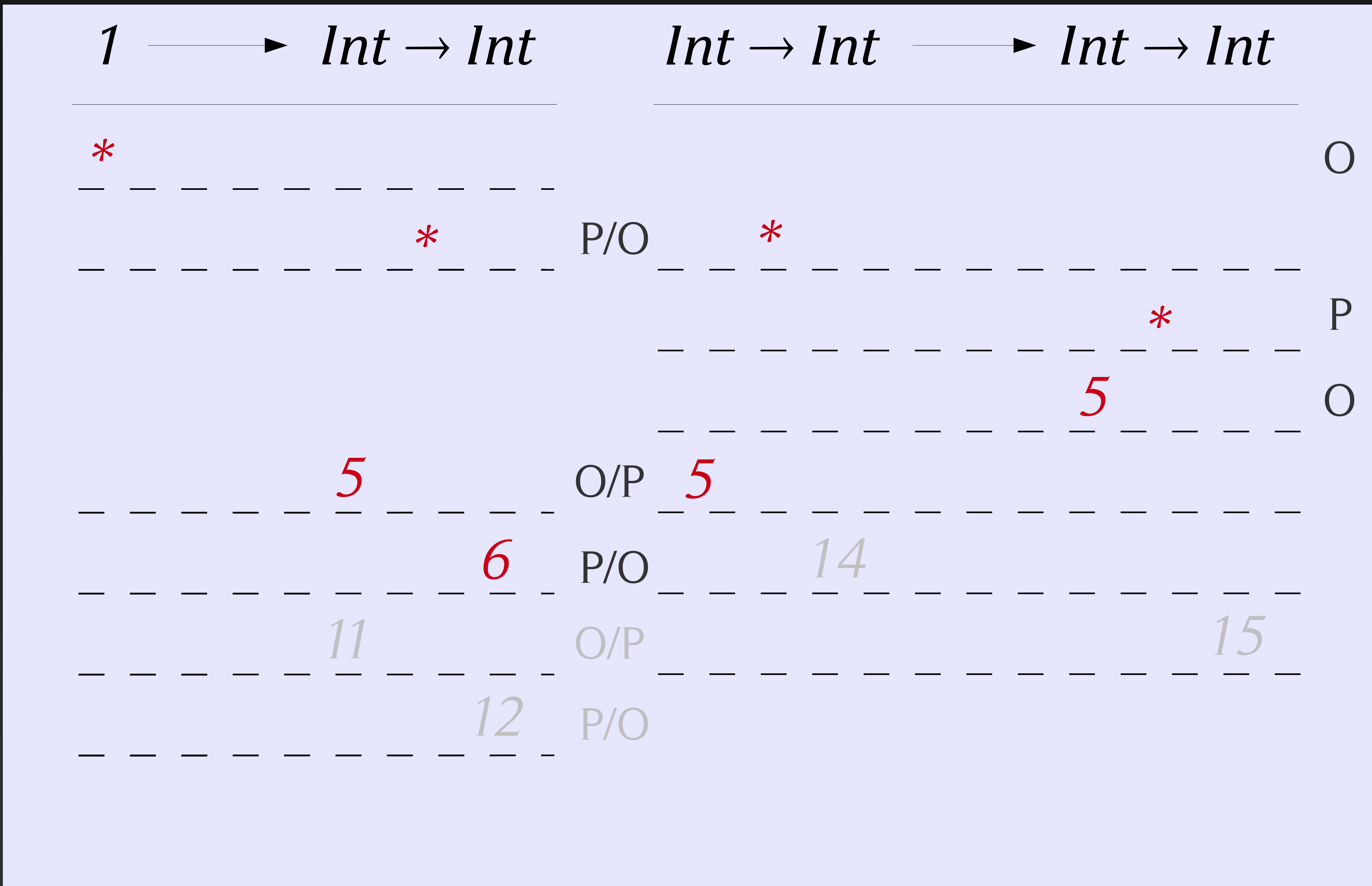

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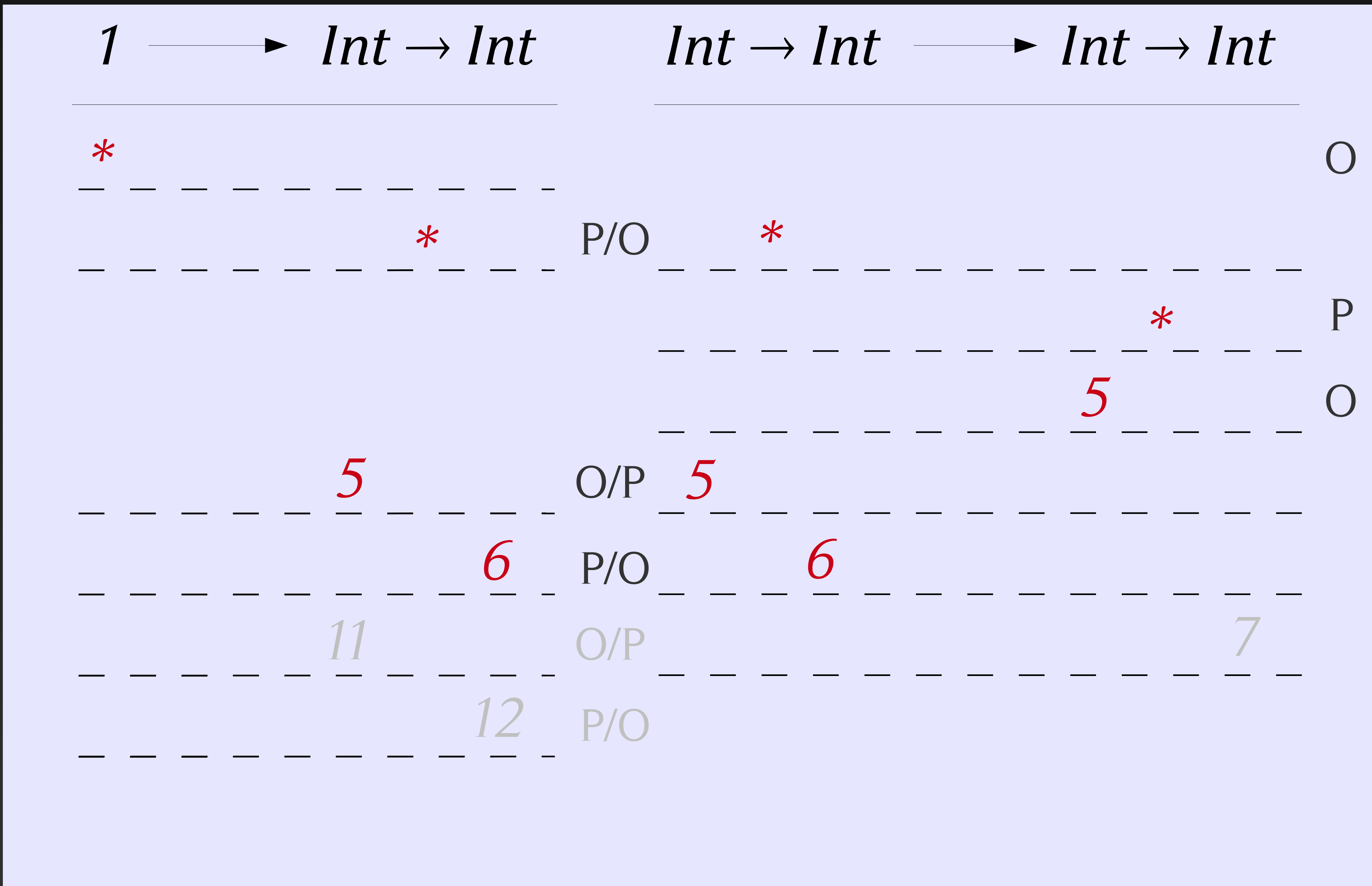

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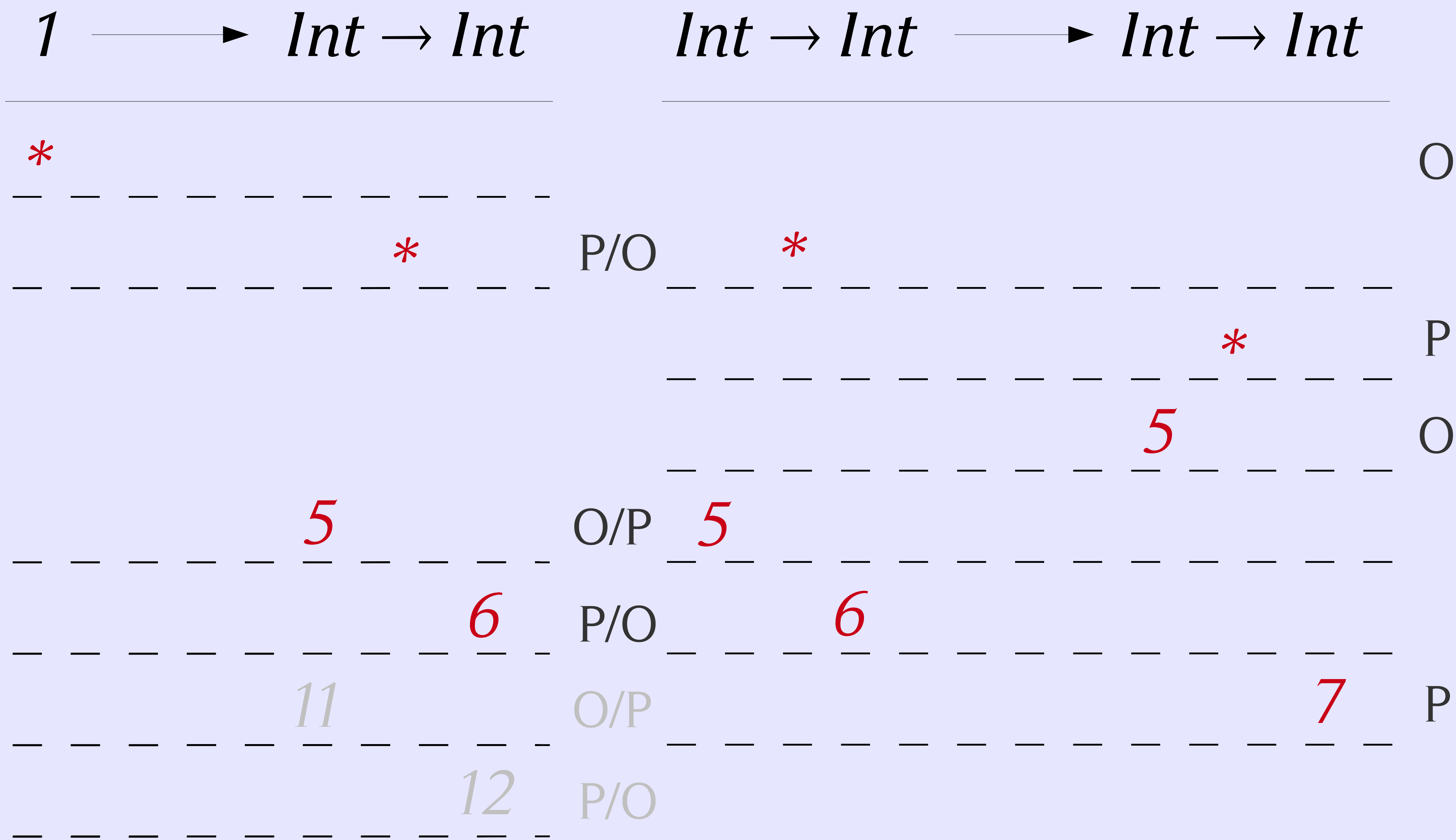

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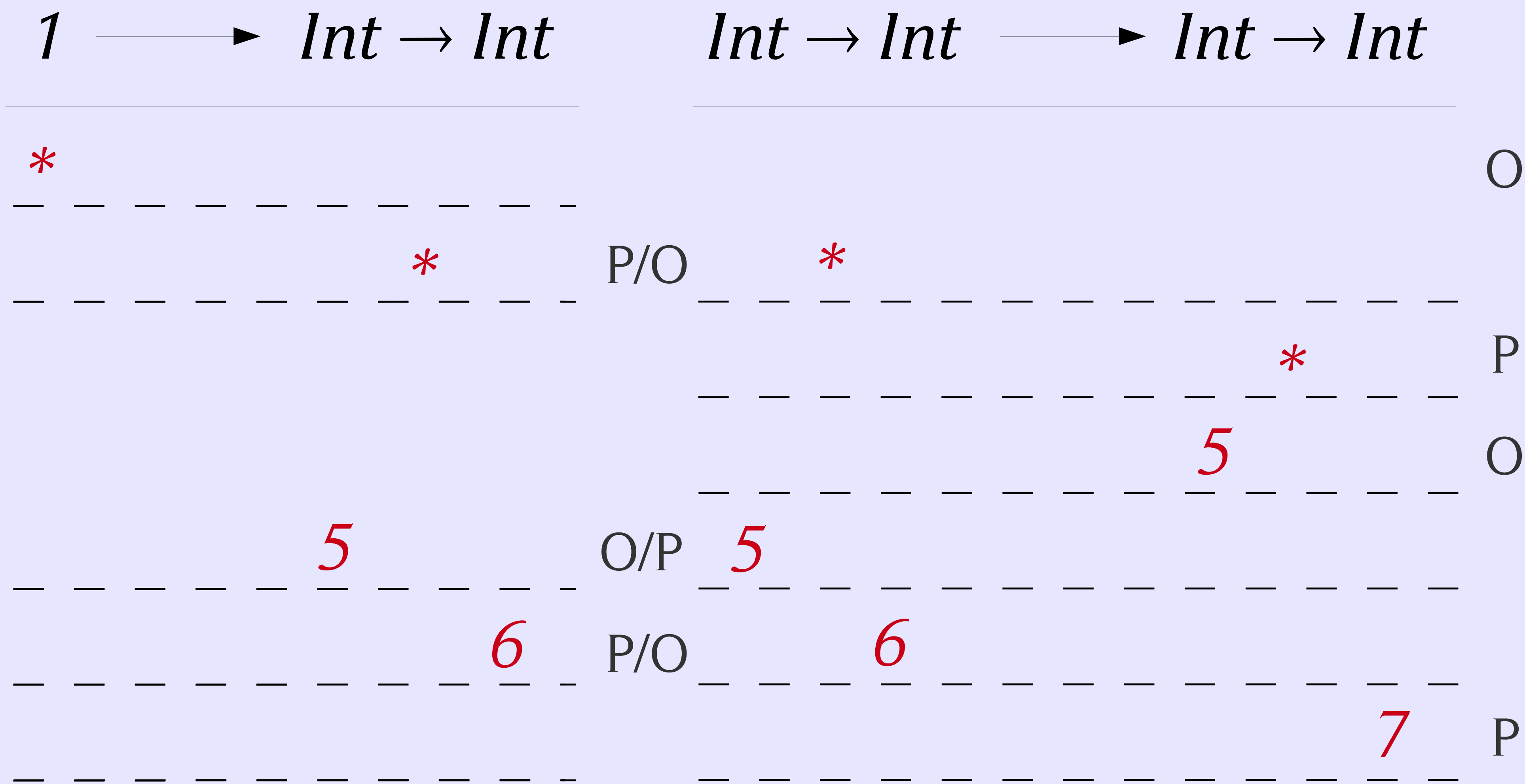

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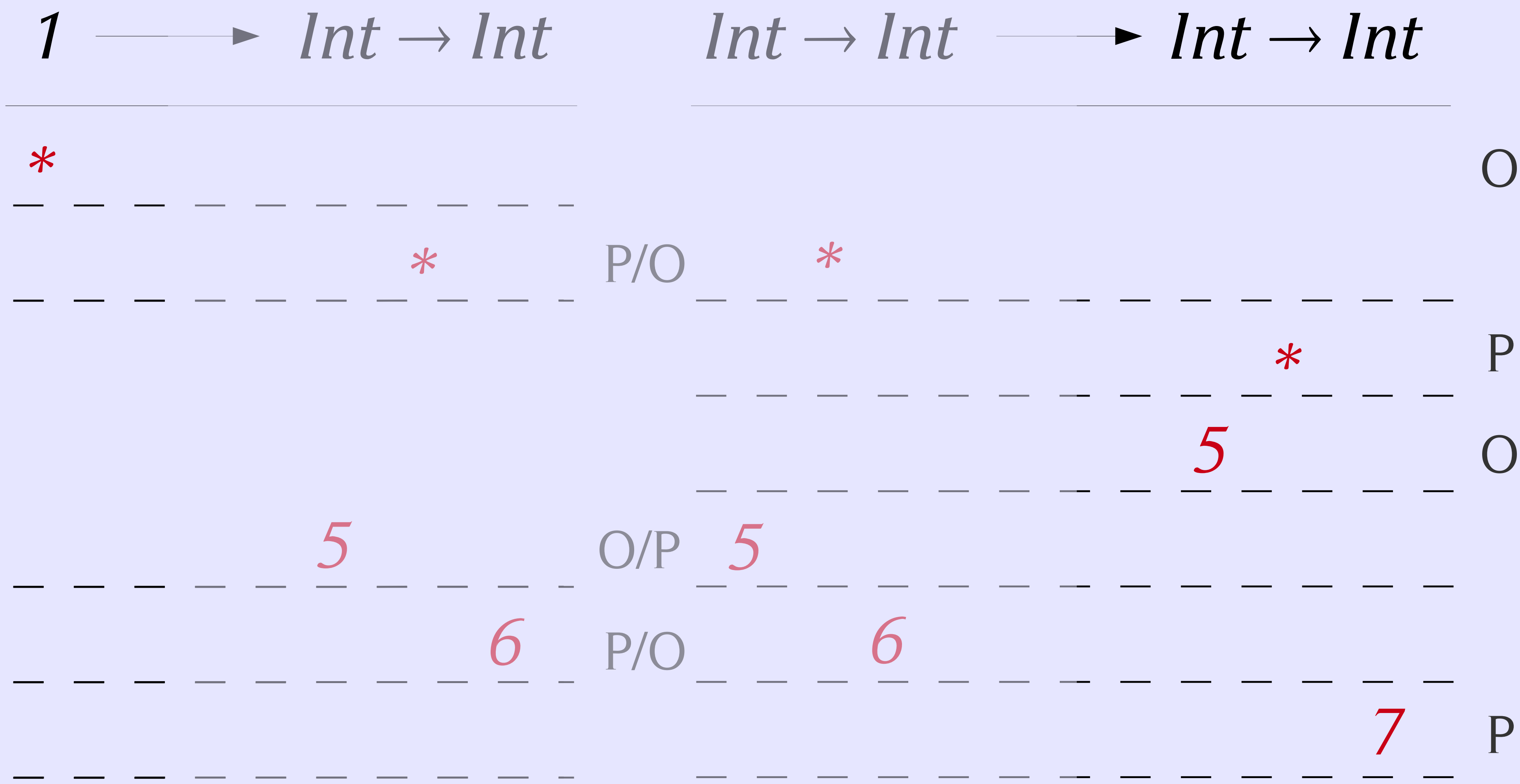

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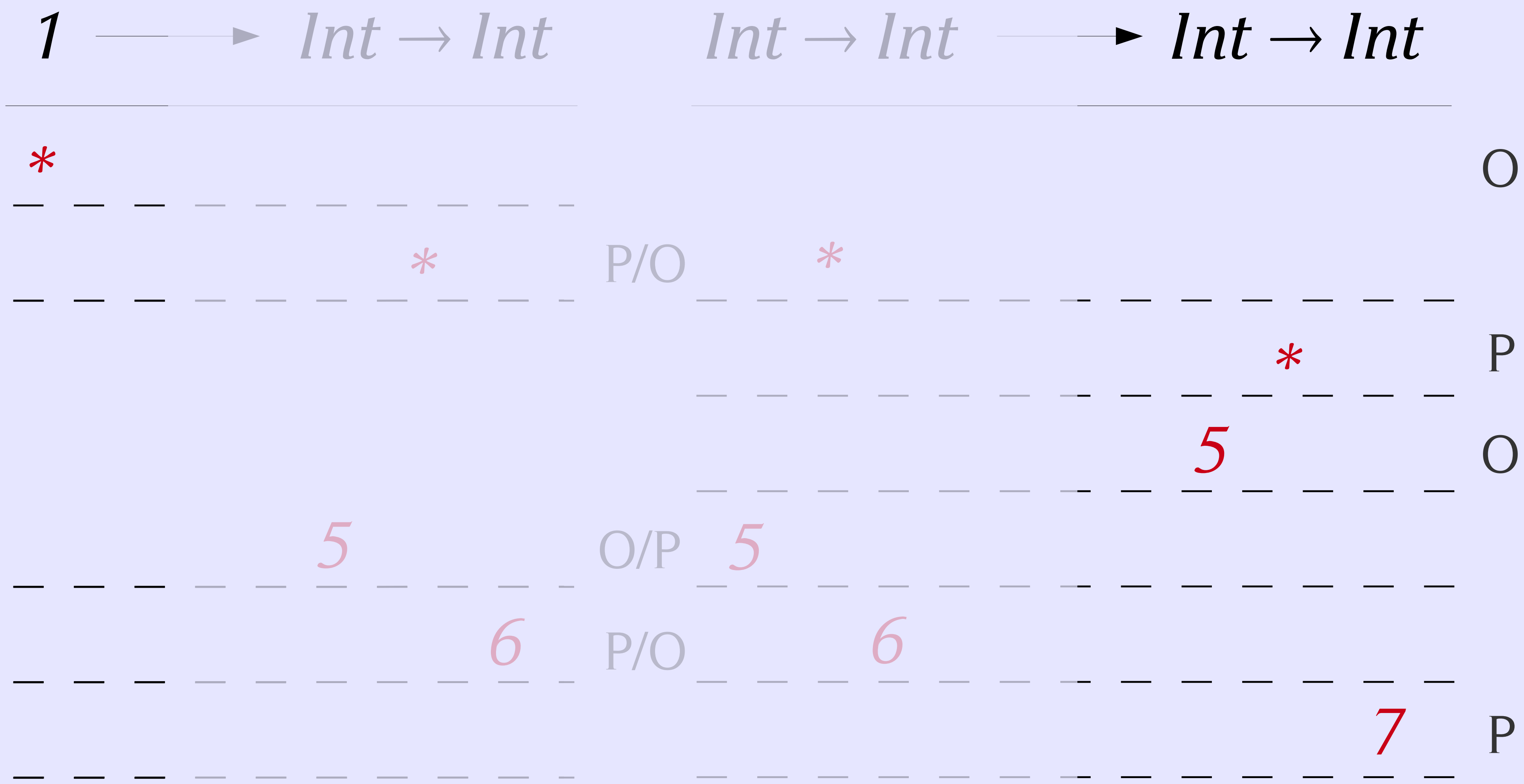

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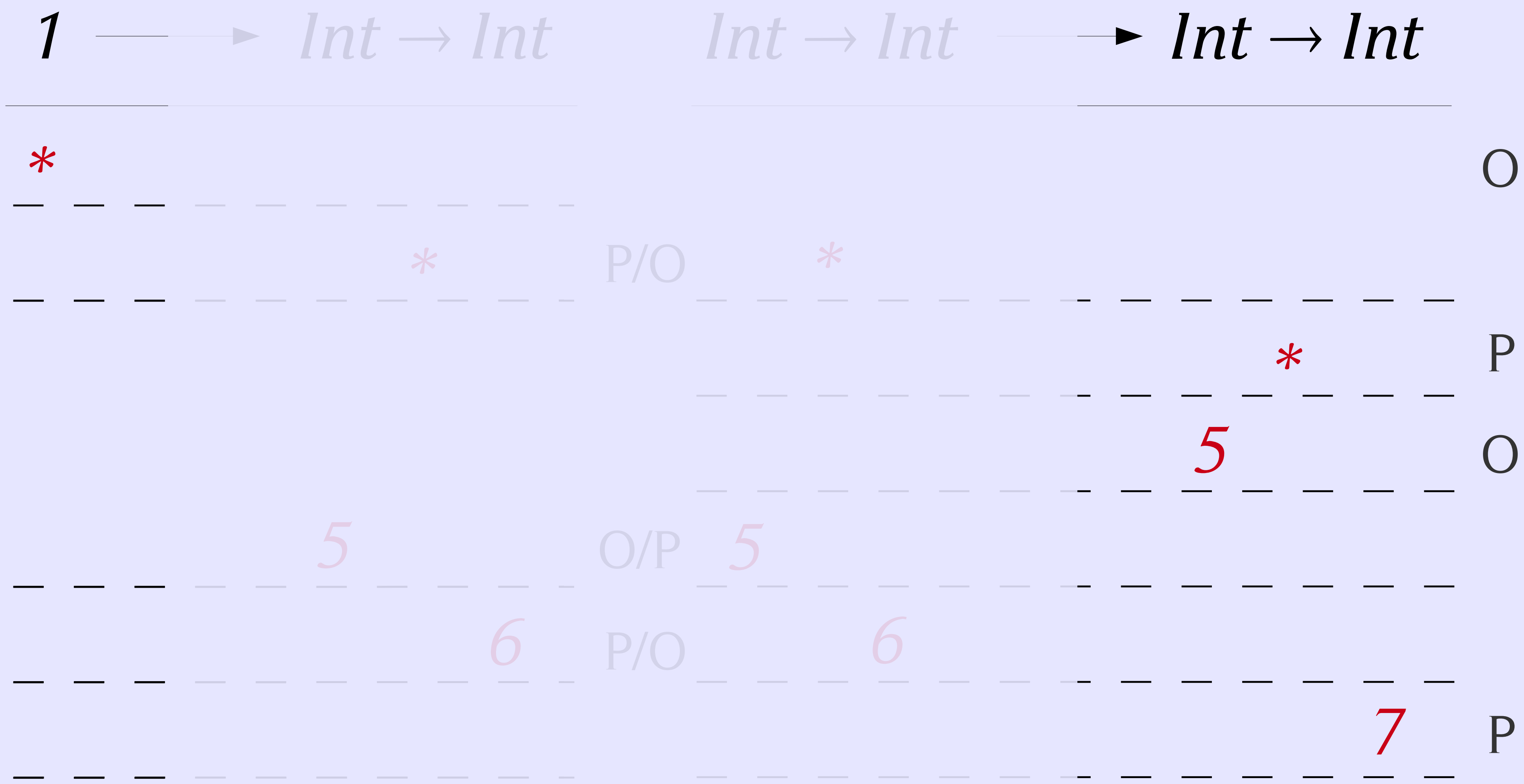

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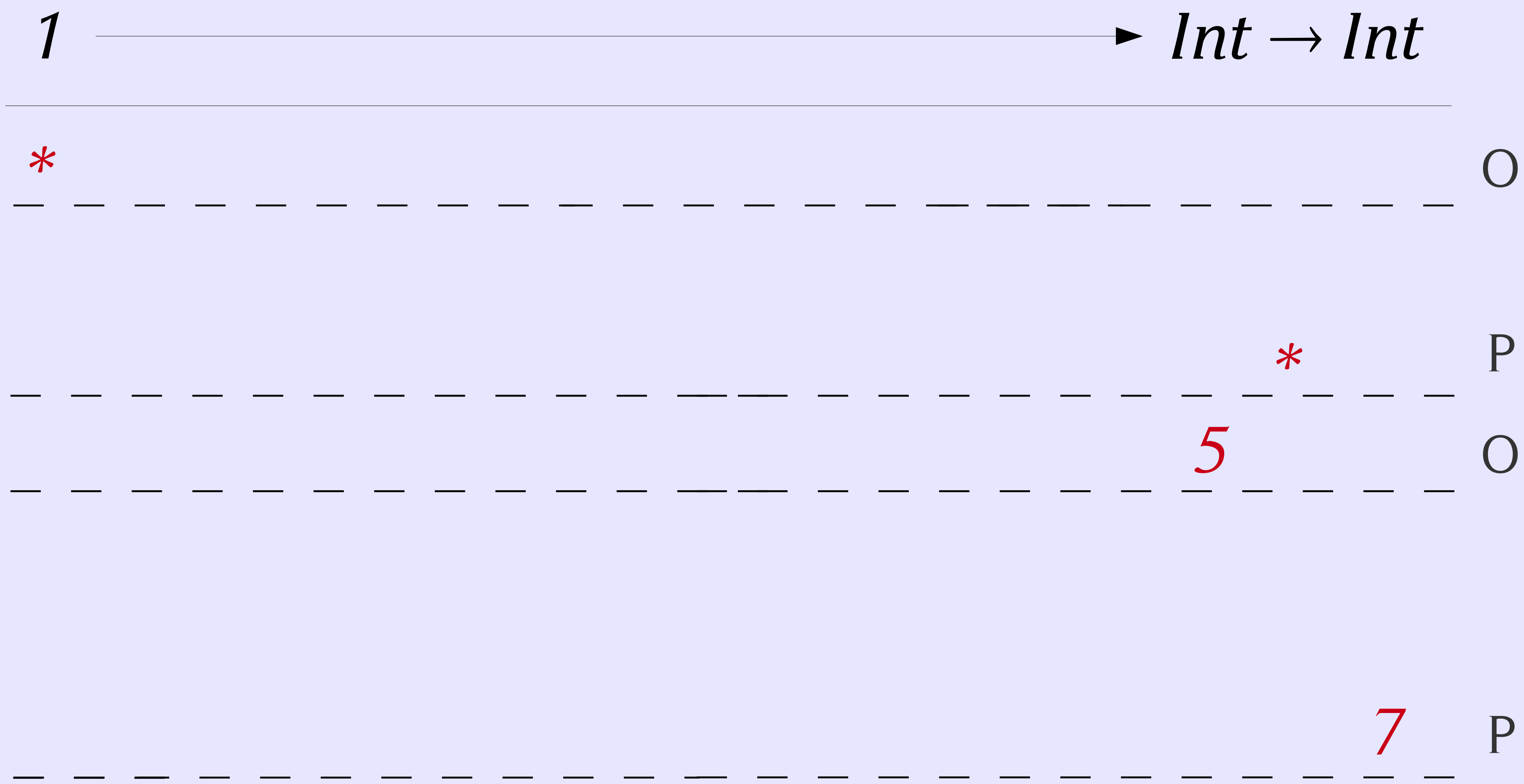

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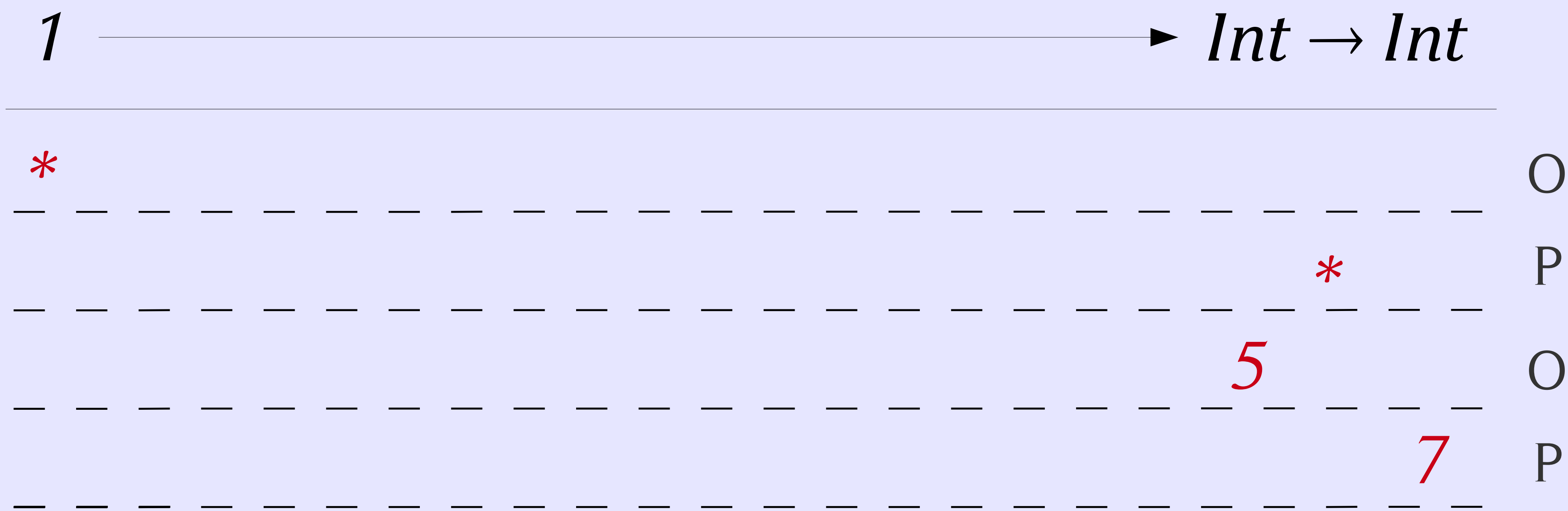

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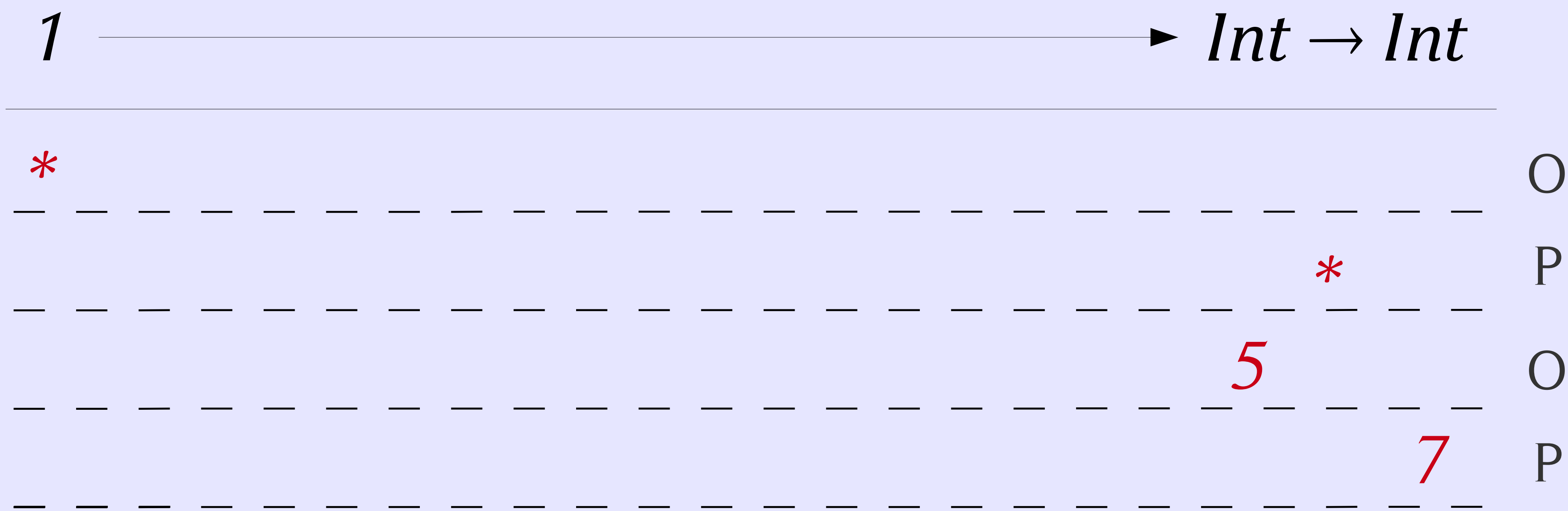
Composition



⊢ λx.x+1 : int ⟶ int ; f : int ⟶ int ⊢ λx.f(x)+1 : int ⟶ int

= ⊢ λx.x+2 : int ⟶ int

`⊢ λx.x+1 : int ⟶ int` `f : int ⟶ int ⊢ λx.f(x)+1 : int ⟶ int`

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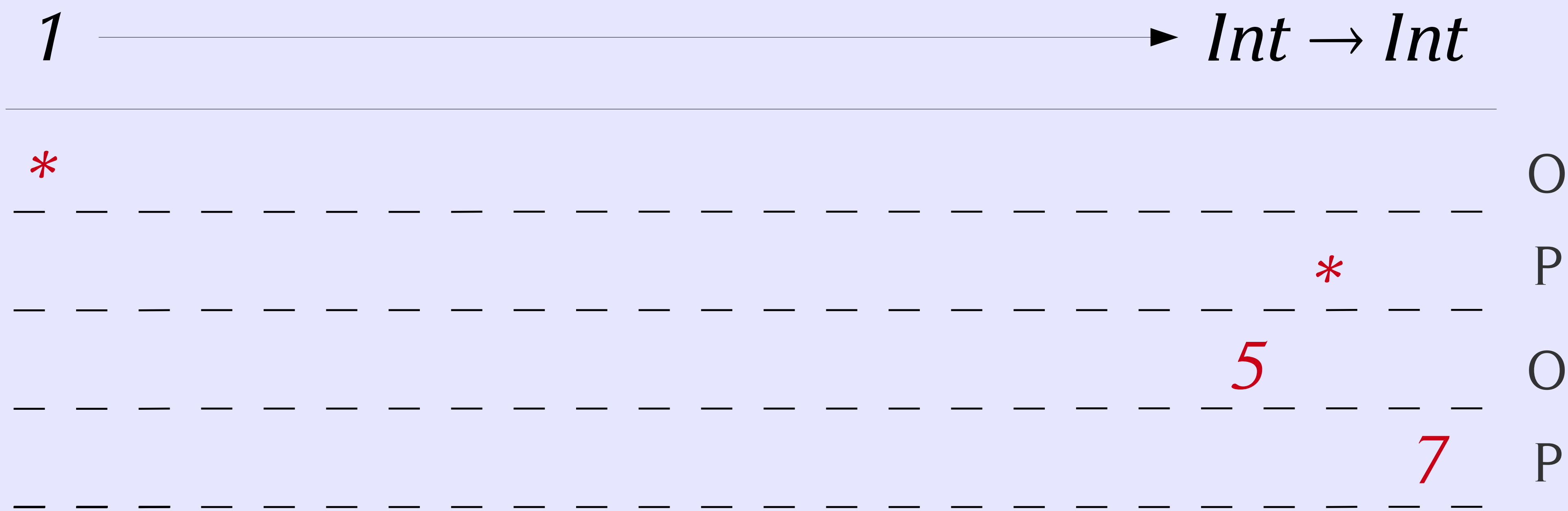

`⊢ λx.x+1 : int ⟶ int` **;** `f : int ⟶ int ⊢ λx.f(x)+1 : int ⟶ int`

= `⊢ λx.x+2 : int ⟶ int`

$$A \xrightarrow{\sigma} B \xrightarrow{\tau} C \quad = \quad A \xrightarrow{\sigma;\tau} C$$

Game Semantics

- Computation is modelled as a 2-player game between:
  - *Opponent* (the environment)
  - *Proponent* (the program)
- Qualitative games
- Programs = *strategies* for Proponent
- Families (i.e. *categories*) of games

Road to nominal games

Full Abstraction for PCF (early 90's)

- Two groups in the UK, one in Germany
- Roots in Mathematical Logic

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- Program analysis

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Nominal game semantics (2004-)

Nominal sets

Foundation for doing maths with names and name-binding

- Countably infinite set $\mathcal{N}$ of names

Nominal sets

Foundation for doing maths with names and name-binding

- Countably infinite set $\mathcal{N}$ of names
- A nominal set X consists of:
  - a carrier set X
  - an action $\bullet : \mathrm{PERM}(\mathcal{N}) \times X \longrightarrow X$

    e.g. $(n_1\ n_2) \bullet n_1 n_1 n_2 n_1 = n_2 n_2 n_1 n_2$
  - all elements of X have finite support
- NOM: nominal sets and equivariant functions

Nominal games

Nominal games

$\vdash$ `λx.ref(0)` : `com` $\rightarrow$ `intref`

```
let f = [_] in { f() == f() }
```

Nominal games

$\vdash \lambda x.\mathtt{ref(0)} : \mathtt{com} \to \mathtt{intref}$

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let f = [_] in { f() == f() }
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Nominal games

Games in nominal sets

names

$\vdash \lambda x.\mathtt{ref(0)} : \mathtt{com} \to \mathtt{intref}$

```
let f = [_] in { f() == f() }
```

Examples

$\vdash \lambda x.\mathtt{ref(0)} : \mathtt{com} \to \mathtt{intref}$

$$1 \longrightarrow 1 \to \mathit{Ref}$$

O

P

O

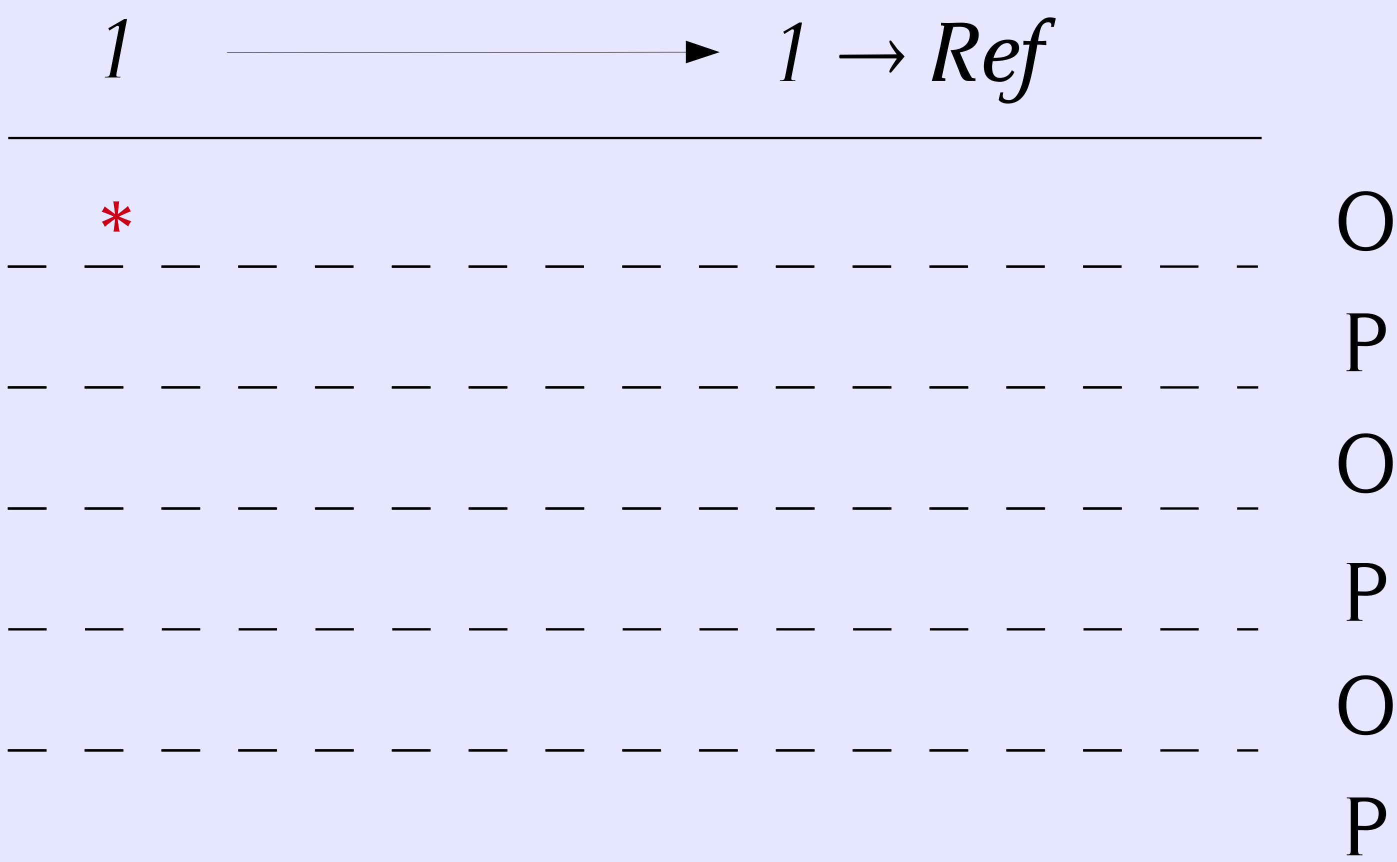
P

O

P

Examples

$\vdash$ `λx.ref(0)` : `com` $\rightarrow$ `intref`



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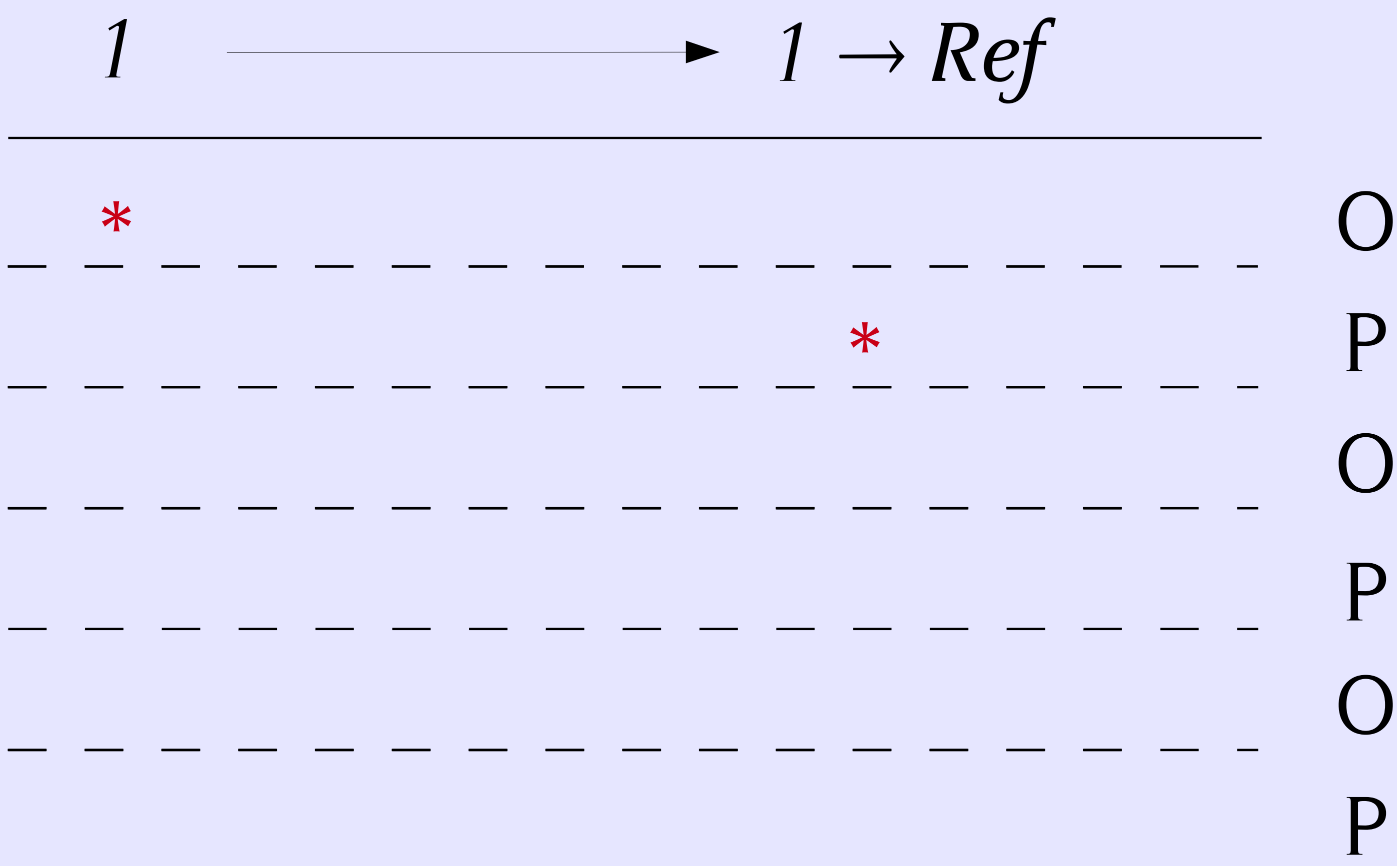

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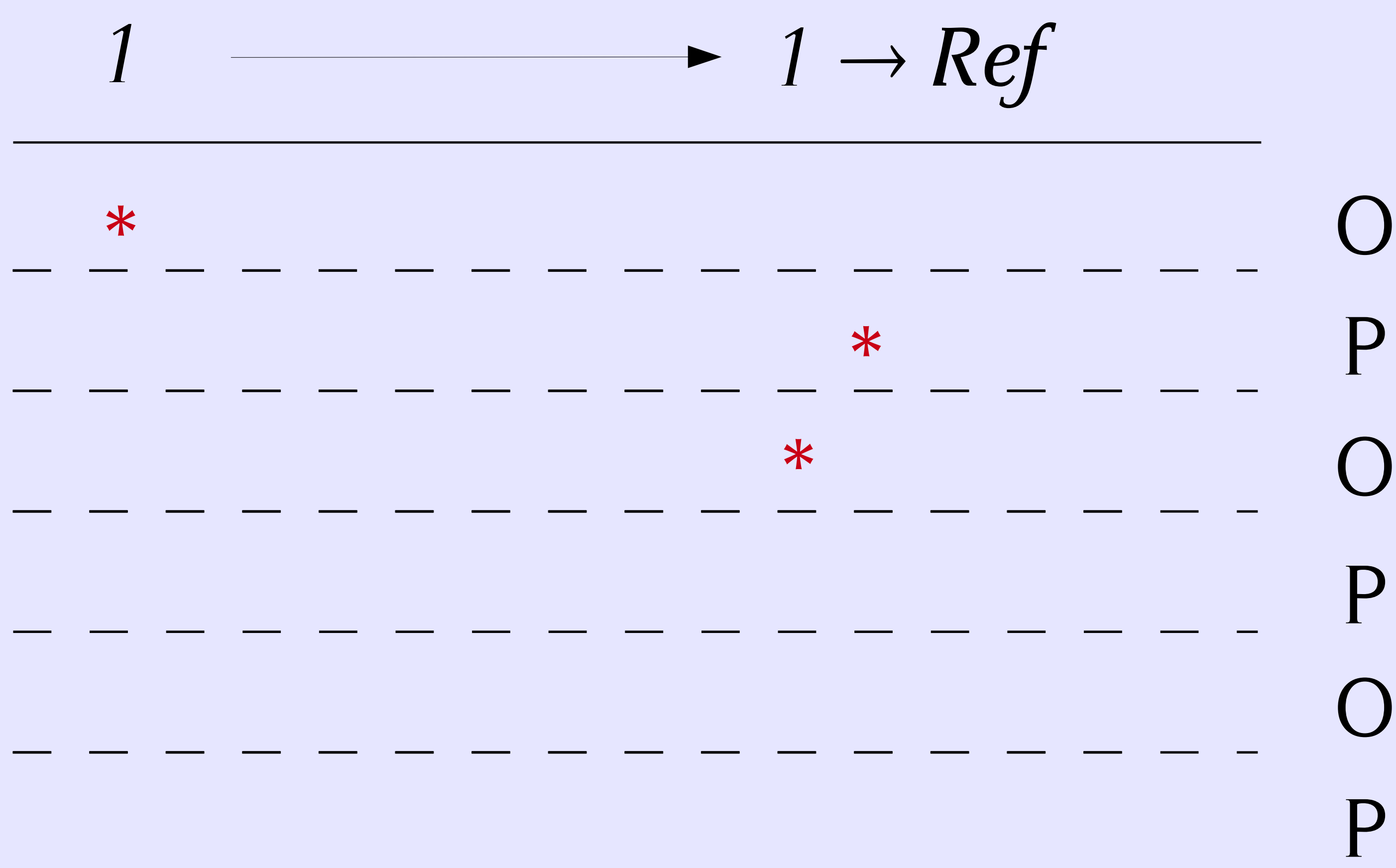

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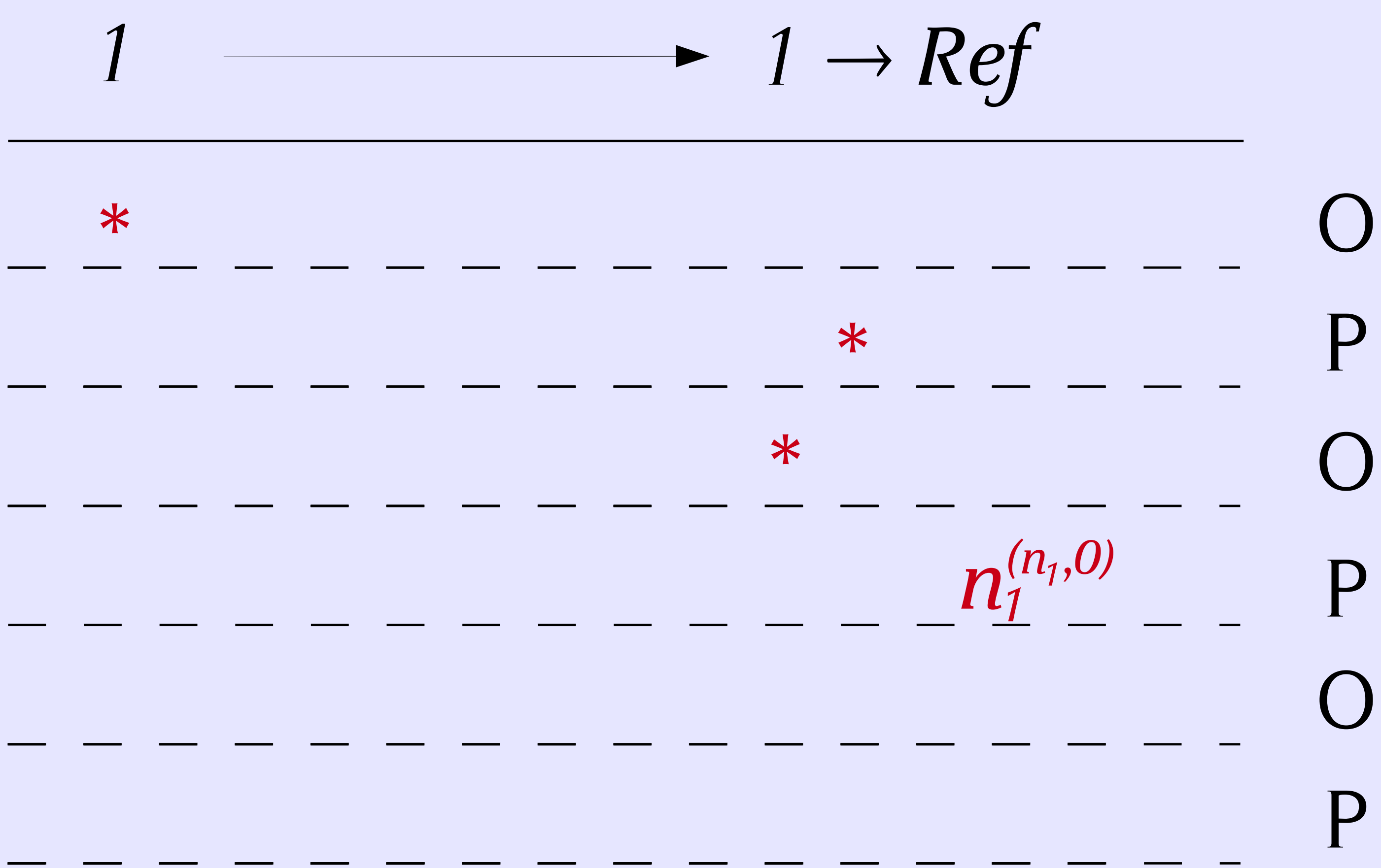

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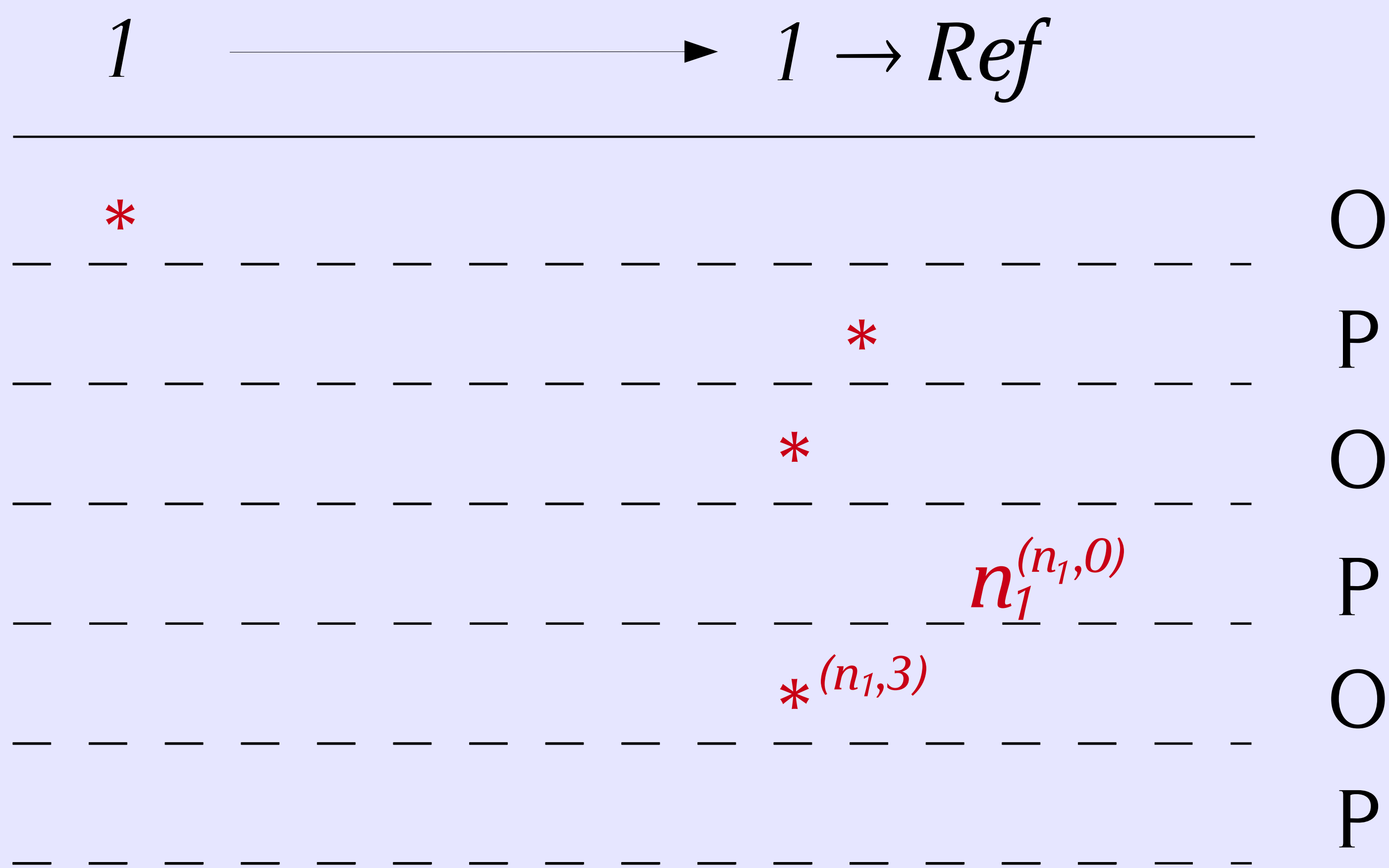

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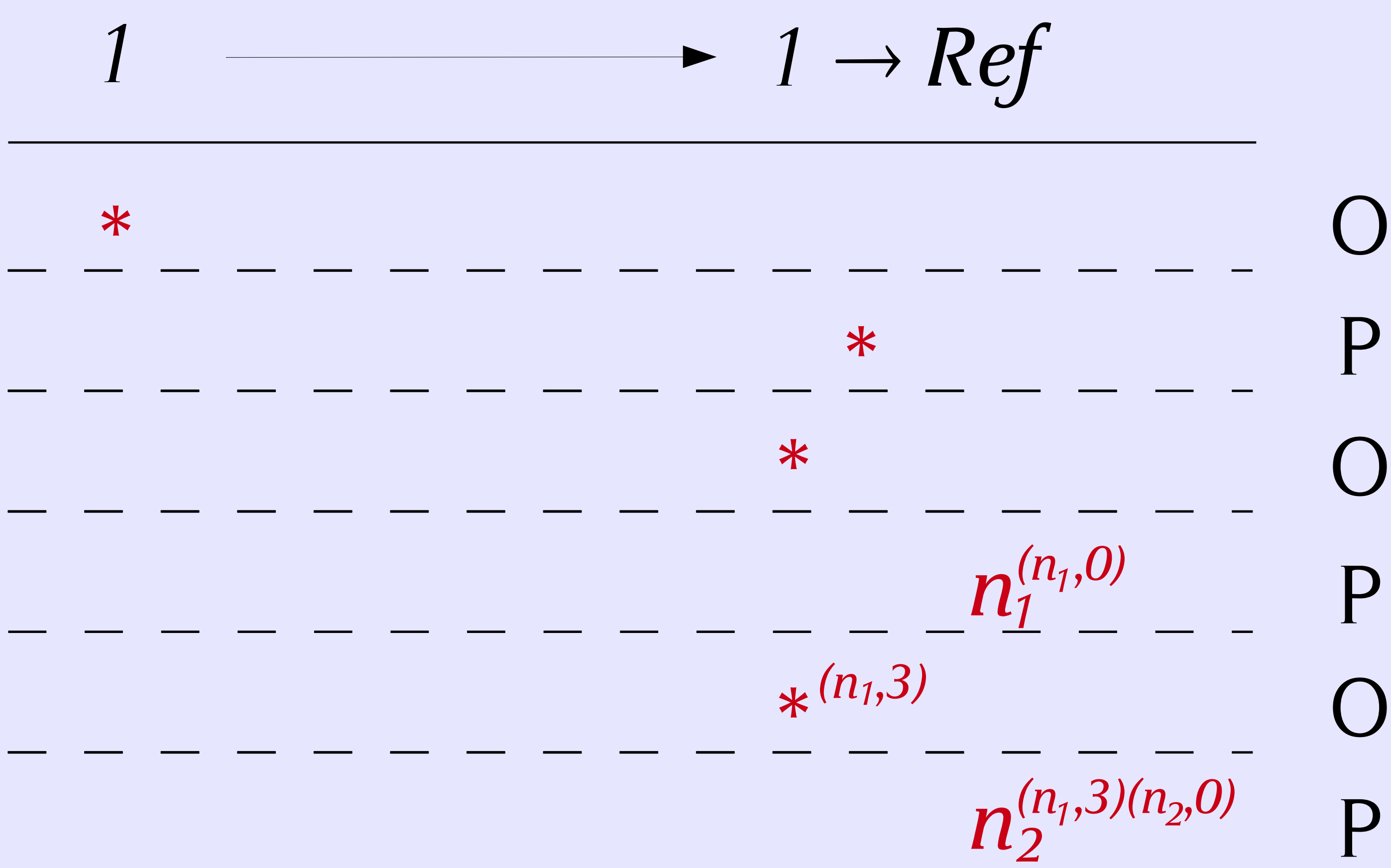

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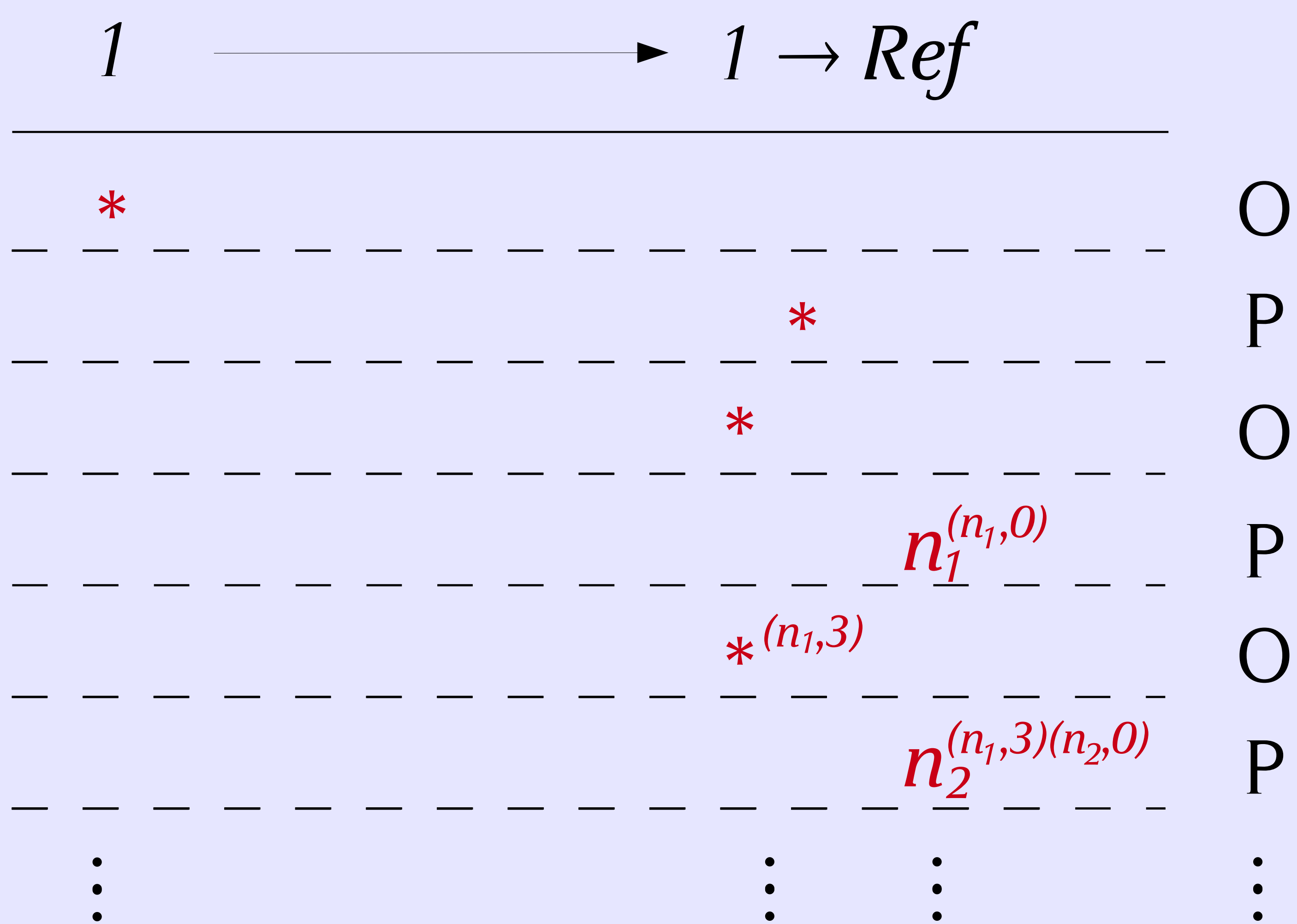

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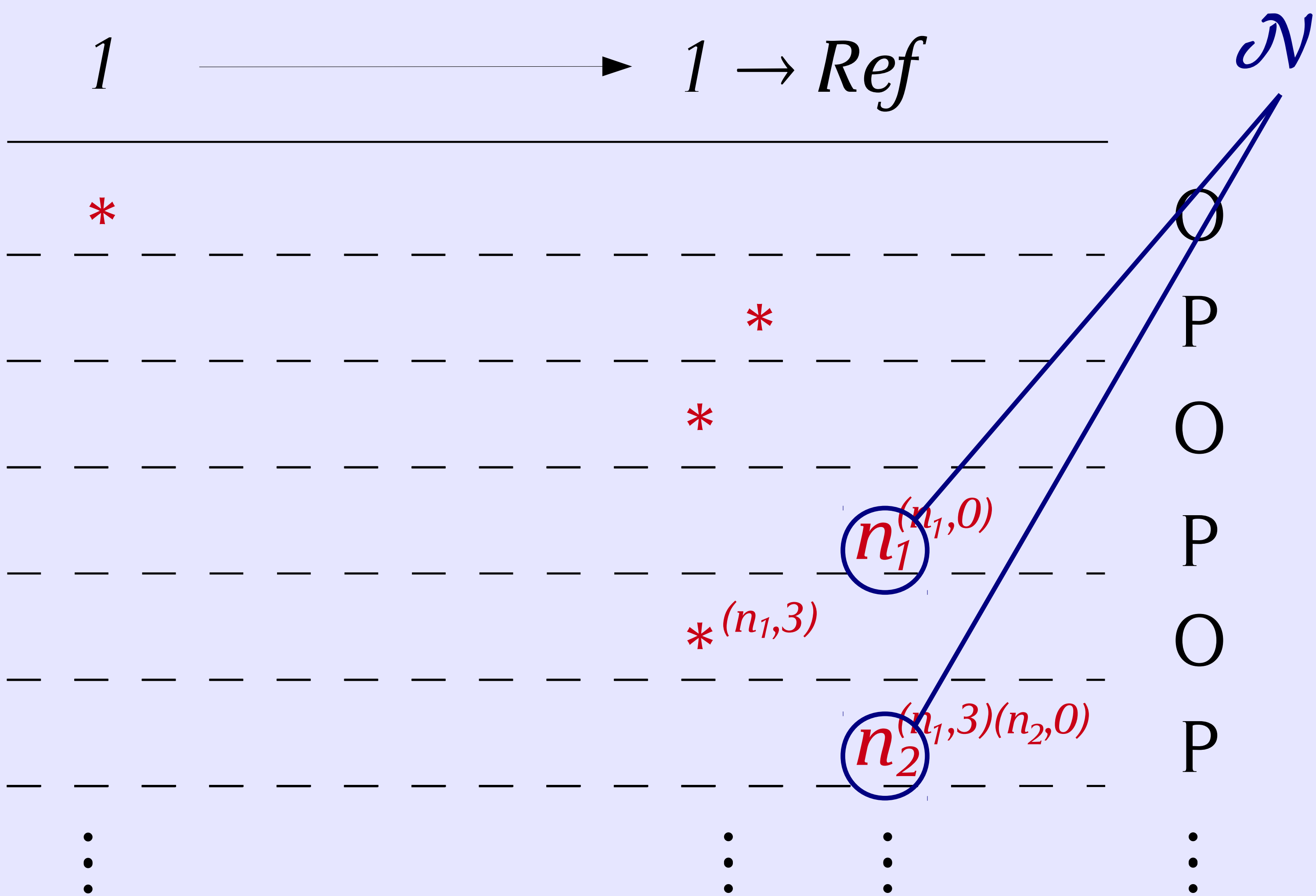
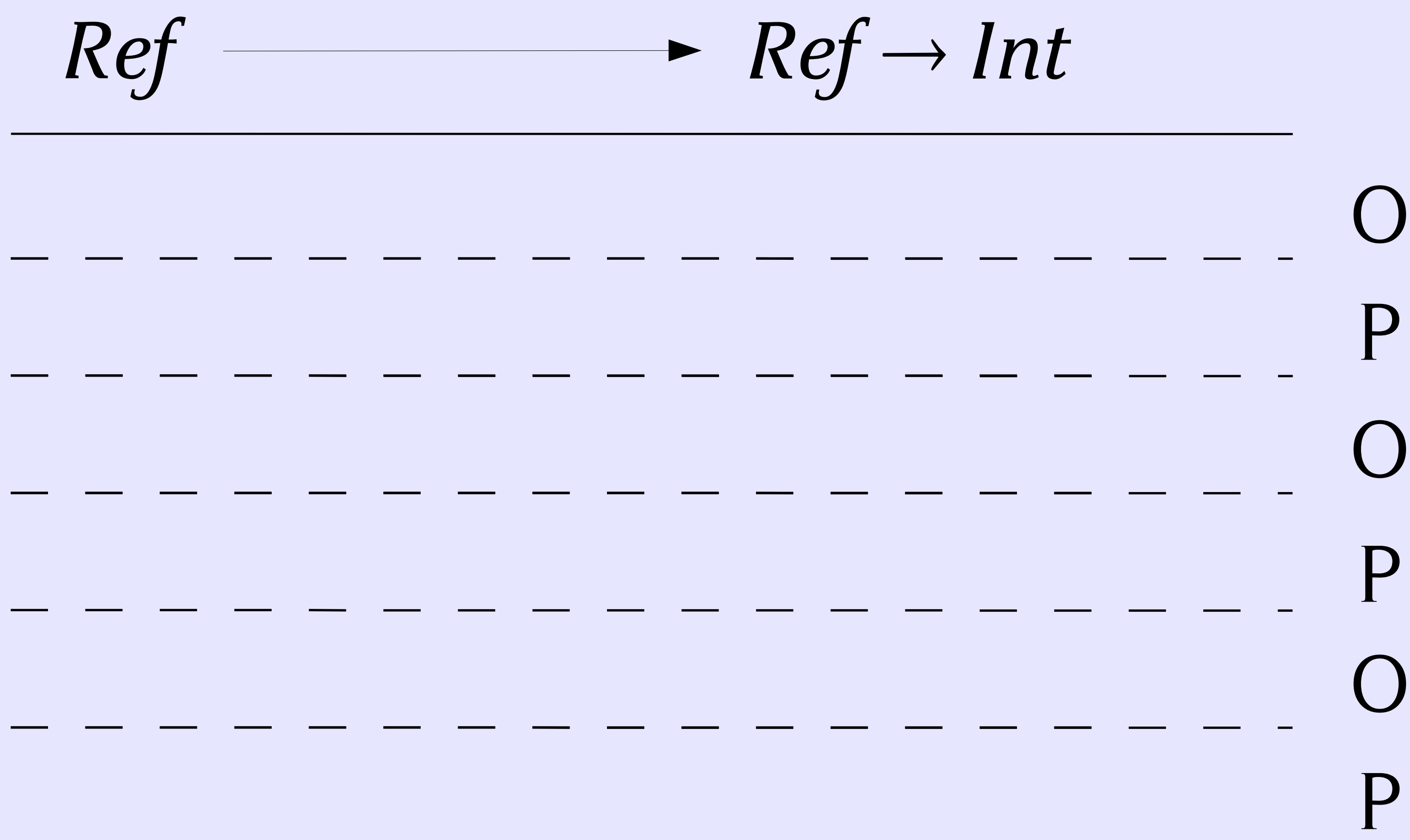


Examples

$\texttt{x : intref} \vdash \lambda\texttt{y.(x == y) : intref} \to \texttt{int}$



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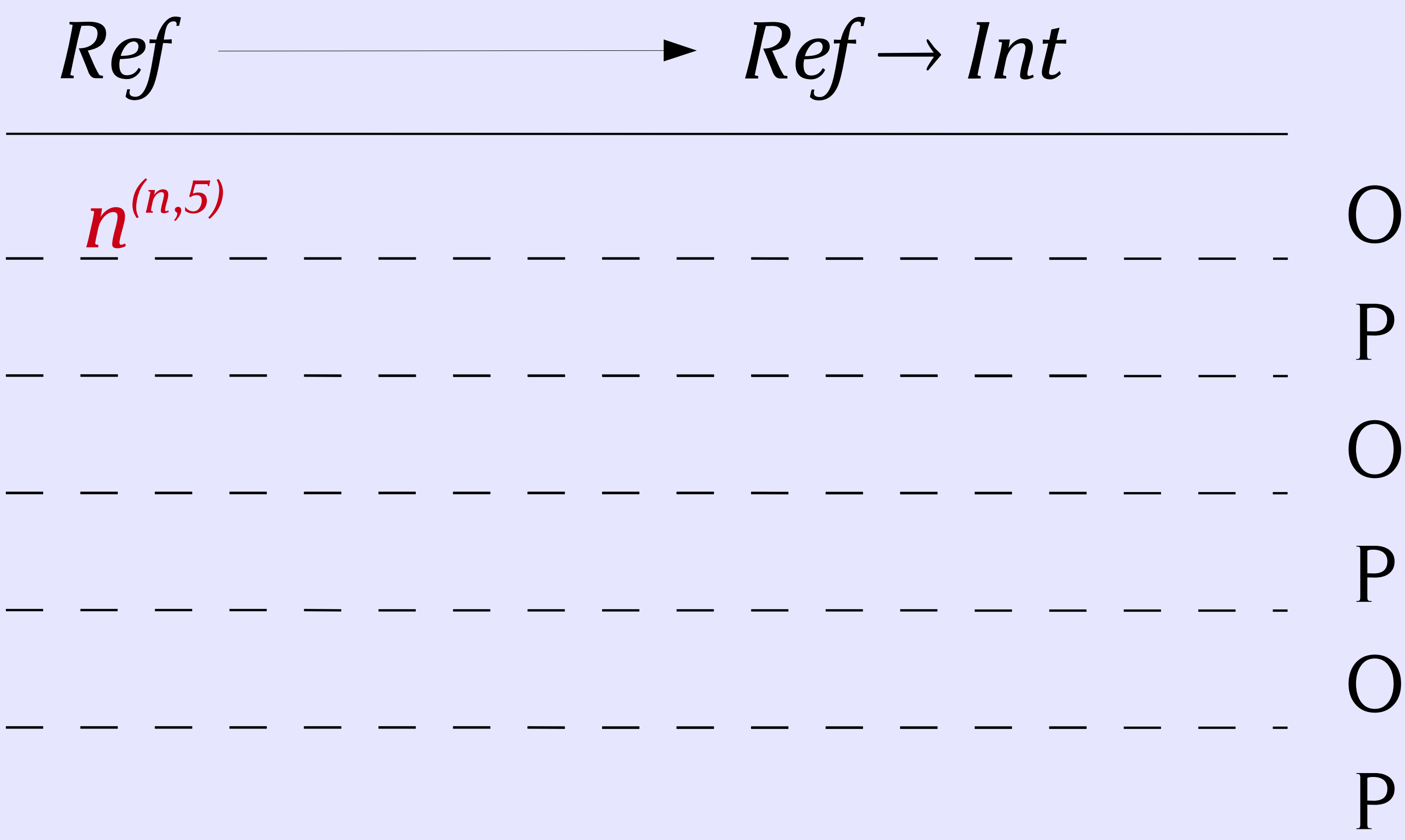

Examples

$\texttt{x : intref} \vdash \lambda\texttt{y.(x == y) : intref} \to \texttt{int}$

$$Ref \longrightarrow Ref \to Int$$

$n^{(n,5)}$		O
	$*^{(n,5)}$	P
		O
		P
		O
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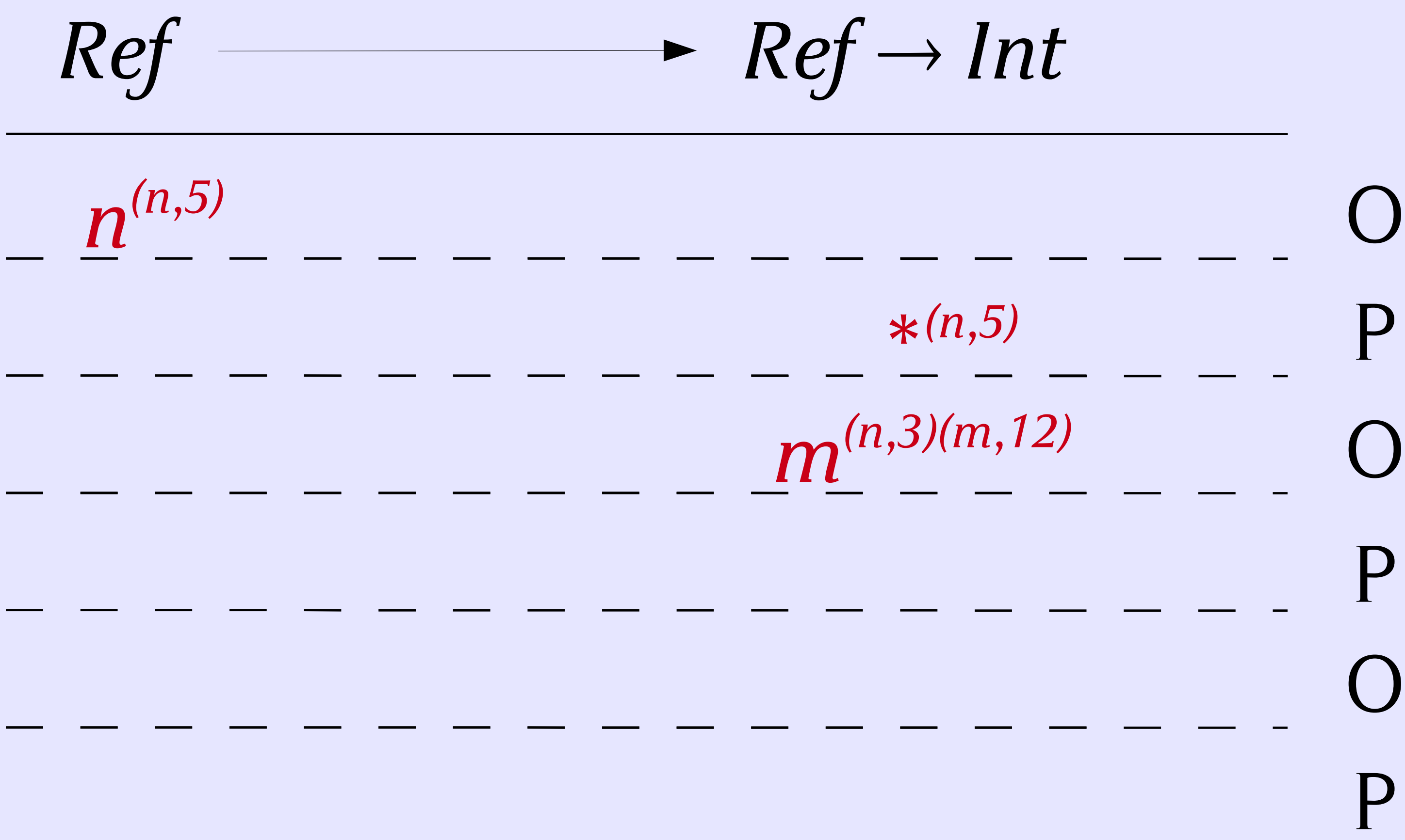

Examples

`x : intref` $\vdash$ `λy.(x == y) : intref` $\to$ `int`

$$Ref \longrightarrow Ref \to Int$$

Ref	$Ref \to Int$	
$n^{(n,5)}$		O
	$*^{(n,5)}$	P
	$m^{(n,3)(m,12)}$	O
	$0^{(n,3)(m,12)}$	P
		O
		P

Examples

$\texttt{x : intref} \vdash \lambda\texttt{y.(x == y) : intref} \to \texttt{int}$

$$Ref \longrightarrow Ref \to Int$$

Move	
$n^{(n,5)}$	O
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$m^{(n,3)(m,12)}$	O
$0^{(n,3)(m,12)}$	P
$n^{(n,13)}$	O
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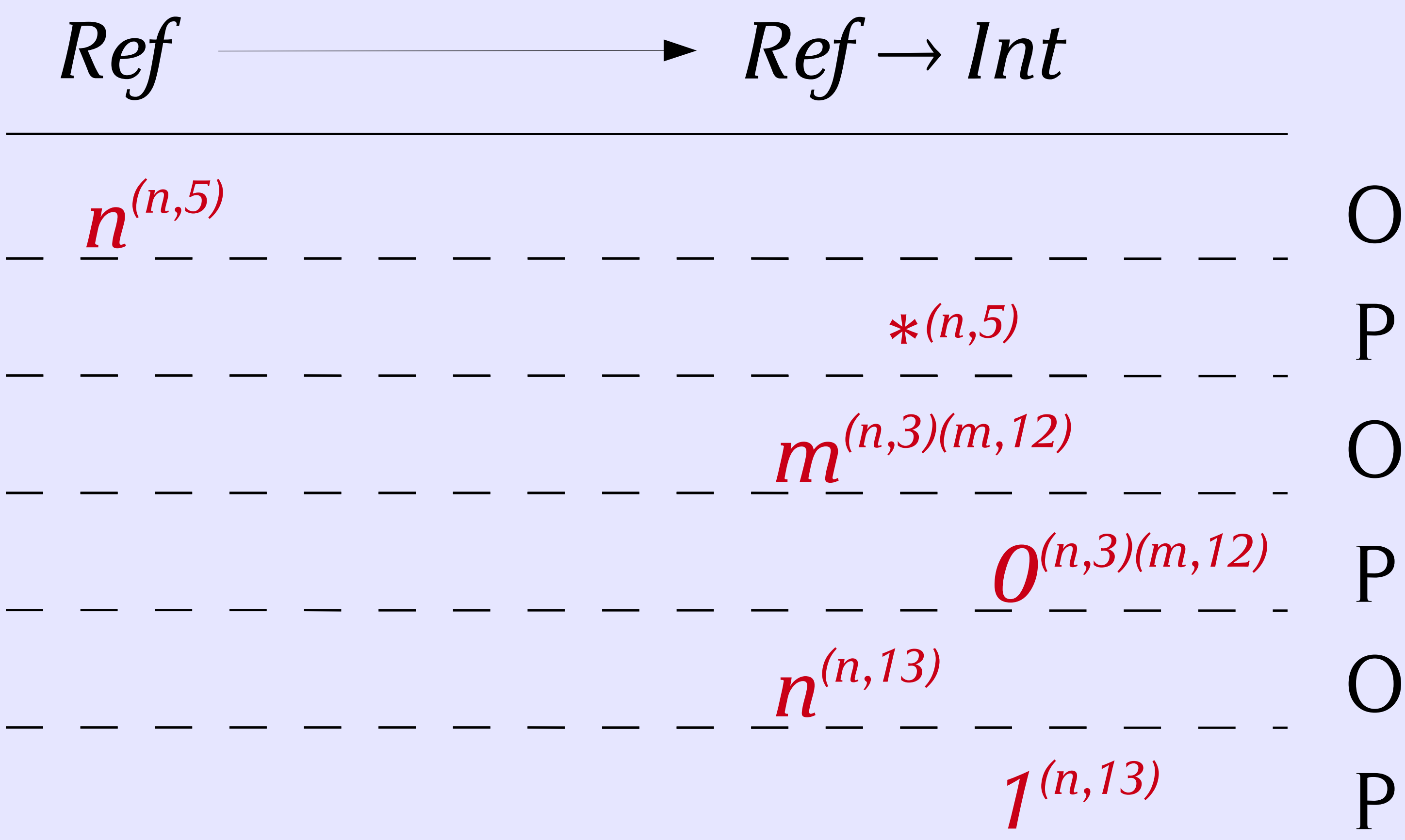
Examples

$x : \texttt{intref} \vdash \lambda y.(x == y) : \texttt{intref} \to \texttt{int}$



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$\vdots$		$\vdots$		$\vdots$	$\vdots$

Achievements

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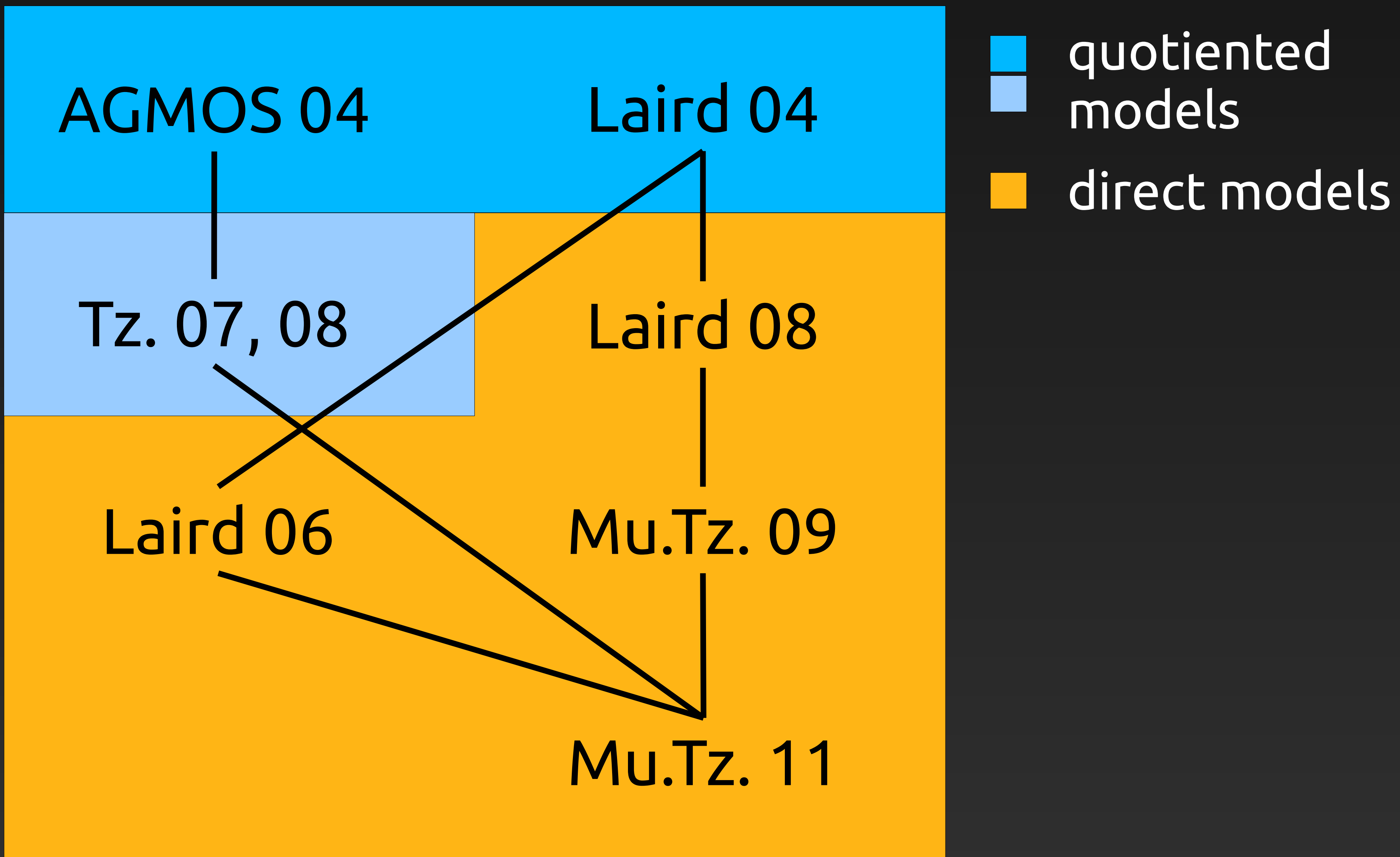

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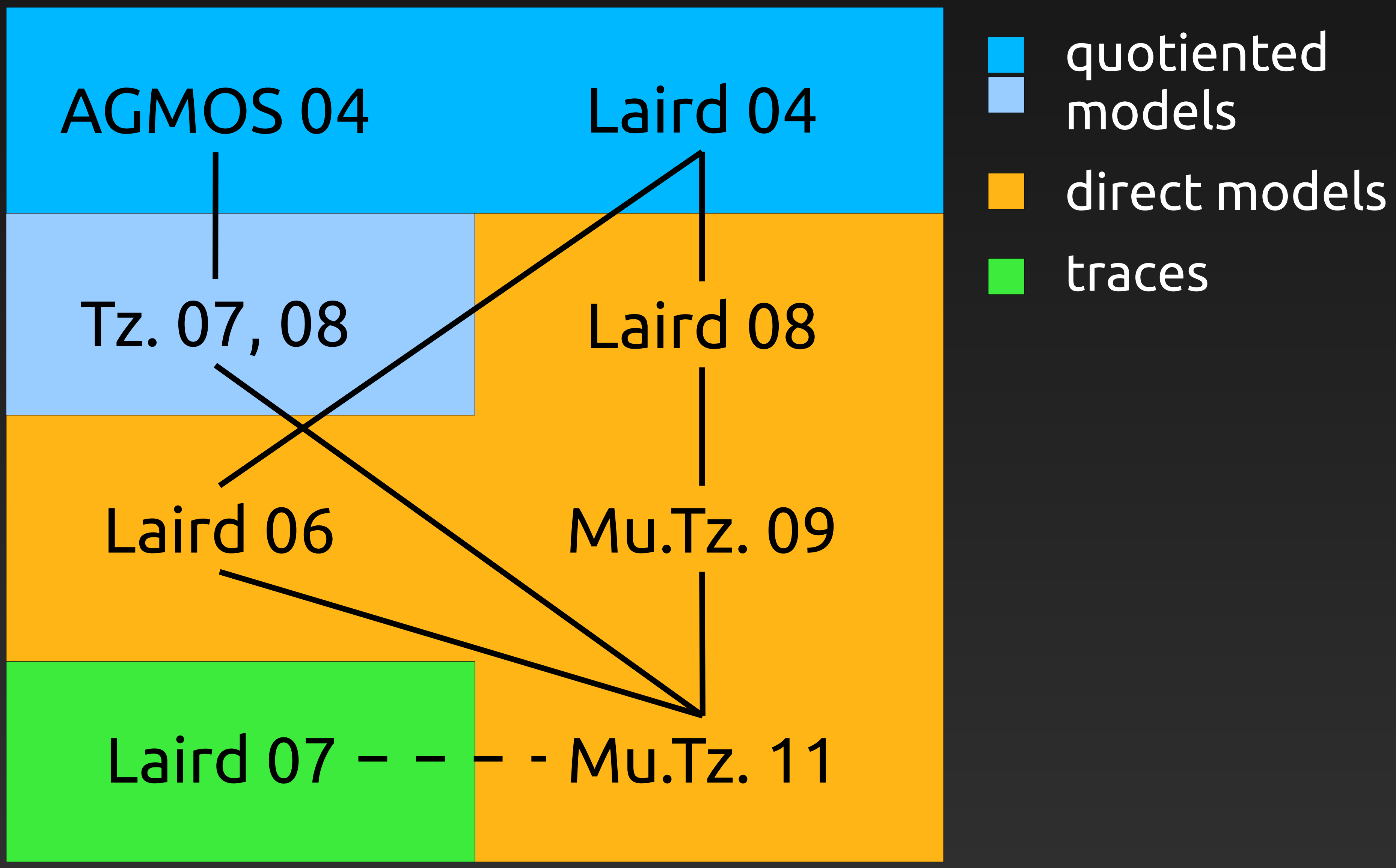

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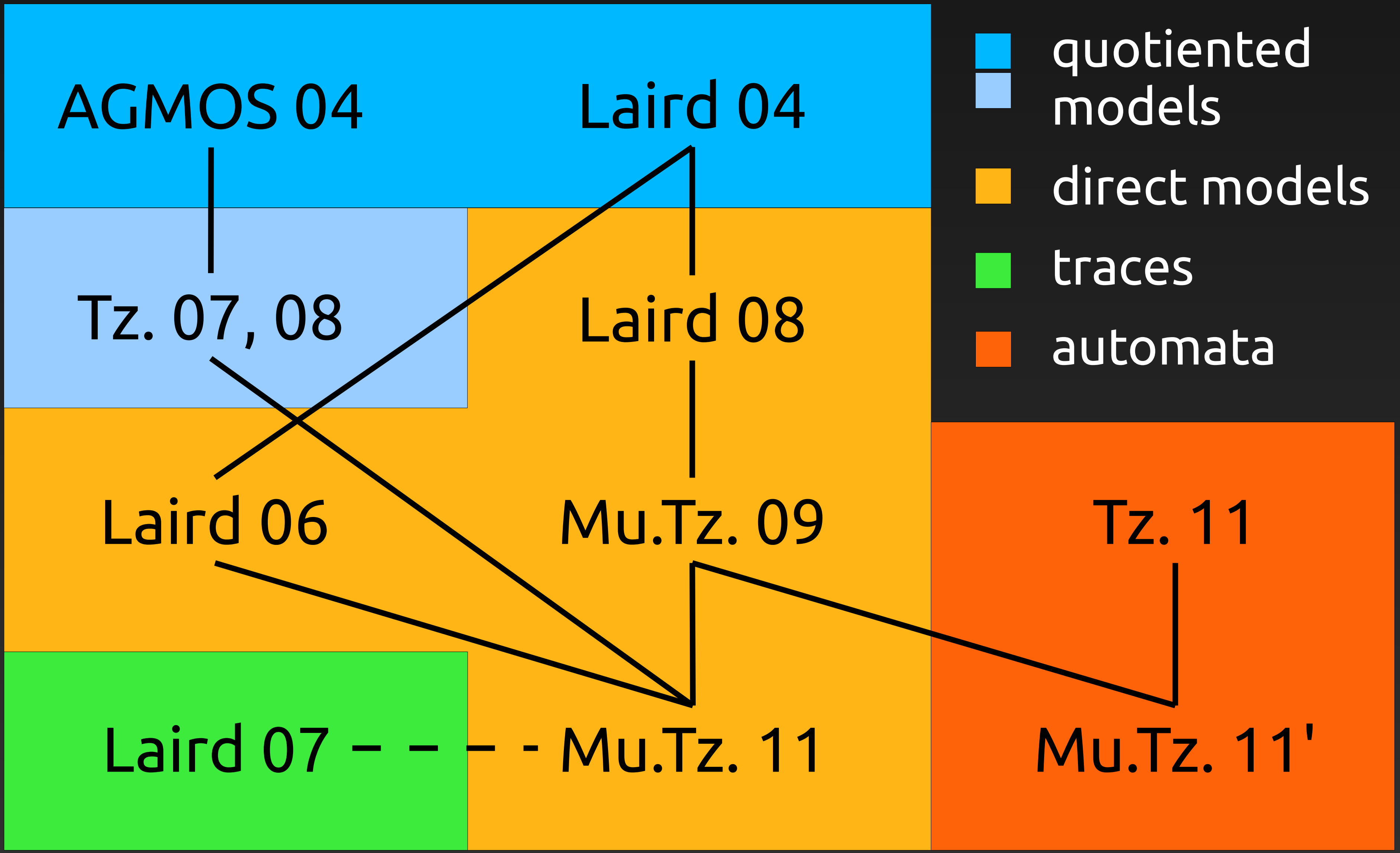

Quotiented models

- AGMOS 04, Laird 04: nu-calculus
- Tz. 07, 08: HO references, exceptions, ...

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$$[\![A \to B]\!] = [\![A]\!] \Rightarrow (S \Rightarrow [\![B]\!] \otimes S)$$

$$S = \bigotimes_A (\mathsf{N}_A \Rightarrow [\![A]\!])$$

Quotiented models

- AGMOS 04, Laird 04: nu-calculus
- Tz. 07, 08: HO references, exceptions, ...

$$M \cong N \iff [\![M]\!] \cong [\![N]\!]$$

Characteristics:

- moves involving names
- moves-with-state (a set/list of names)
- “functional” behaviour + monads for effects

Direct models

- Laird 06: higher-order channels
  Laird 08: pointers
- Mu.Tz. 09: integer references (Reduced ML)
  Mu.Tz. 11: HO references

$$M \cong N \iff \mathsf{comp}(\llbracket M \rrbracket) = \mathsf{comp}(\llbracket N \rrbracket)$$

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Characteristics:

- moves involving names/ moves-with-store
- name-availability conditions/ “direct” effects

Direct models

- Laird 06: hi
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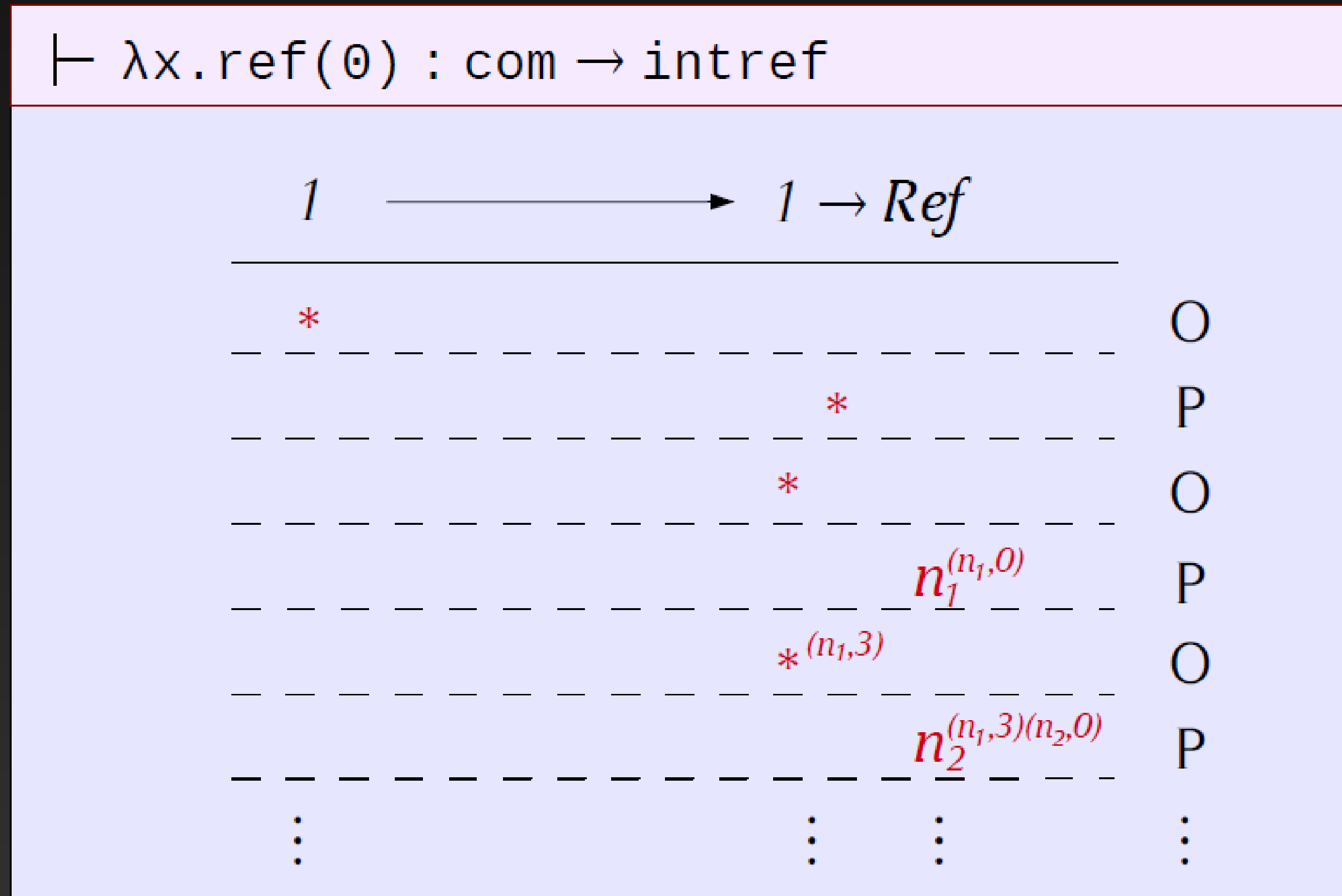

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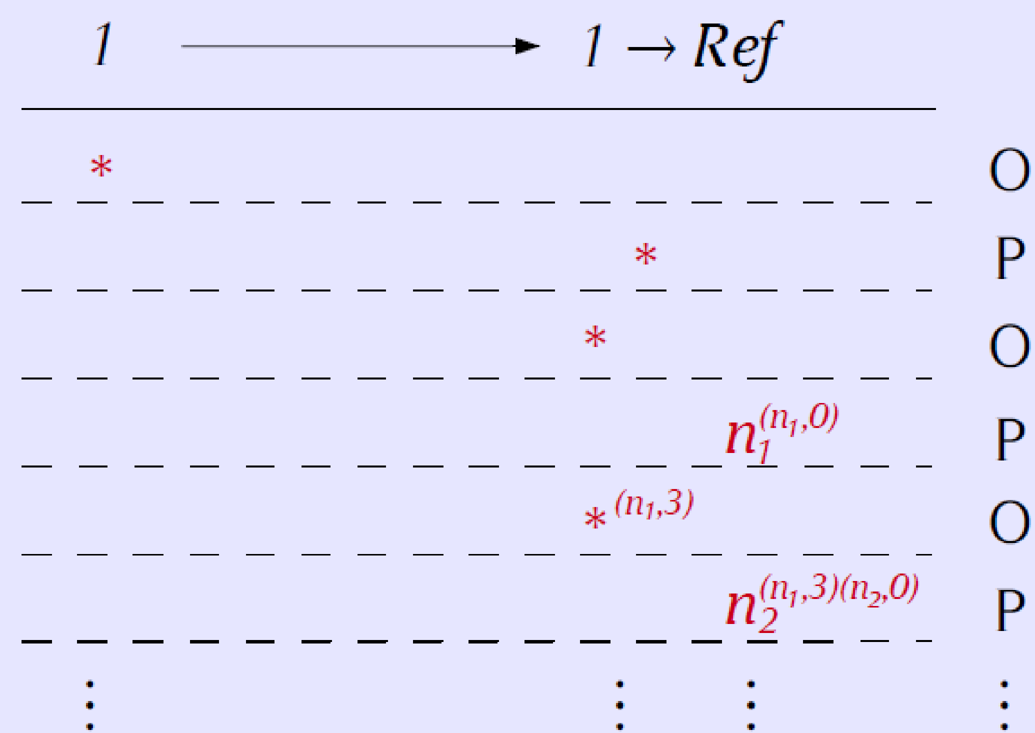

Characteristics

- moves involving names/ moves-with-store
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Algorithmics

Fresh-register automata

$\texttt{λz.ref(0)} \longmapsto \{\, * * * n_1 * n_2 * n_3 \ldots \mid n_i\text{'s distinct} \,\}$

Fresh-register automata

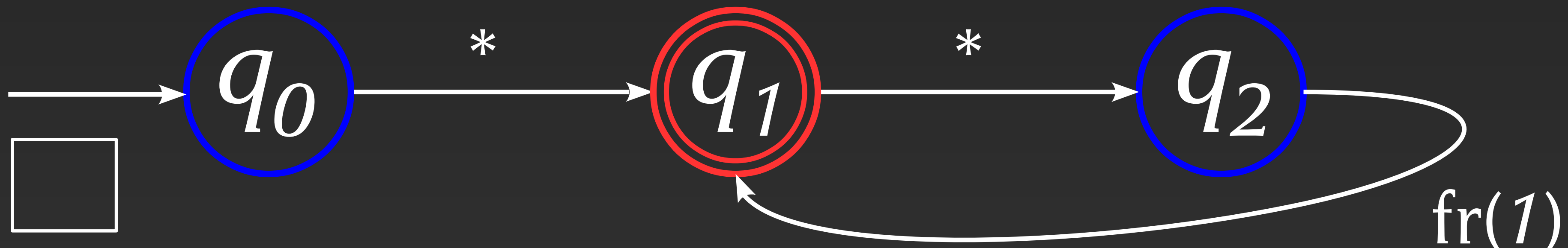
`λz.ref(0)` $\longmapsto \{ * * * n_1 * n_2 * n_3 \ldots \mid n_i\text{'s distinct} \}$

Automata with names

- Infinite alphabet $\mathcal{N}$
- Freshness recognition

Finite-state machines with registers

Fresh-register automata



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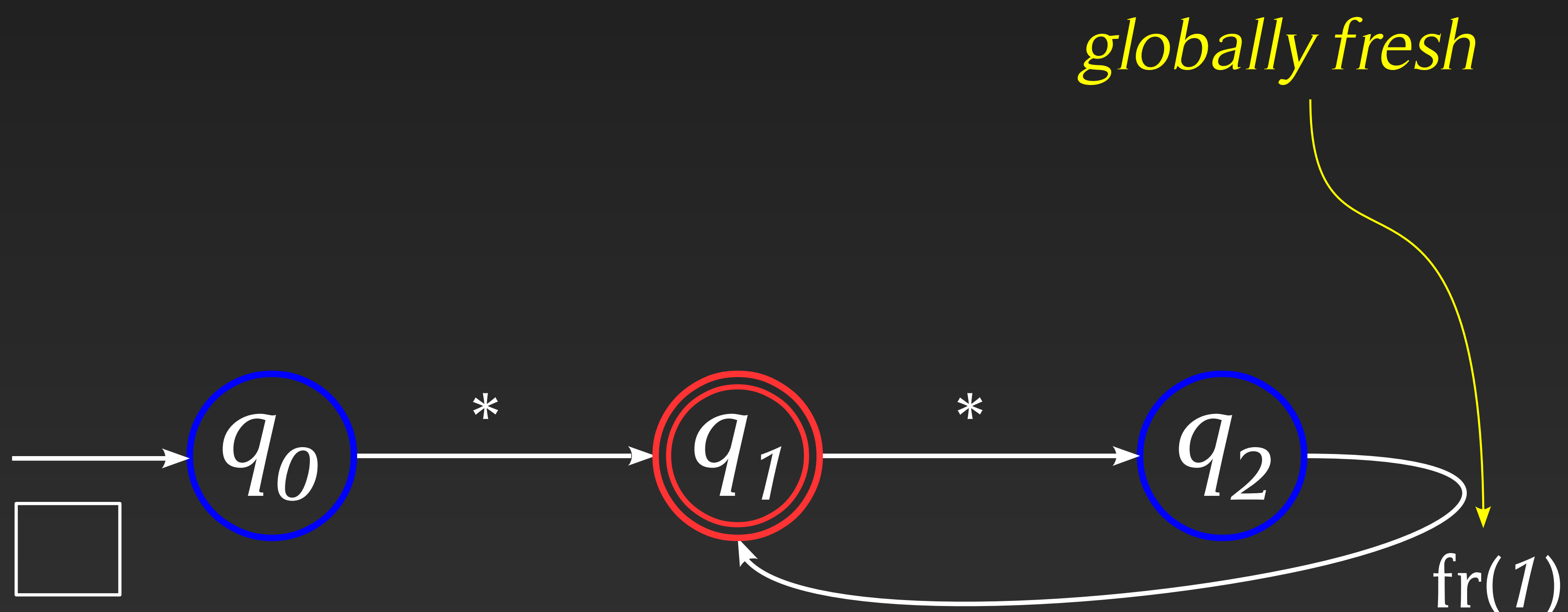

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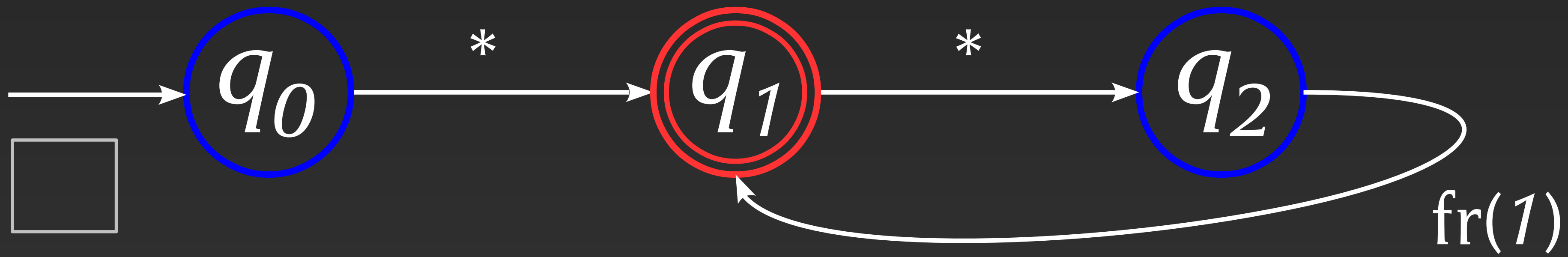

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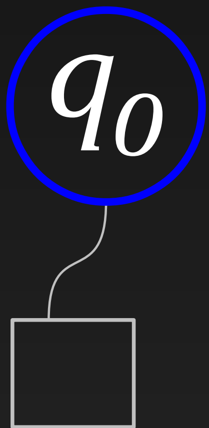

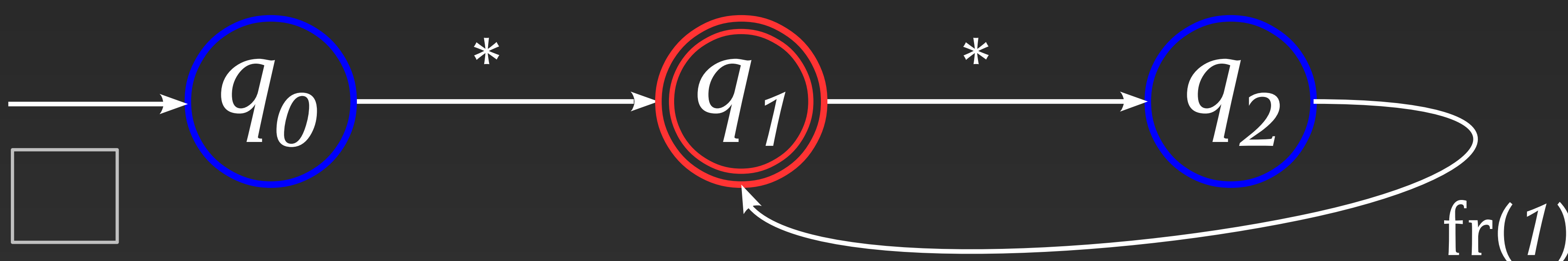

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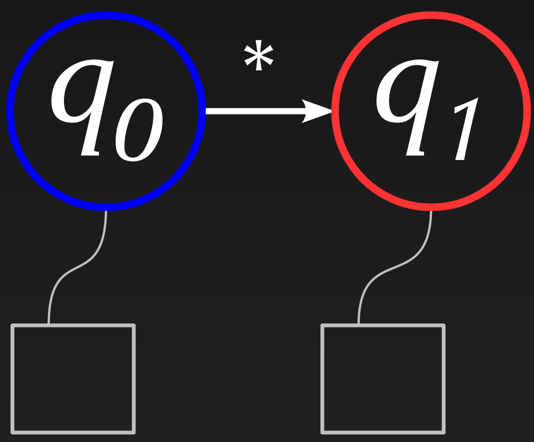

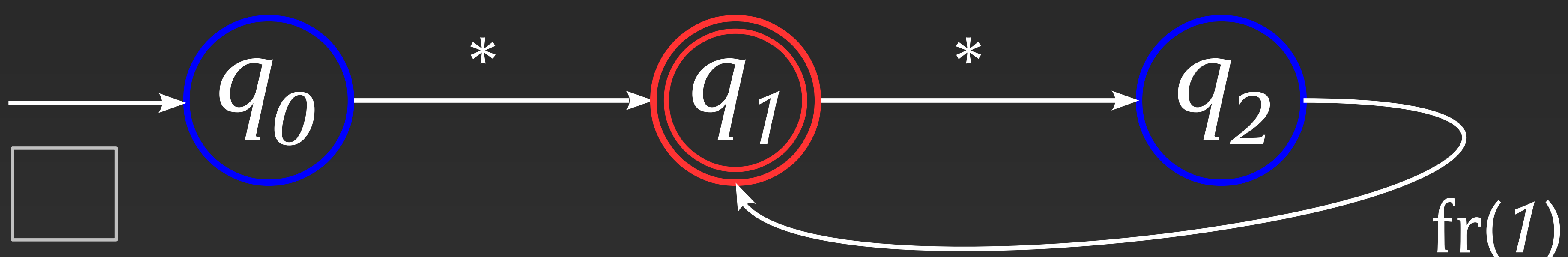

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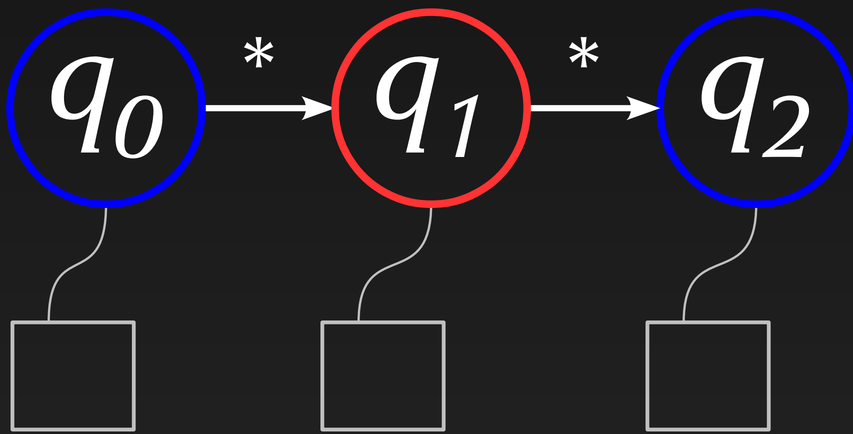

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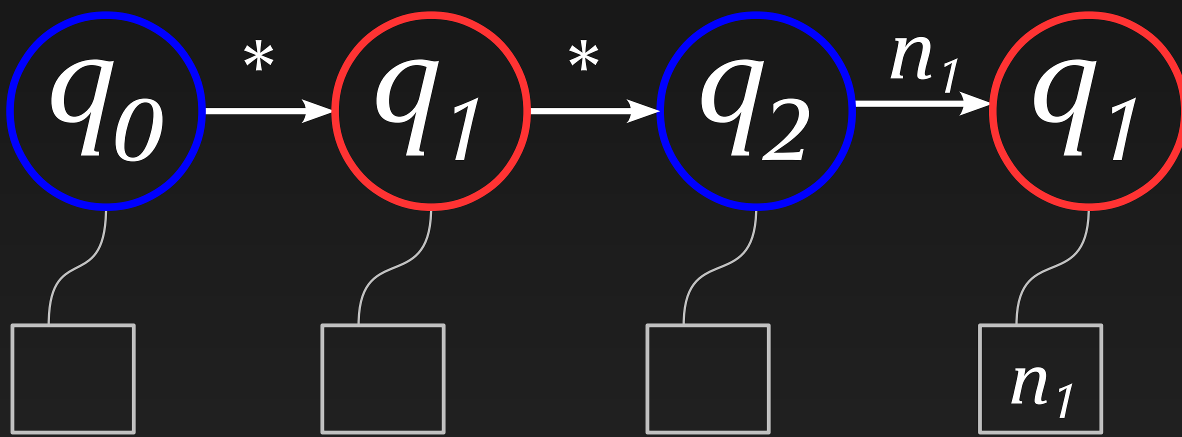

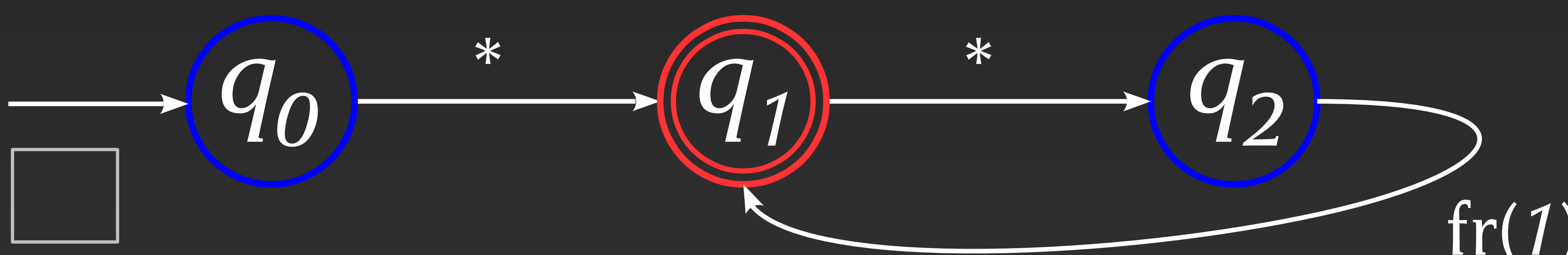

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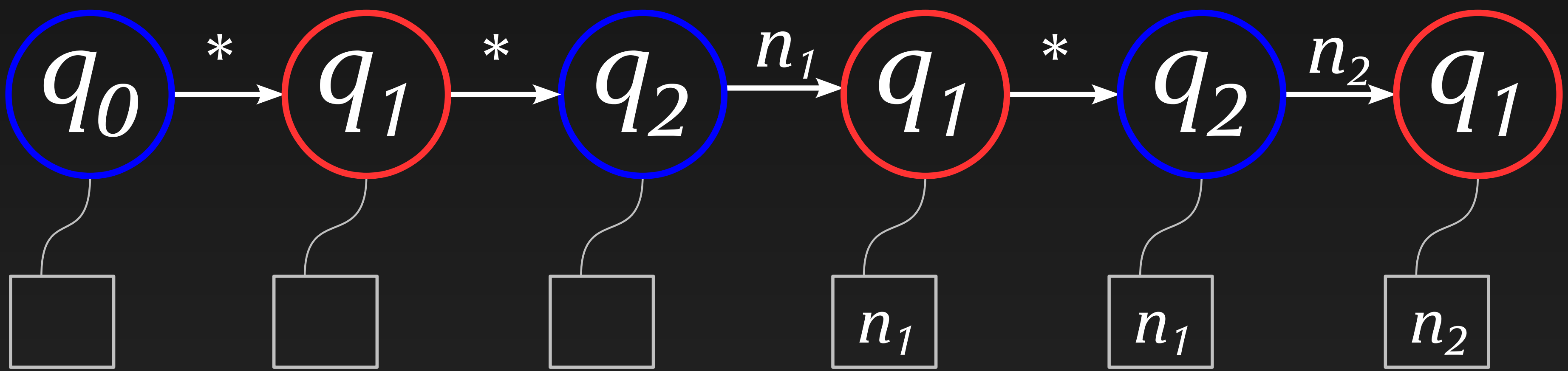

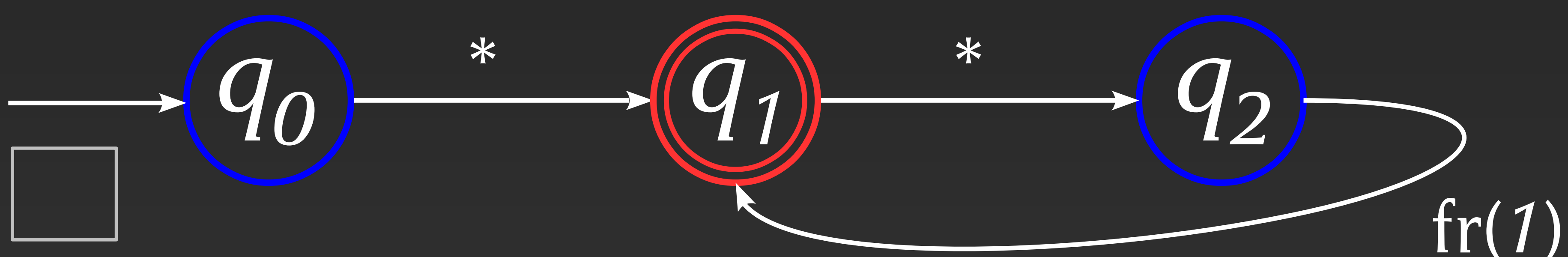

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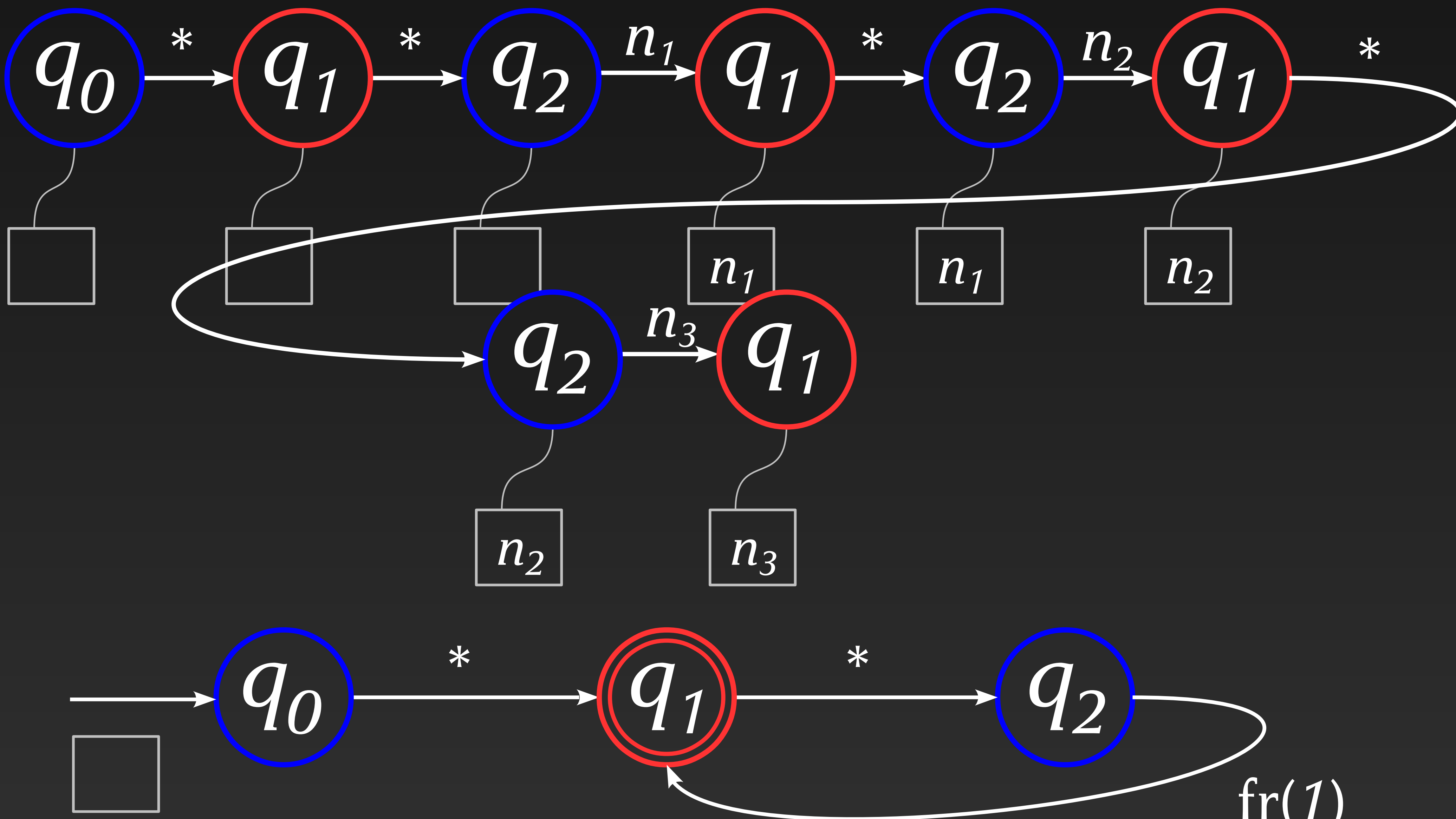

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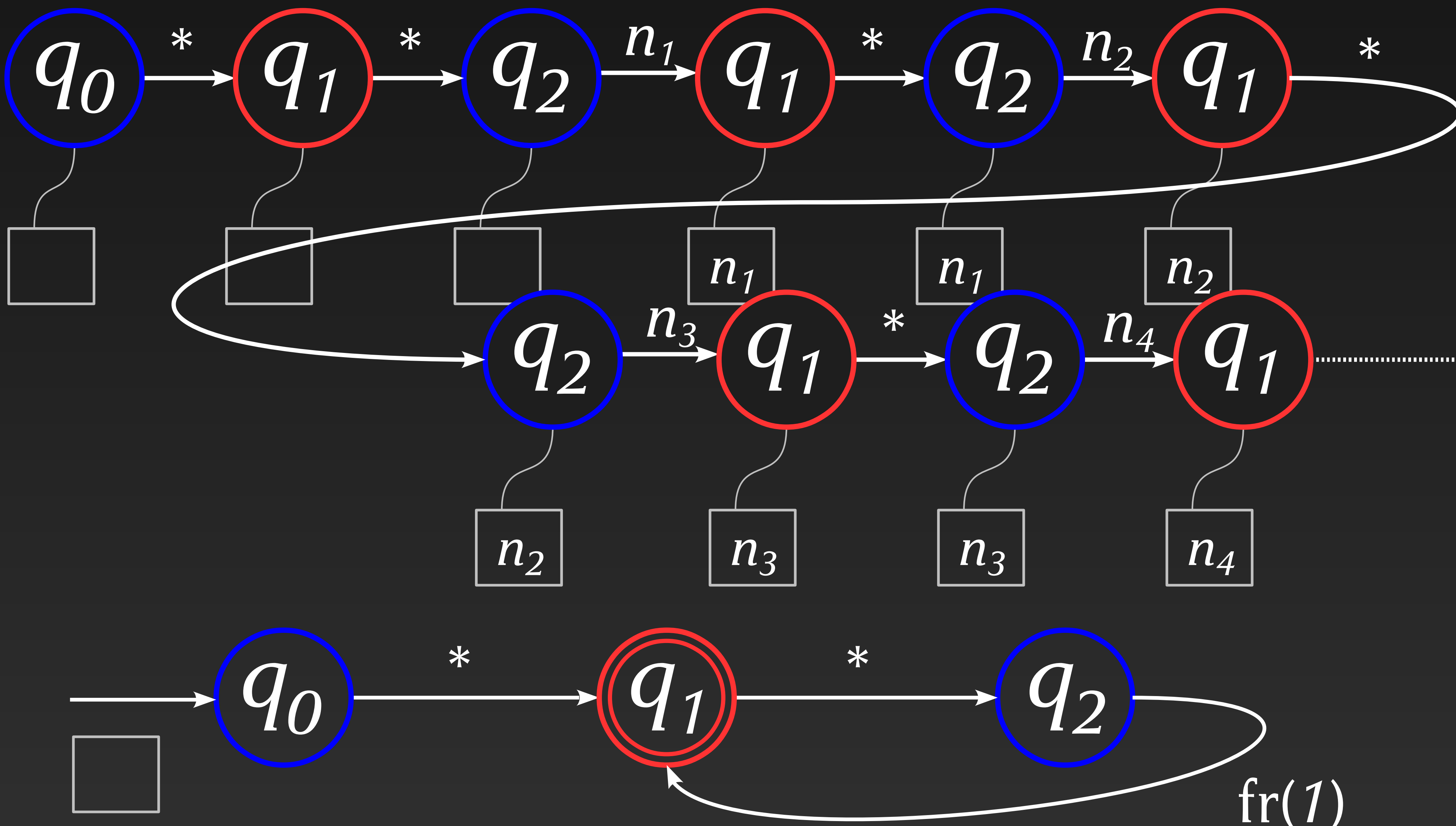

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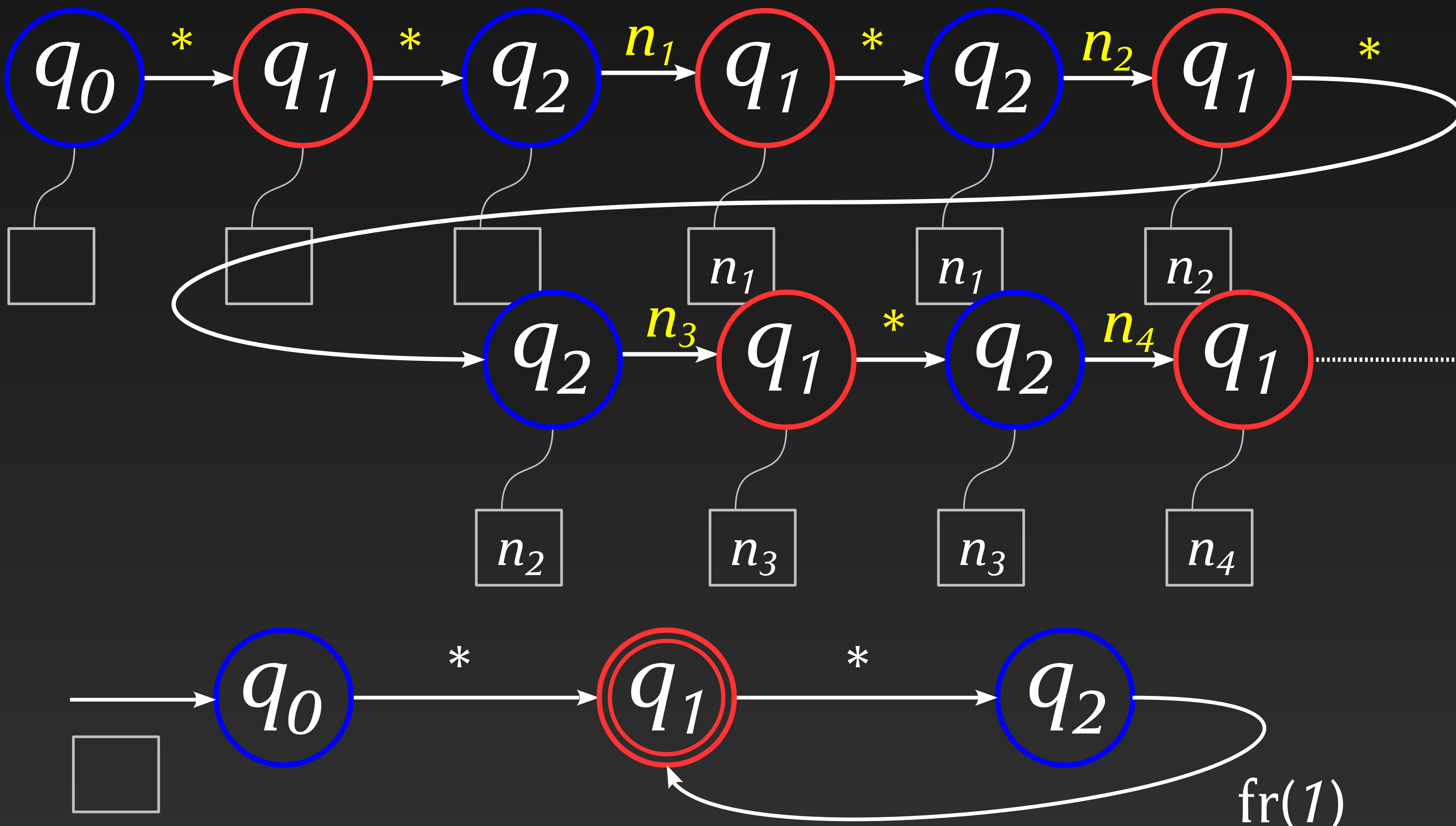

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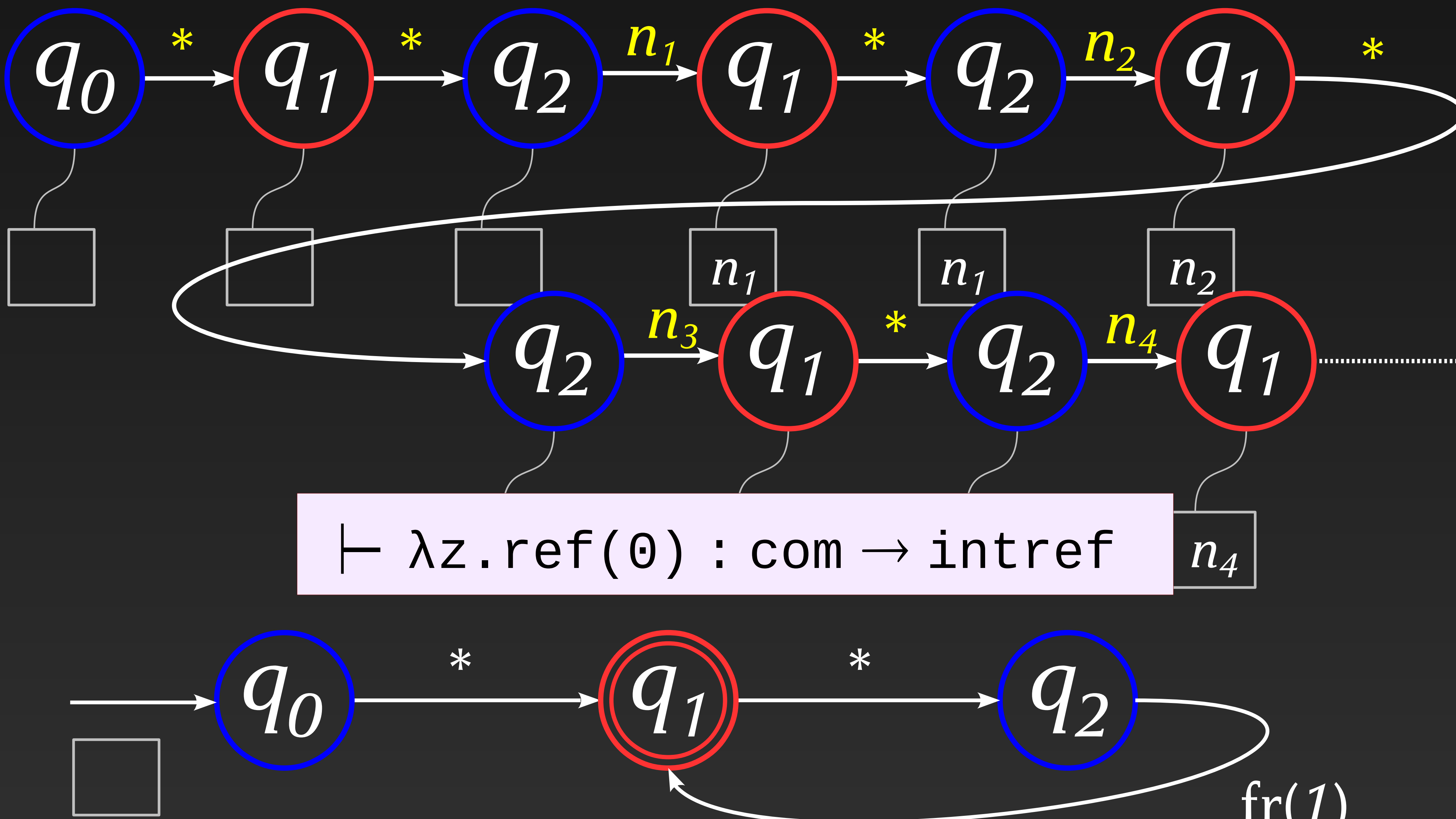
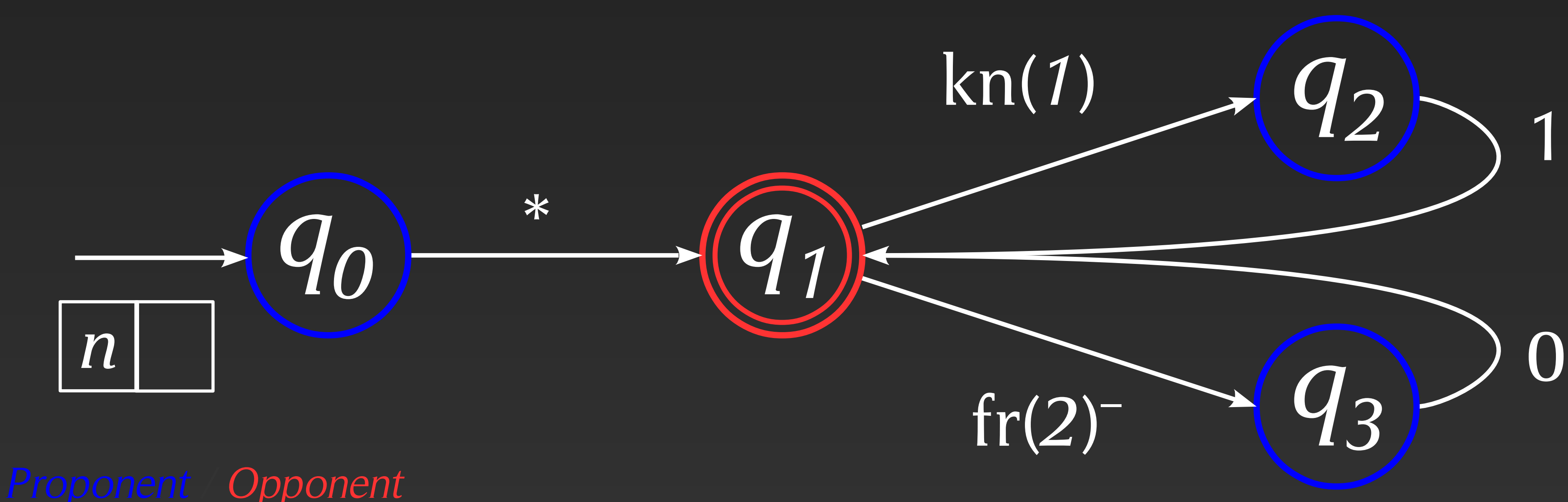


$\vdash \lambda z.\mathtt{ref}(0) : \mathtt{com} \to \mathtt{intref}$

Proponent *Opponent*

Fresh-register automata



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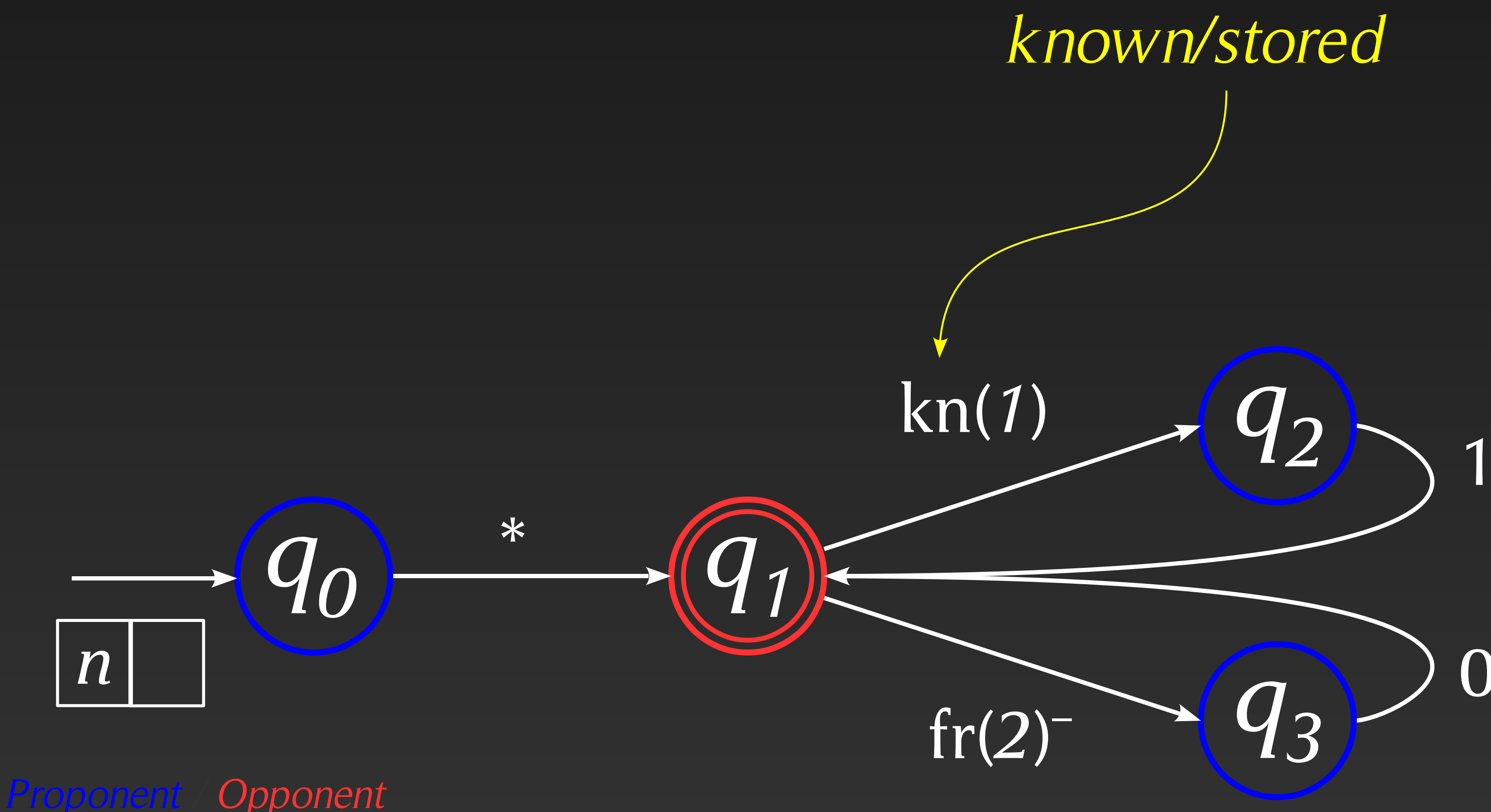

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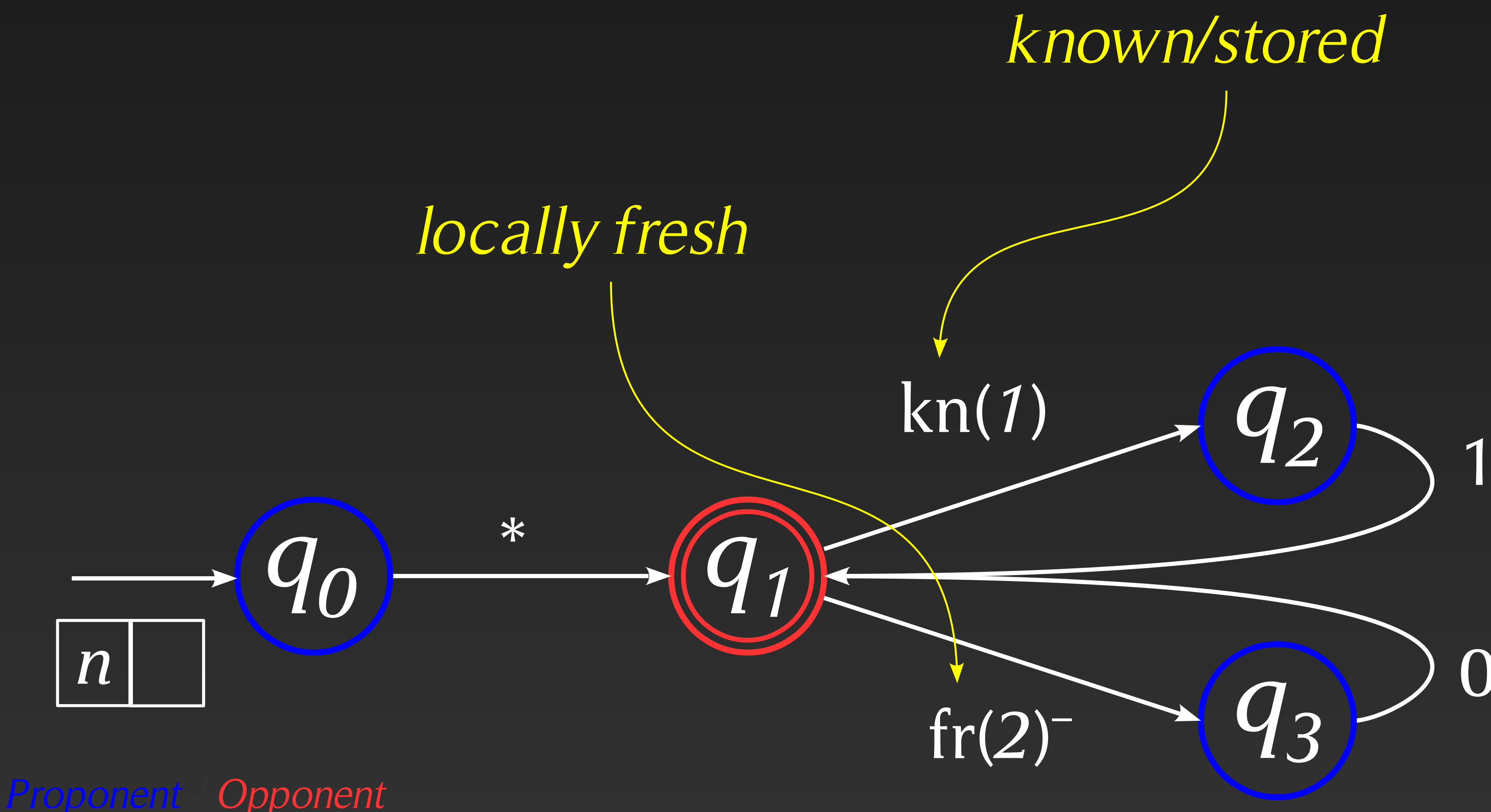

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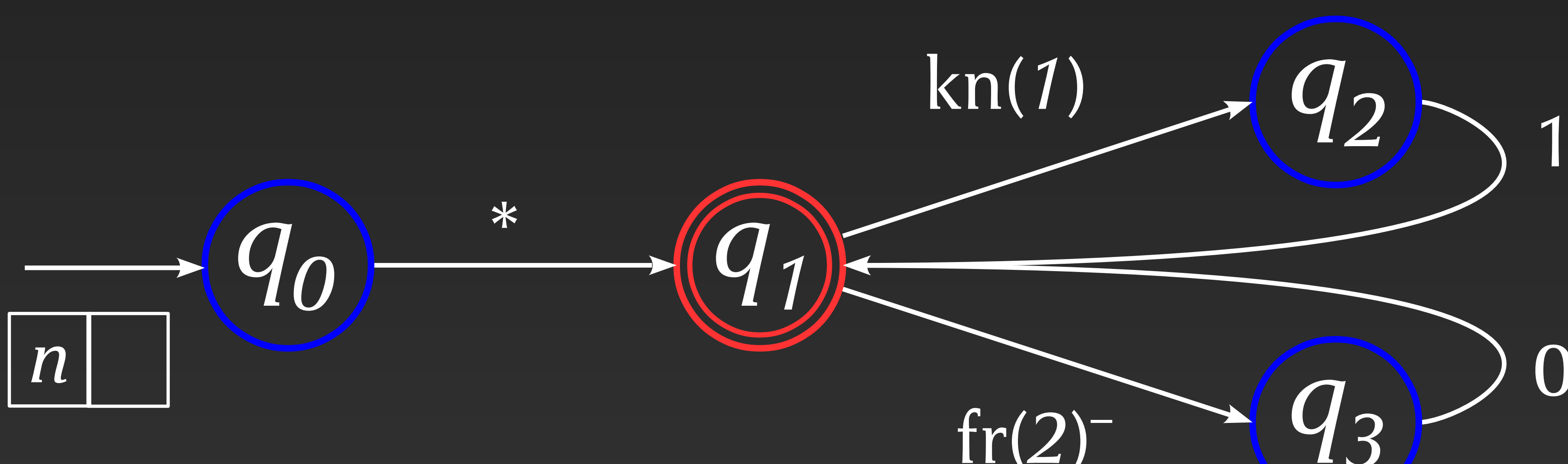

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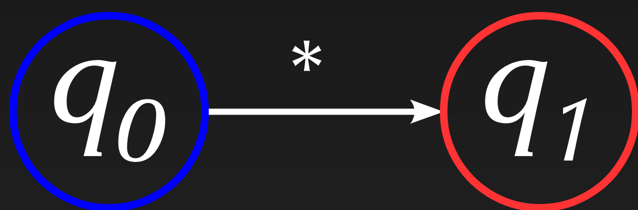

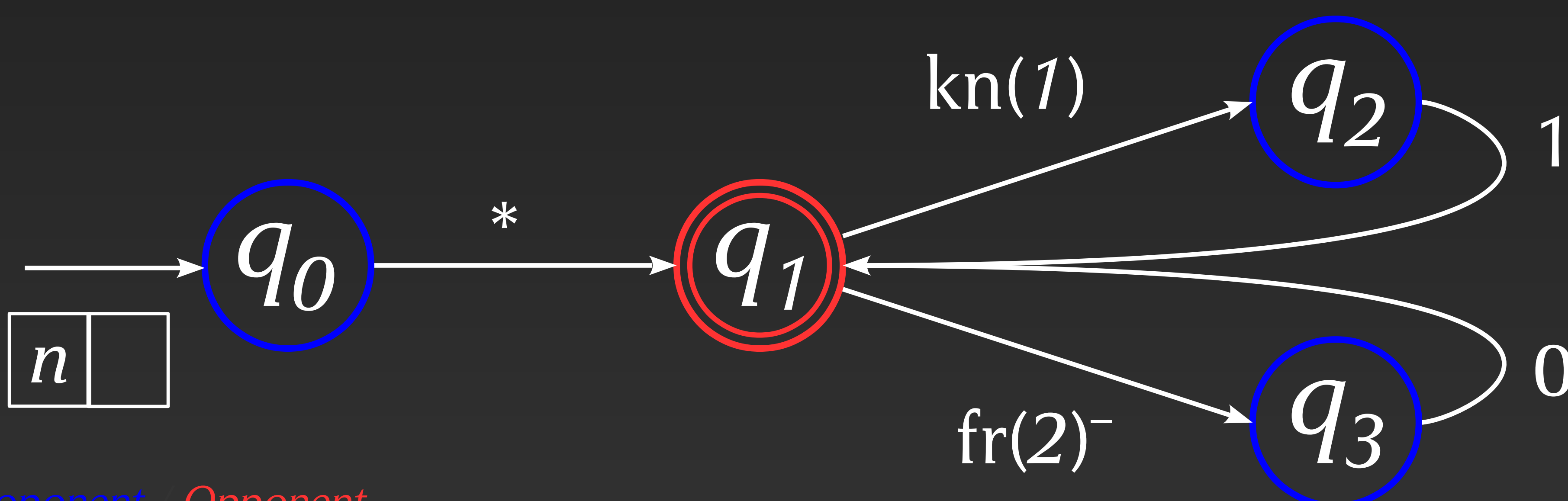

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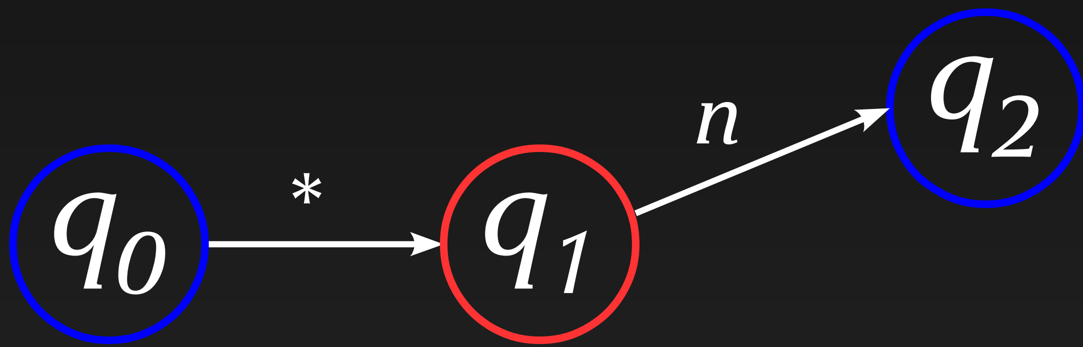

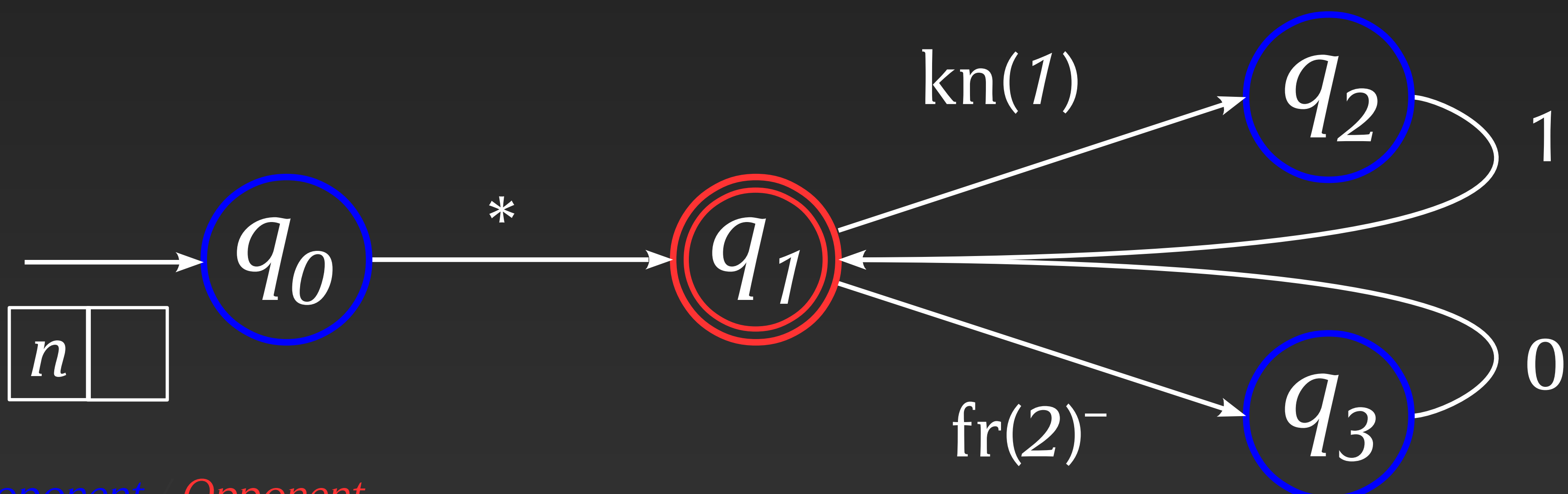

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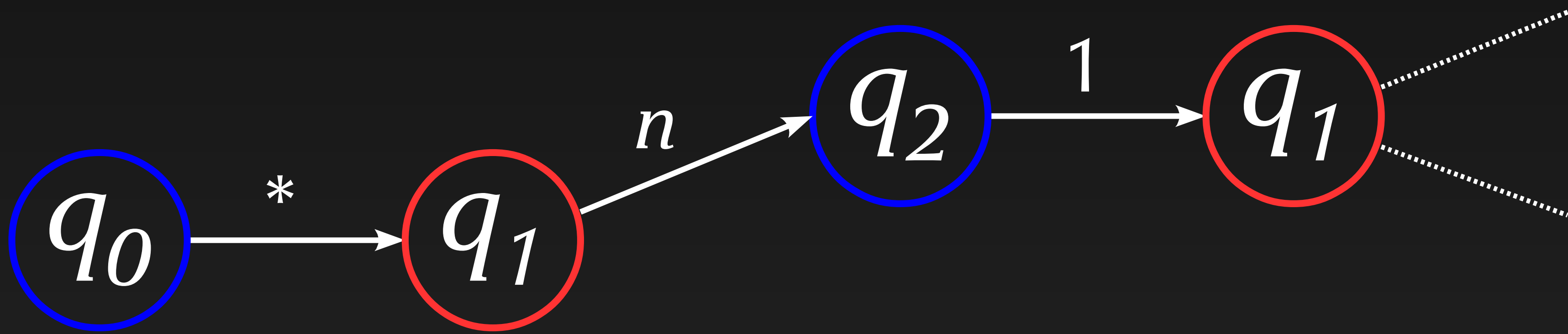

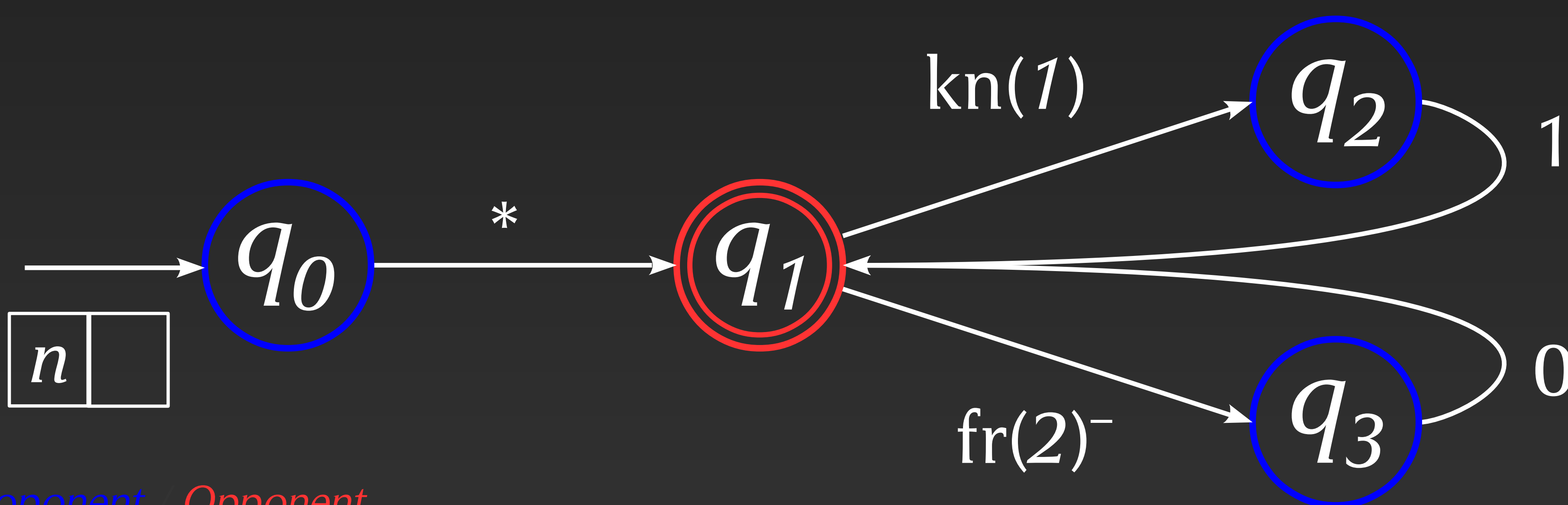

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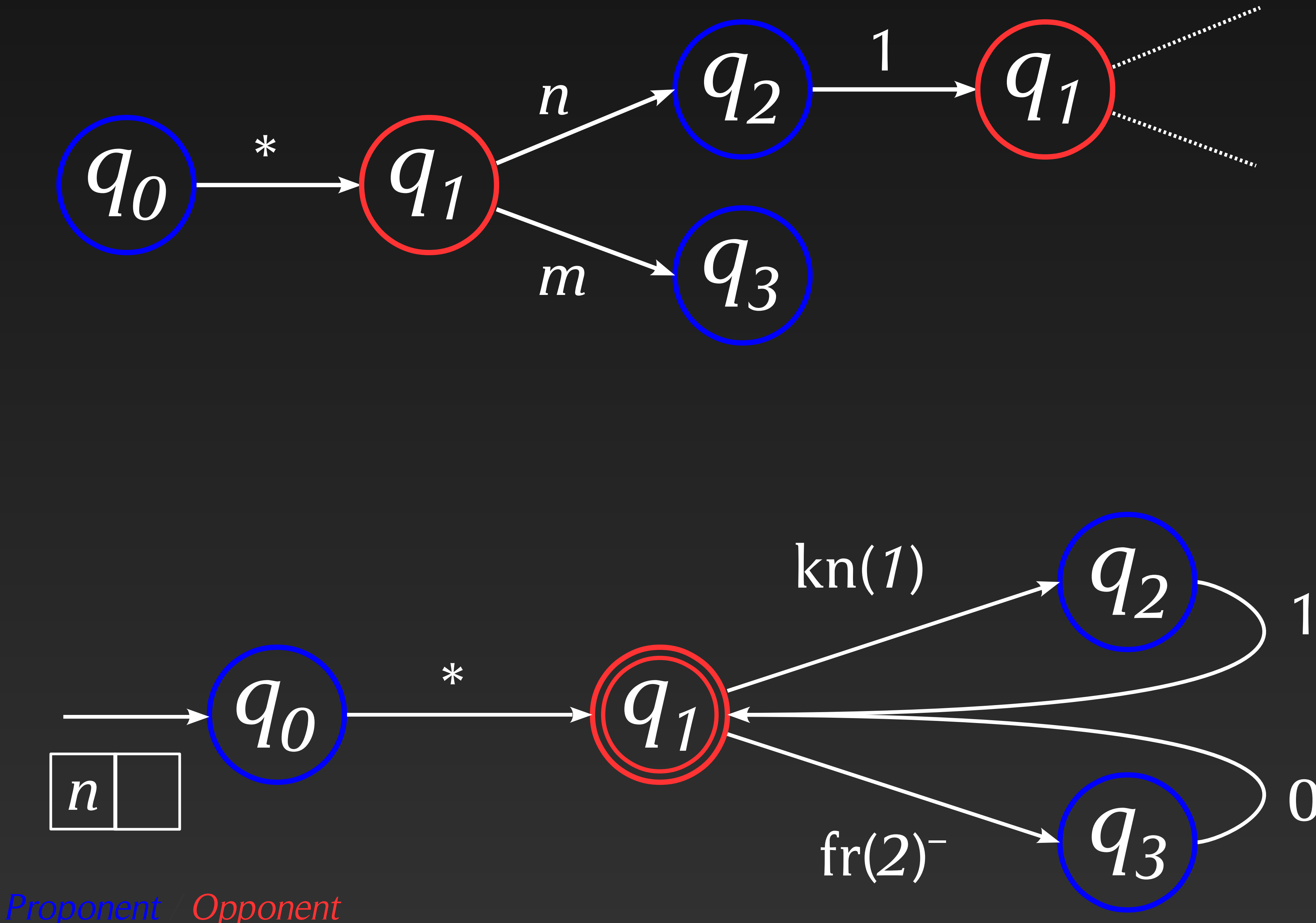

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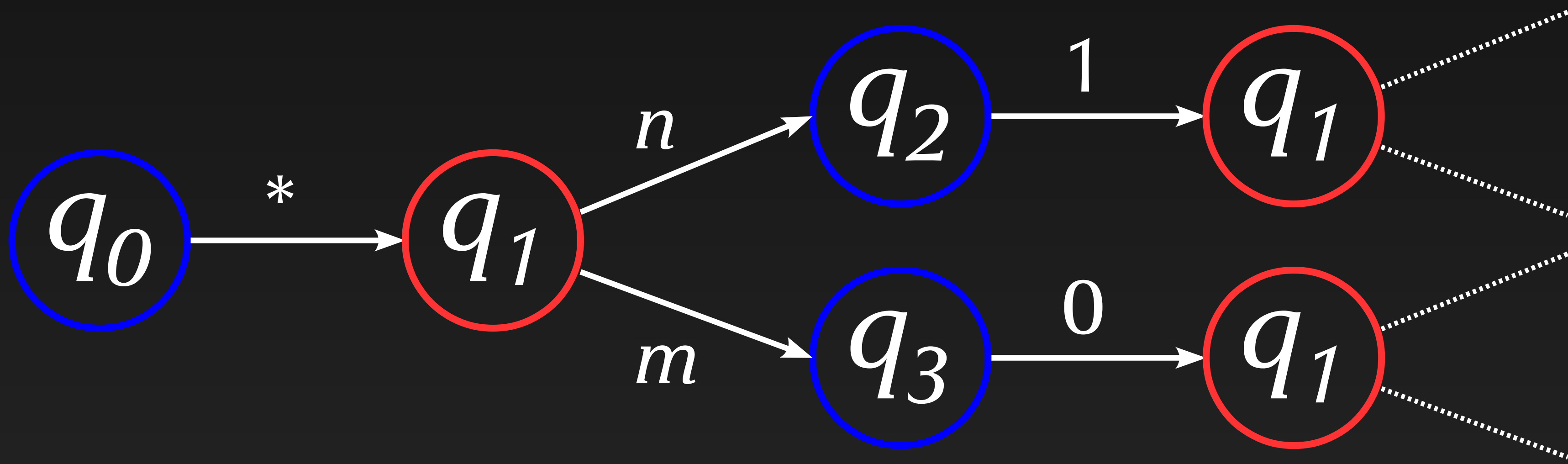

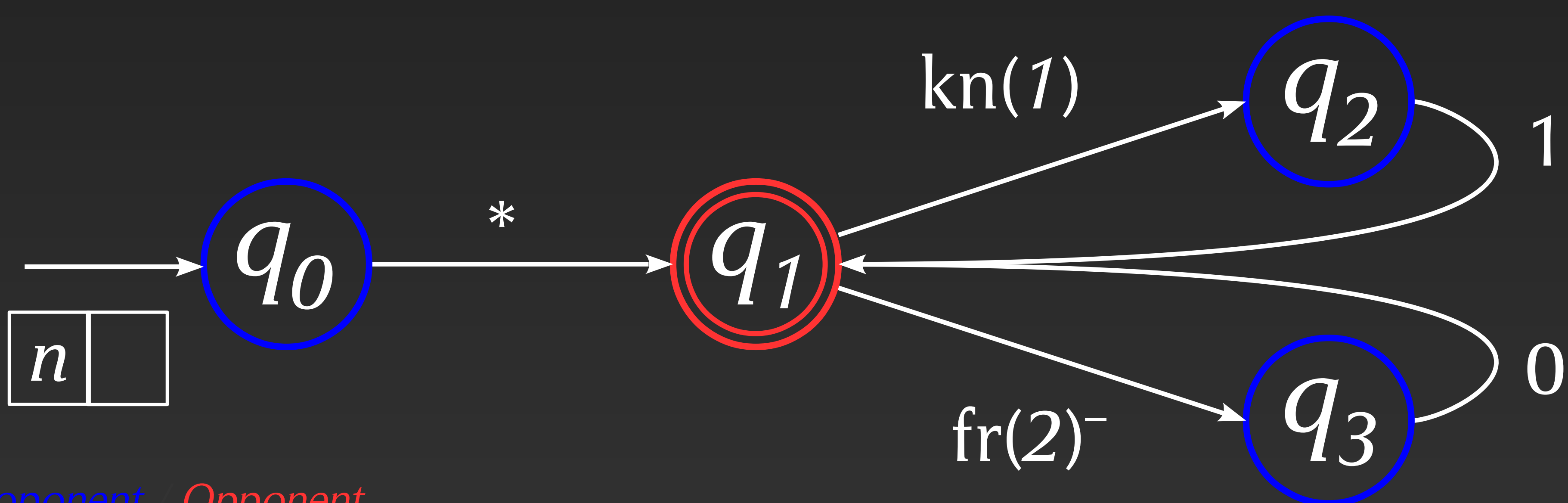

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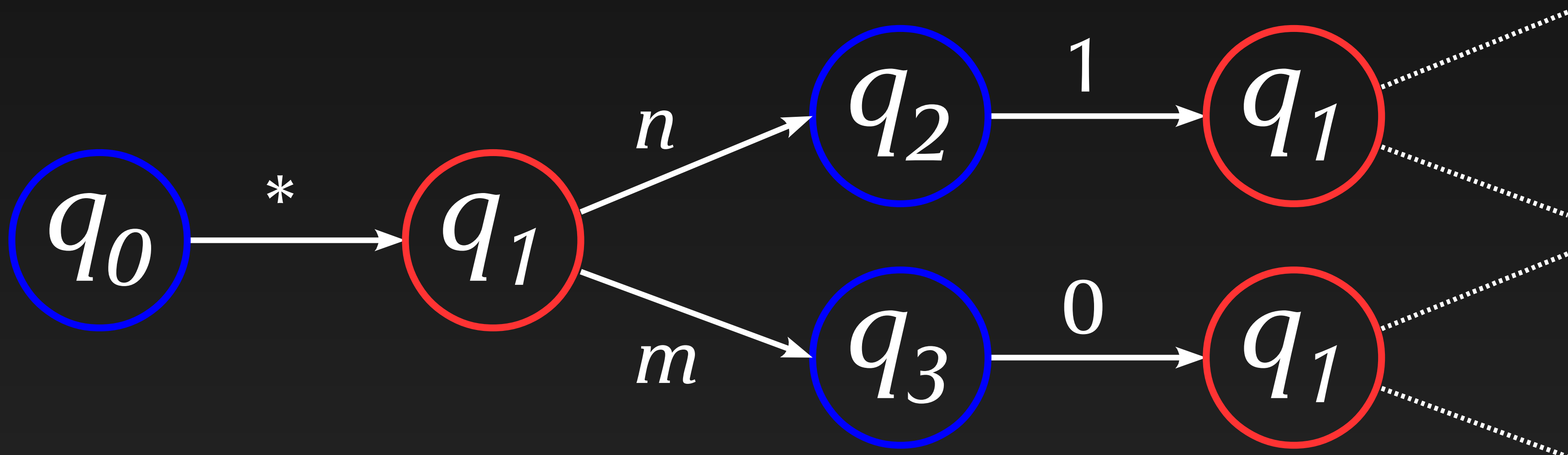

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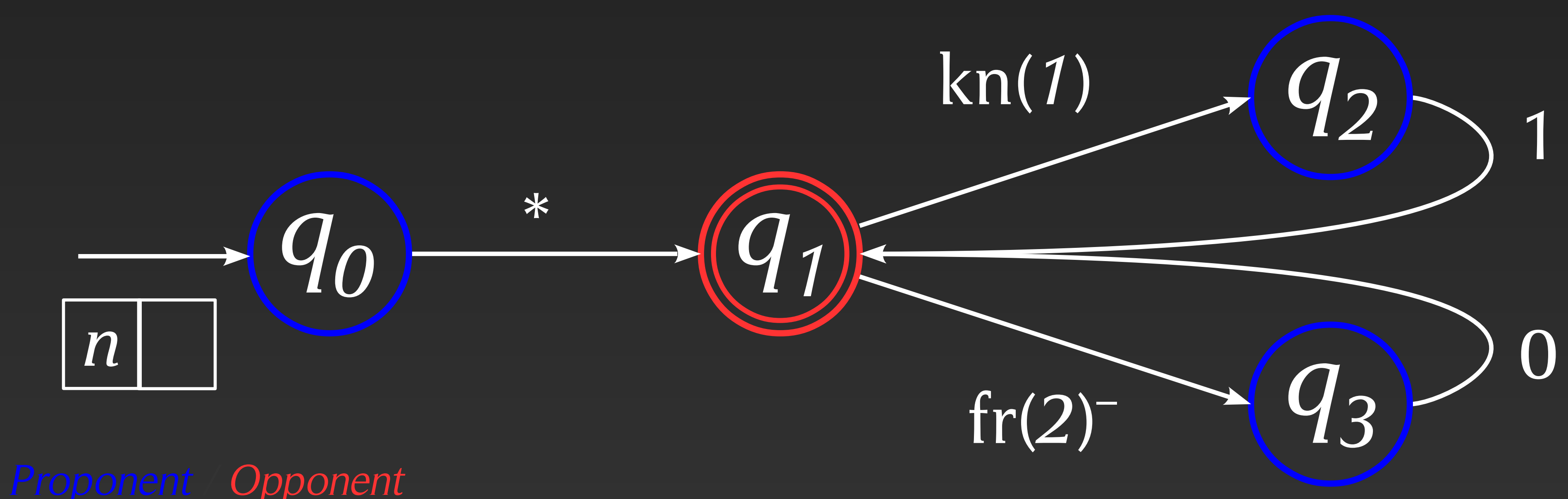

Automata

- Tz.11: Fresh-Register Automata
  - *Basic automata for names*
  - Closure ($\cap$, $\cup$), non-closure ($\bullet$, * , $\overline{}$)
  - Decidability (emptiness, bisimilarity)
  - Undecidability (universality, containment)

Automata

- Tz.11: Fresh-Register Automata
- Mu.Tz. 11': Algorithmic games for RedML*

$$M \cong N \iff \mathcal{A}_M \sim \mathcal{A}_N$$

Automata

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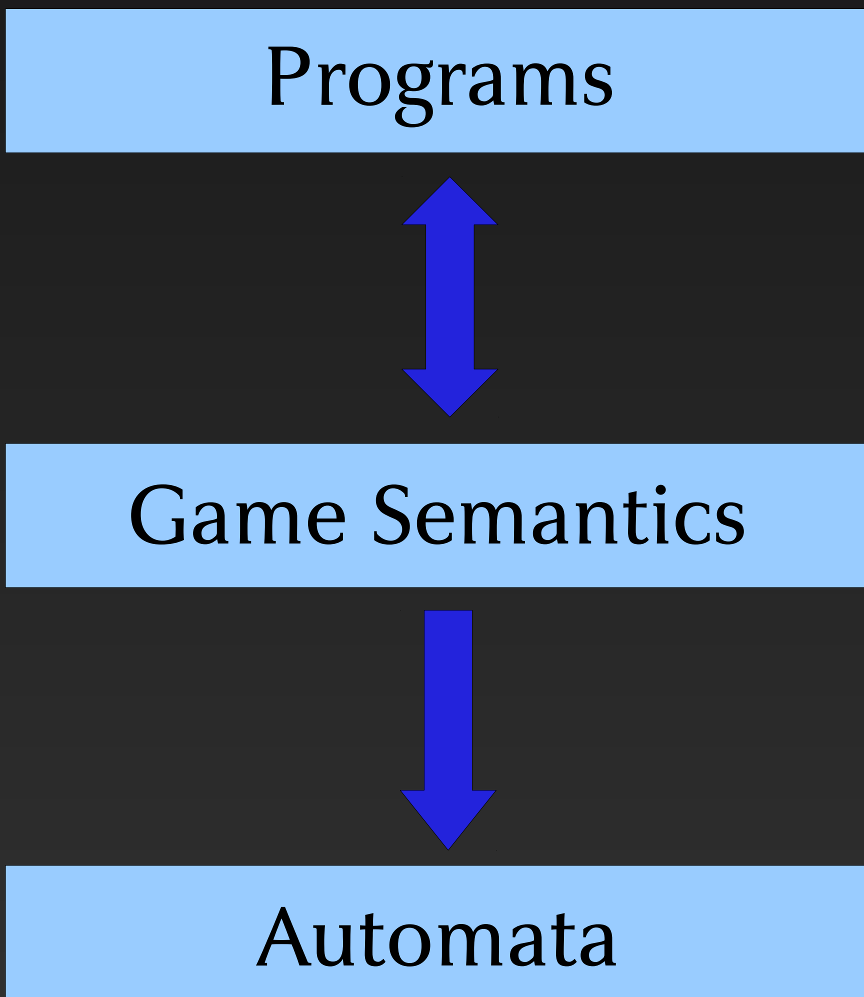
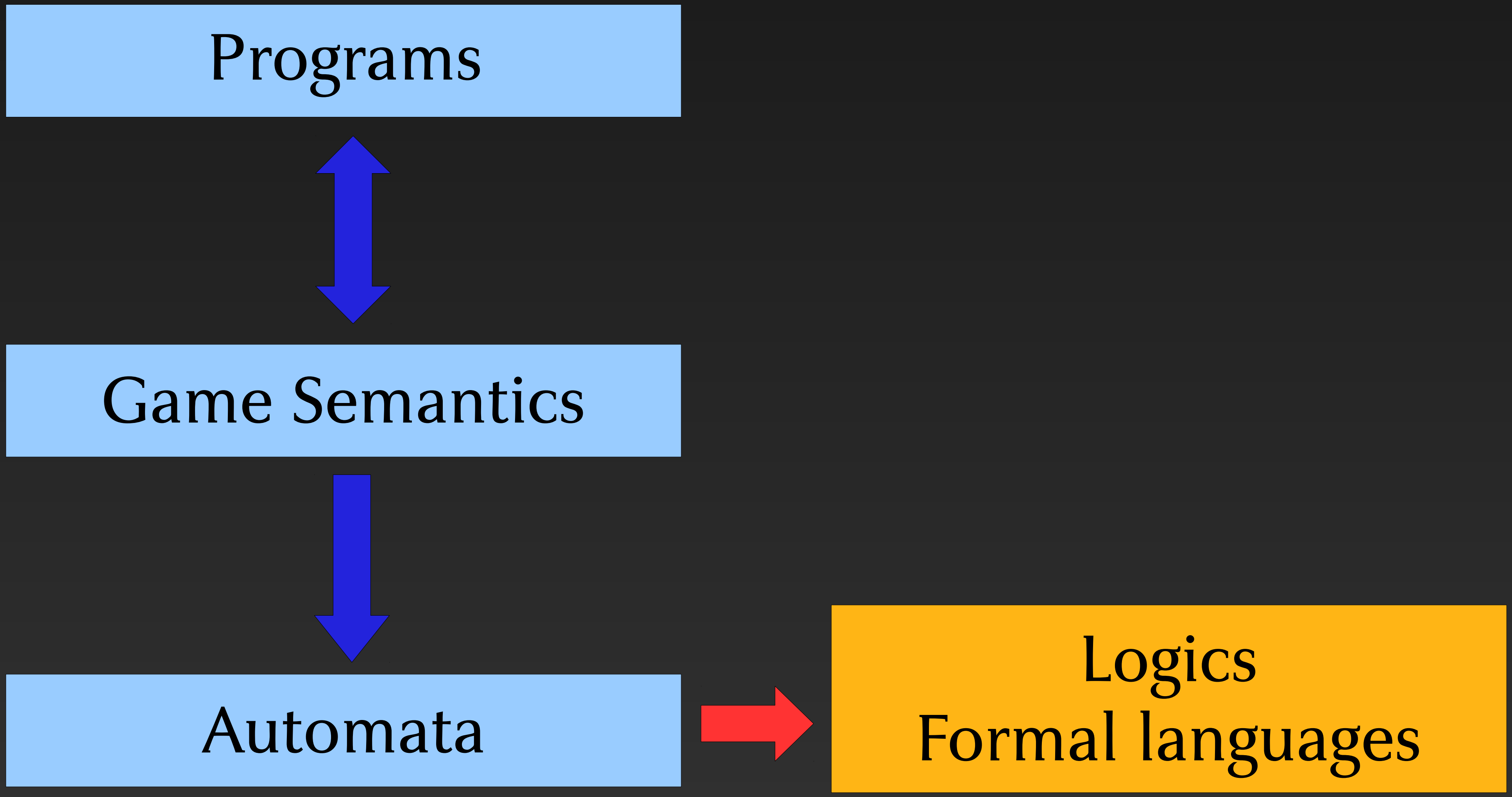
- Mu.Tz. … : Pushdown FRA's

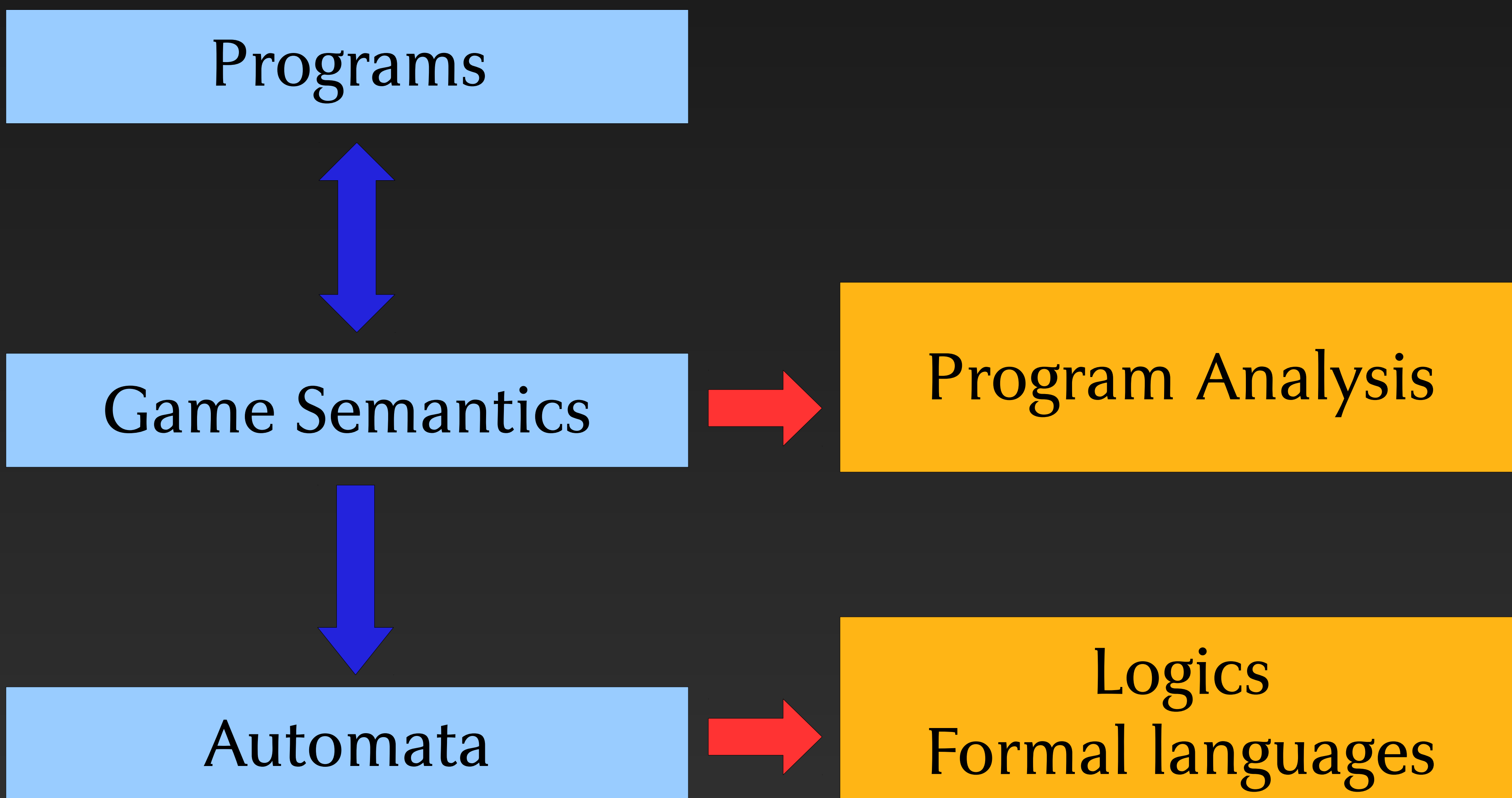
Games with names



Games with names



Games with names



Games with names

thank you!