Nominal game semantics and automata

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What this talk is about

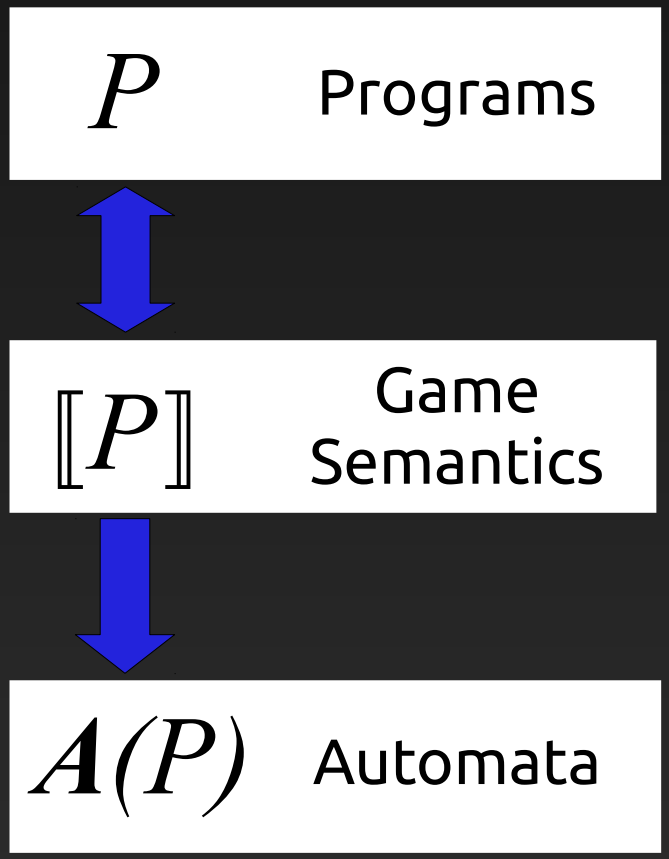
We give an overview of techniques from game semantics and automata for modelling and analysing programs

A distinctive feature of the approach is the handling of dynamic resources:

references, objects, channels, exceptions…

Resources are modelled as names with associated effects → nominal games & automata

Overview



$$P \cong P' \Leftrightarrow [\![P]\!] = [\![P']\!]$$

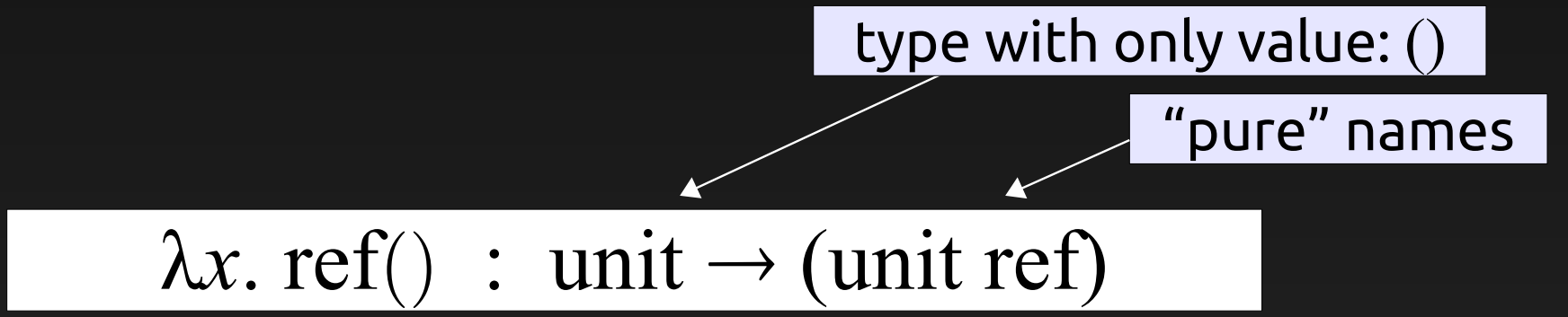
$$P \vdash \varphi \;\Leftrightarrow\; A(P) \vdash \varphi$$

Computation with names (Java)

```
...
  public void foo() {
    // Create new list
    List x = new ArrayList();

    x.add(1); x.add(2);
    Iterator i = x.iterator();
    Iterator j = x.iterator();
    i.next(); i.remove(); j.next();
  }
...
```

Computation with names (ML)



Computation with names (ML)

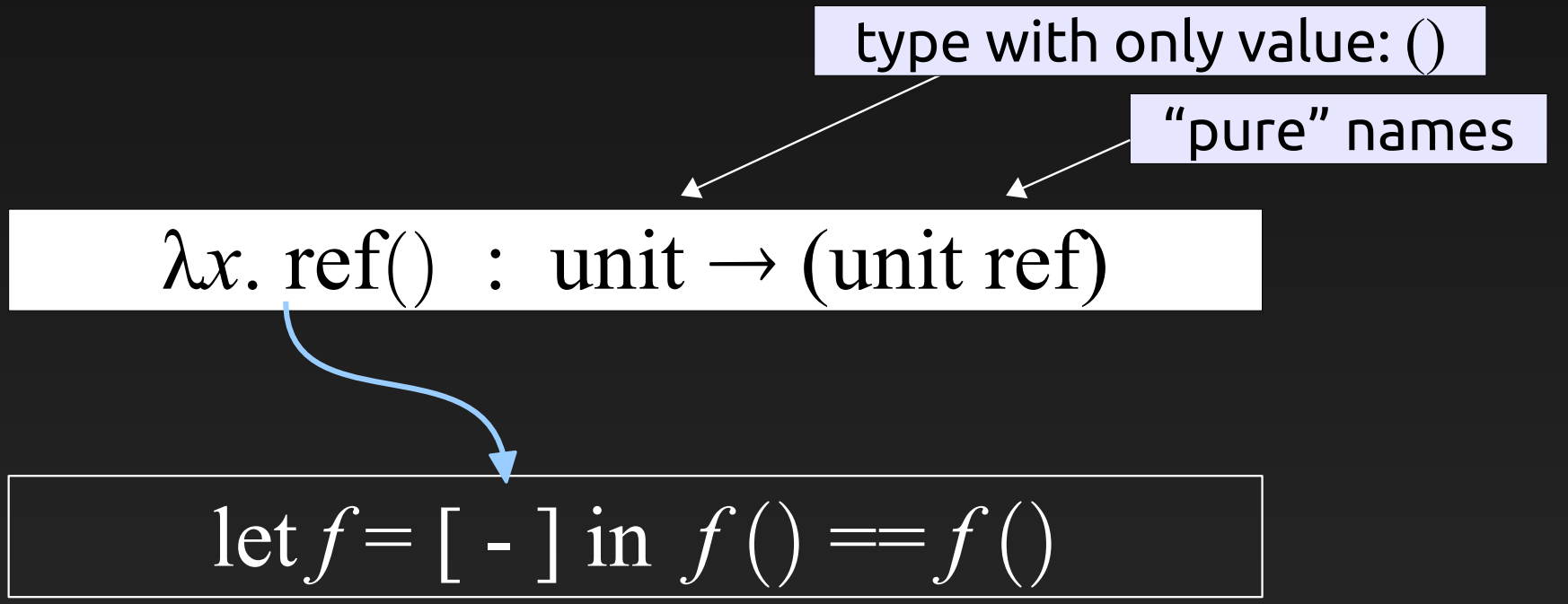
type with only value: ()

“pure” names

$$\lambda x.\ \mathrm{ref}()\ :\ \mathrm{unit} \to (\mathrm{unit\ ref})$$

$$\mathrm{let}\ f = [\ \text{-}\ ]\ \mathrm{in}\ \ f() == f()$$

Computation with names (ML)



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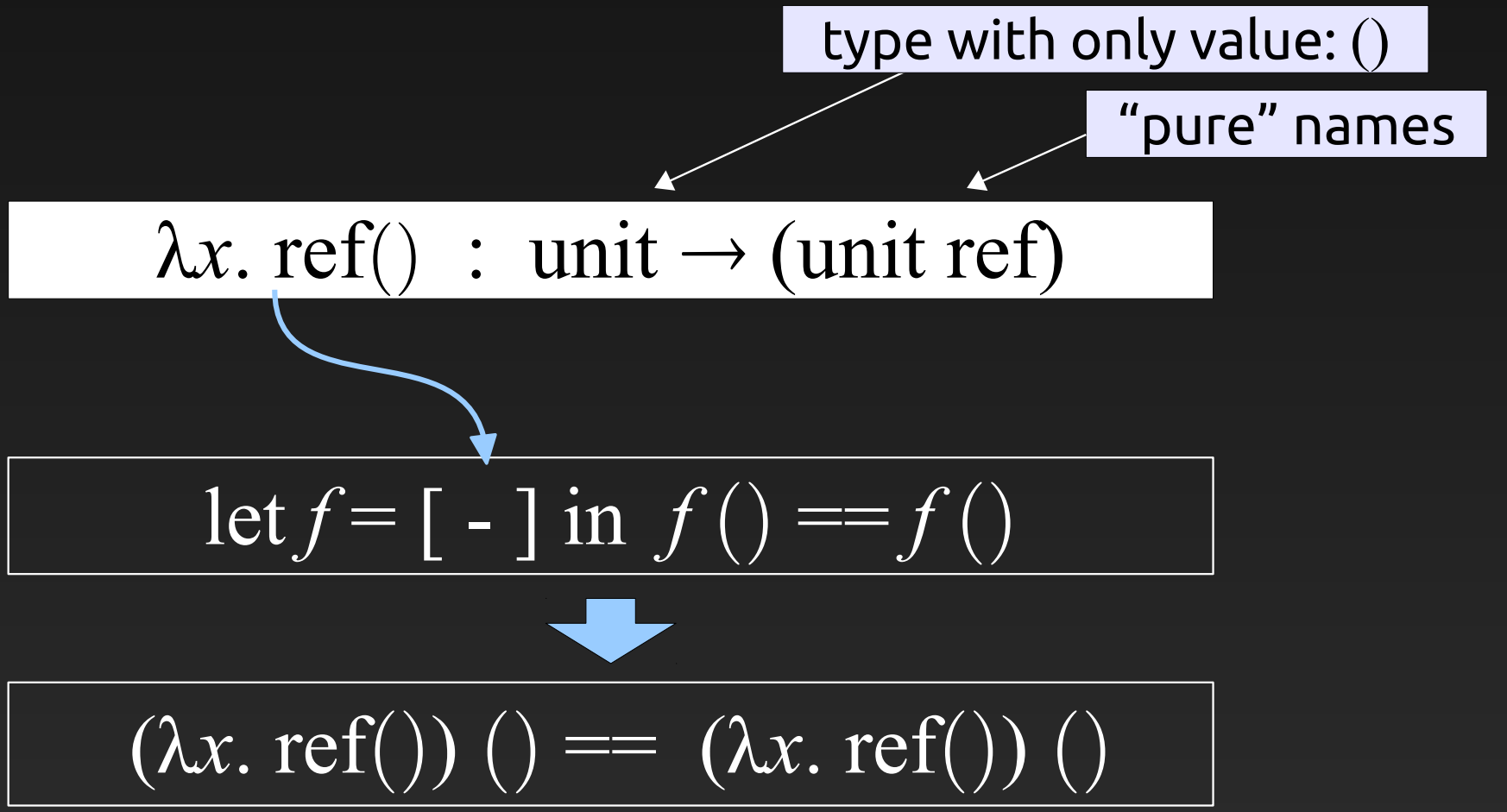

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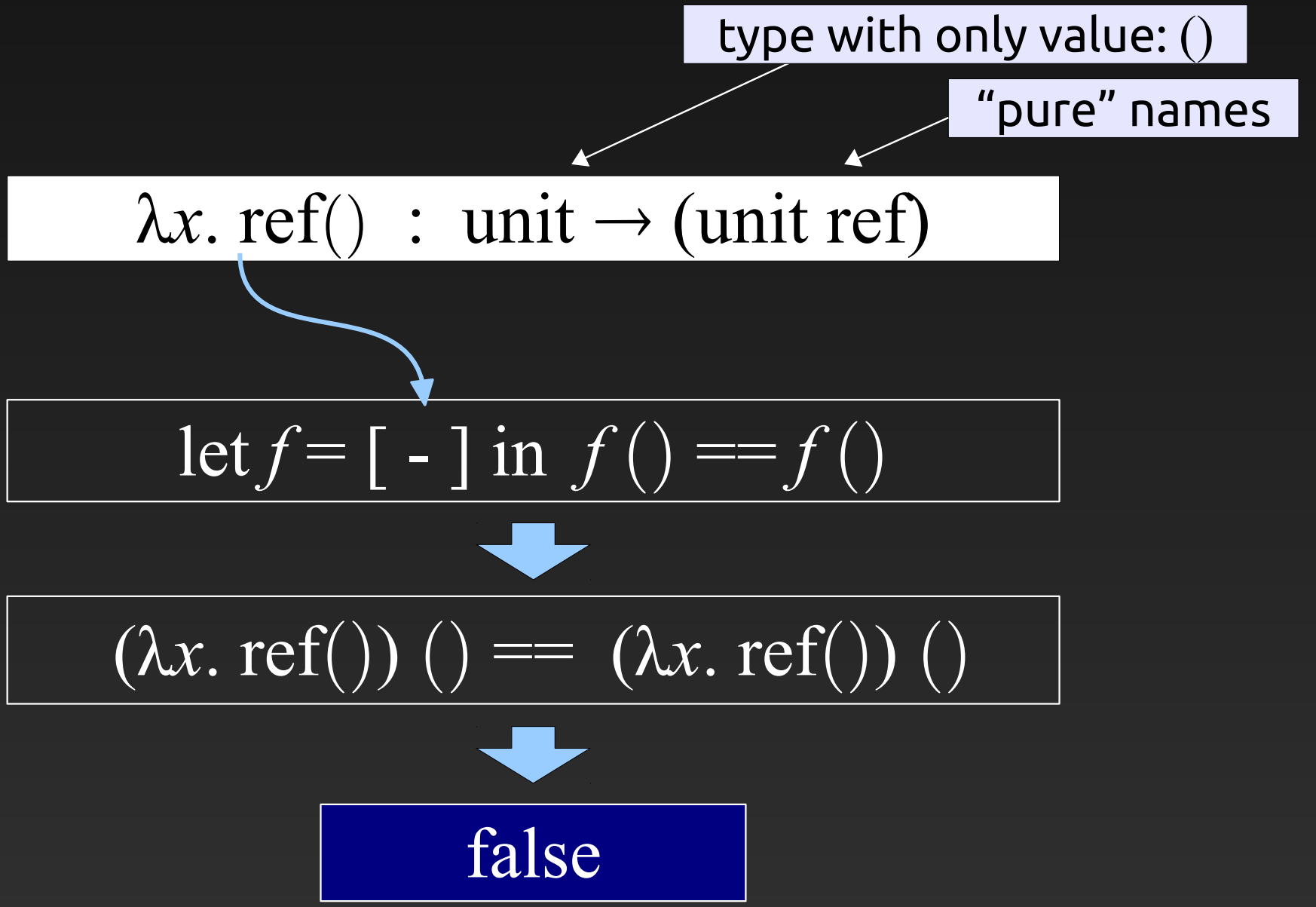

Example (ν-calculus)

Call-by-value language with unit references

$$\lambda y.\,\mathsf{ref}() \quad \not\cong \quad \mathsf{let}\, x = \mathsf{ref}()\, \mathsf{in}\, (\lambda y.\, x) \;:\; \mathsf{unit} \to \mathsf{unit\ ref}$$

$$P \cong P'$$

same observable behaviour in every context

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$$\mathsf{let}\, x = \mathsf{ref}()\, \mathsf{in}\ \lambda y.\, (x == y) \quad \cong \quad \lambda y.\, \mathsf{false} \;:\; \mathsf{unit\ ref} \to \mathsf{bool}$$

$$P \cong P'$$

same observable behaviour in every context

Example (ν-calculus)

Call-by-value language with unit references

"Create a *fresh reference* ; then *compare* input references with the fresh one and return the result of the comparison"

$$\lambda y.\,\mathsf{ref}() \quad \not\cong \quad \mathsf{let}\, x = \mathsf{ref}() \,\mathsf{in}\, (\lambda y.\, x) \;:\; \mathsf{unit} \to \mathsf{unit\ ref}$$

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Full Abstraction

How to assign denotations to programs,

$$[\![\_]\!] : \text{Syntax} \longrightarrow \mathcal{M}$$

such that:

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Full Abstraction

How to assign denotations to programs,

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such that:

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Here the Syntax will be some fragment of ML,
i.e. call-by-value PCF + references + exceptions + …

$$T ::= \text{unit} \mid \text{int} \mid T\,\text{ref} \mid T \to T \mid \ldots$$

$$M ::= () \mid i \mid a \mid x \mid \lambda x.M \mid M\,M \mid \text{ref}\,M \mid M := M \mid !M \mid \boxed{M == M \mid \ldots}$$

Game Semantics

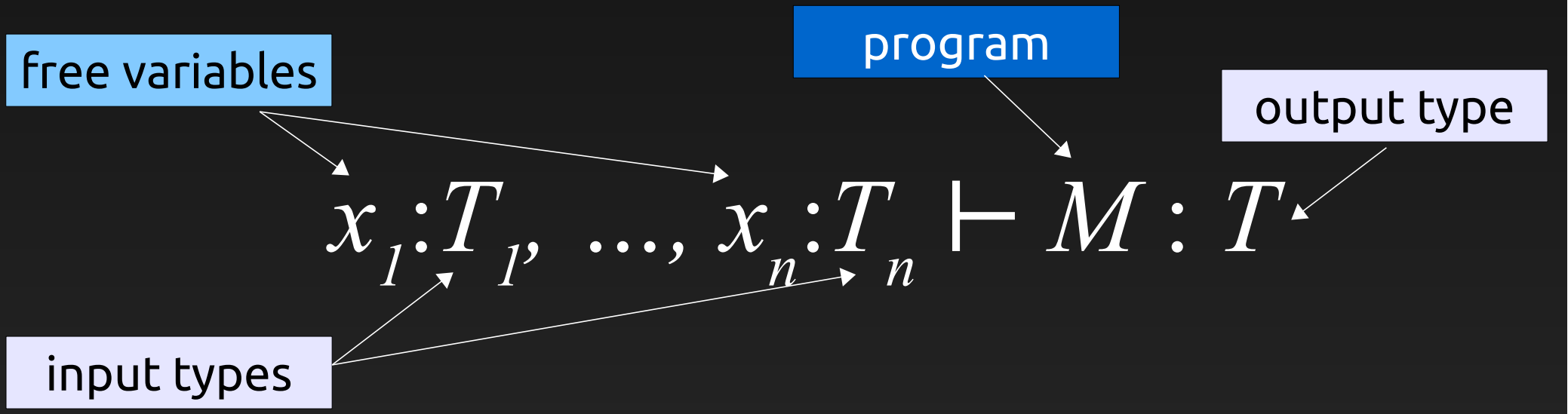
Game Semantics

- Computation is modelled as a 2-player game between:
  - *Opponent* (the environment), aka *O*
  - *Proponent* (the program), aka *P*
- Qualitative games (≠ Game Theory)
- Programs = *strategies* for *P*
- *Categories* of games

Games played in arenas

$$x_1 : T_1,\ \ldots,\ x_n : T_n \vdash M : T$$

Games played in arenas



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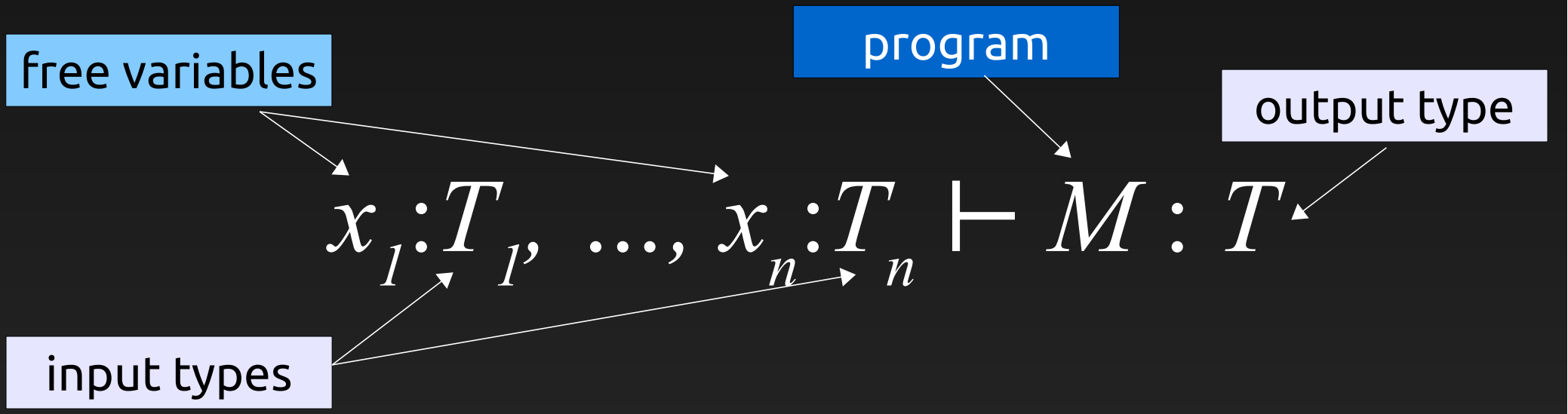

$$[\![M]\!] : [\![T_1, \ldots, T_n]\!] \longrightarrow [\![T]\!]$$

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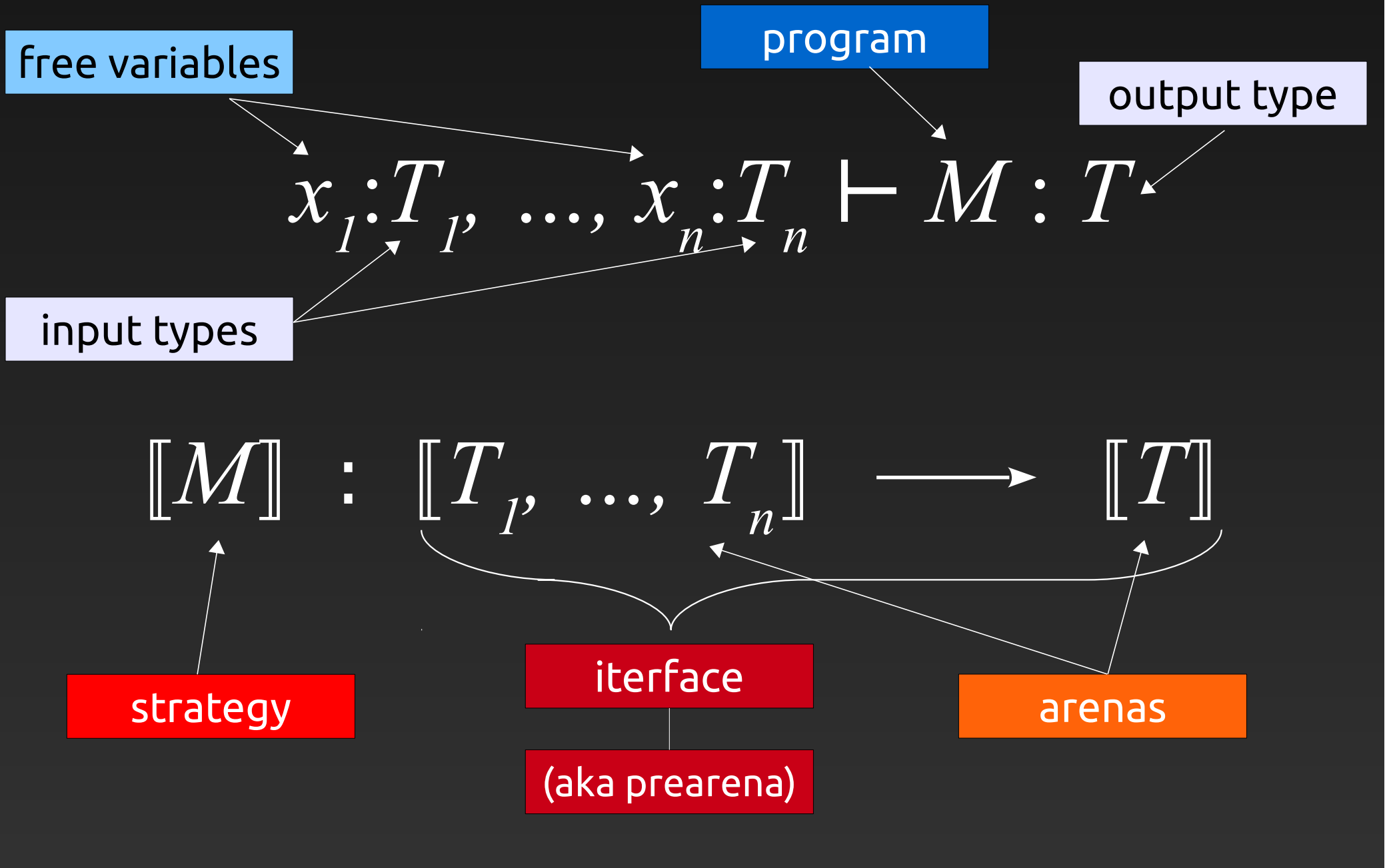
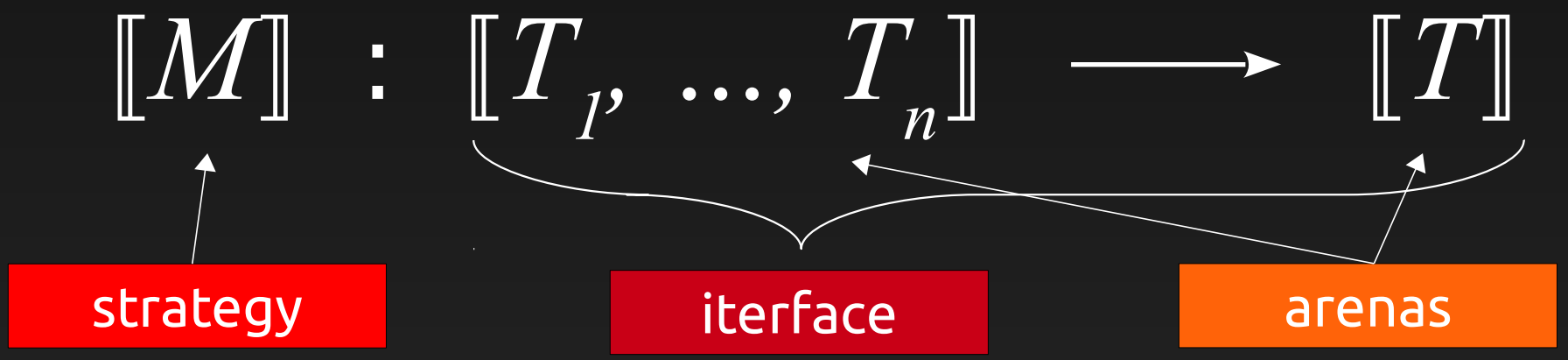


Arenas of moves



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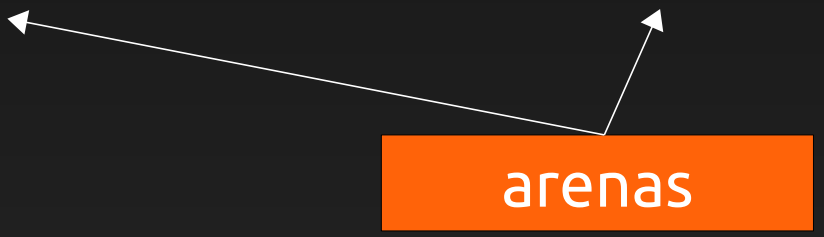

$[\![\text{unit}]\!] = \{ * \}$

$[\![\text{int}]\!] = \{ 0, 1, -1, \ldots \}$

$[\![T \text{ ref}]\!] = \{ a, b, \ldots \}$ $\quad a, b, \ldots \in \mathcal{N}_T$

...

Arenas of moves

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arenas

$\mathcal{N}_T$ a set of *names*:
- Infinitely many
- Comparable for equality only

$$[\![\text{unit}]\!] = \{ * \}$$

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$$a, b, \ldots \in \mathcal{N}_T$$

Arenas of moves

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arenas

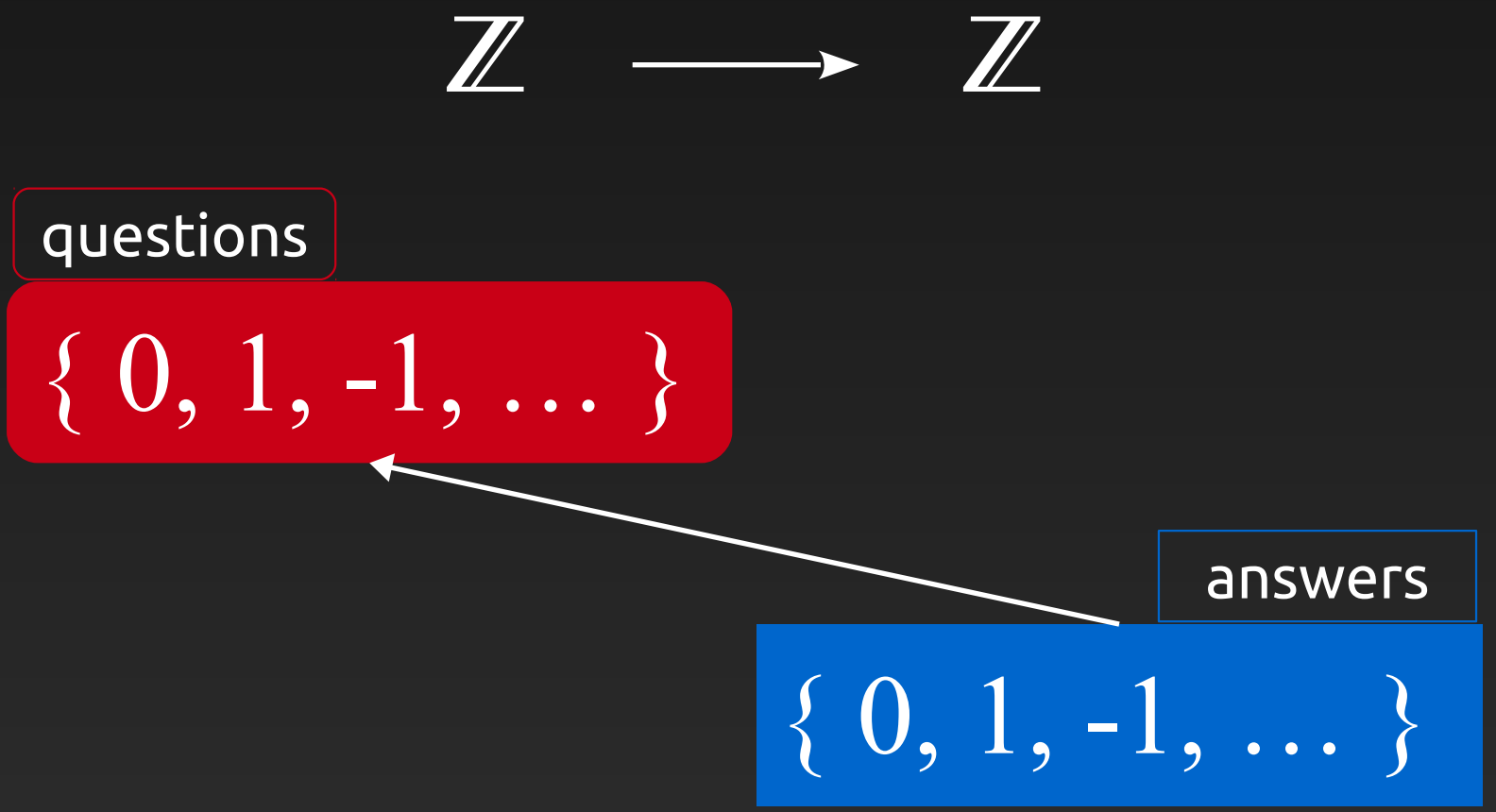
$\mathcal{N}_T$ a set of *names*:
- Infinitely many
- Comparable for equality only

$$[\![\text{unit}]\!] = \{ * \} = \mathbf{1}$$

$$[\![\text{int}]\!] = \{ 0, 1, -1, \ldots \} = \mathbb{Z}$$

$$[\![T \text{ ref}]\!] = \{ a, b, \ldots \} = \mathbb{A}_T \qquad a, b, \ldots \in \mathcal{N}_T$$

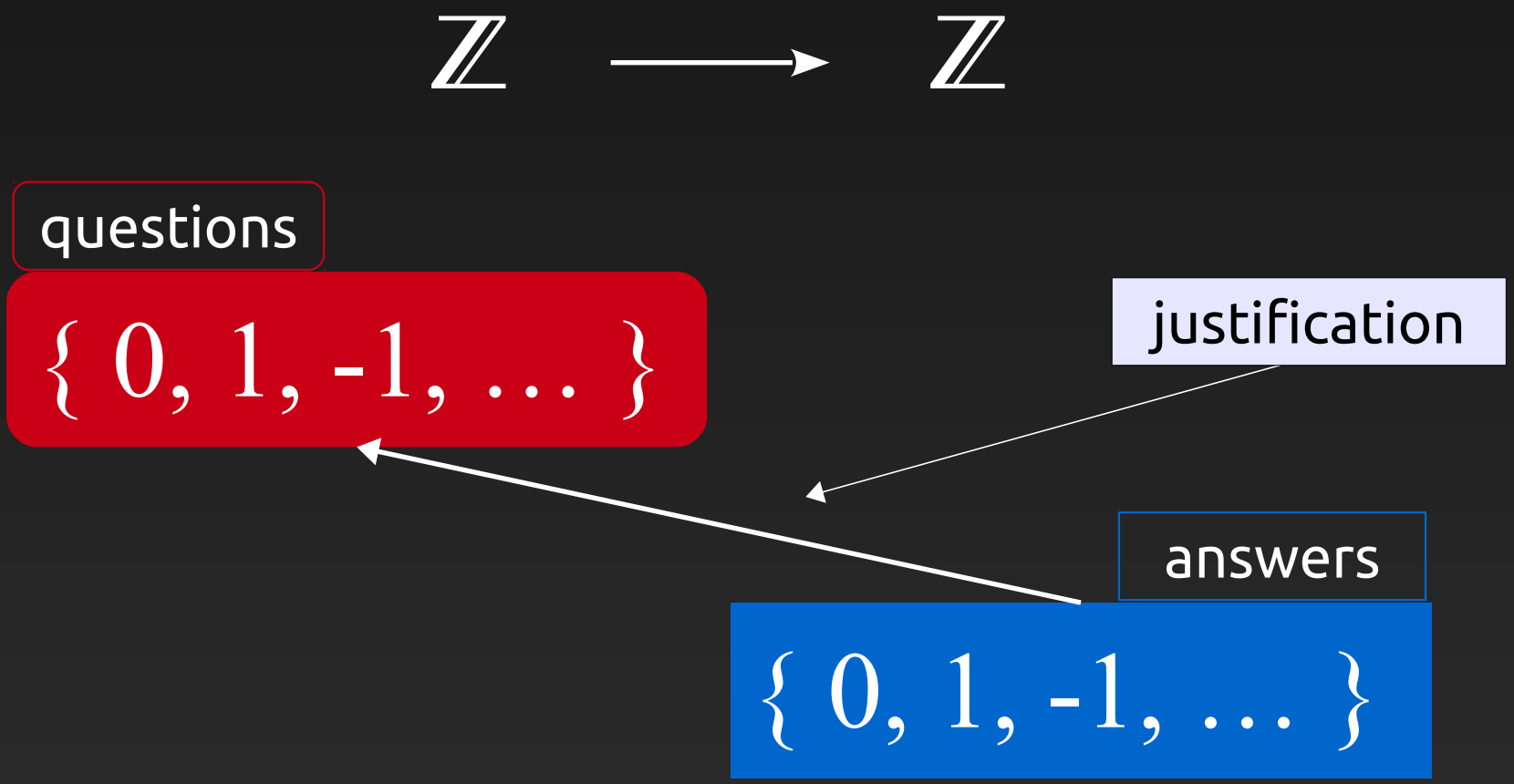
A simple interface



O : Opponent

P : Proponent

A simple interface



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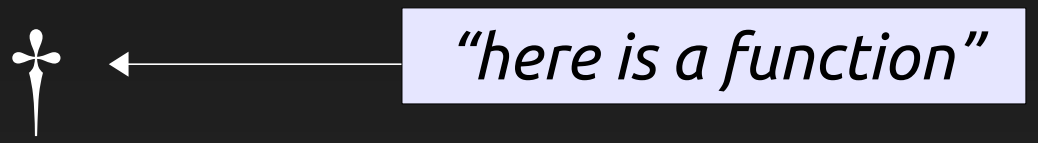
A higher-order arena

$$[\![\text{int} \to \text{int}]\!]$$



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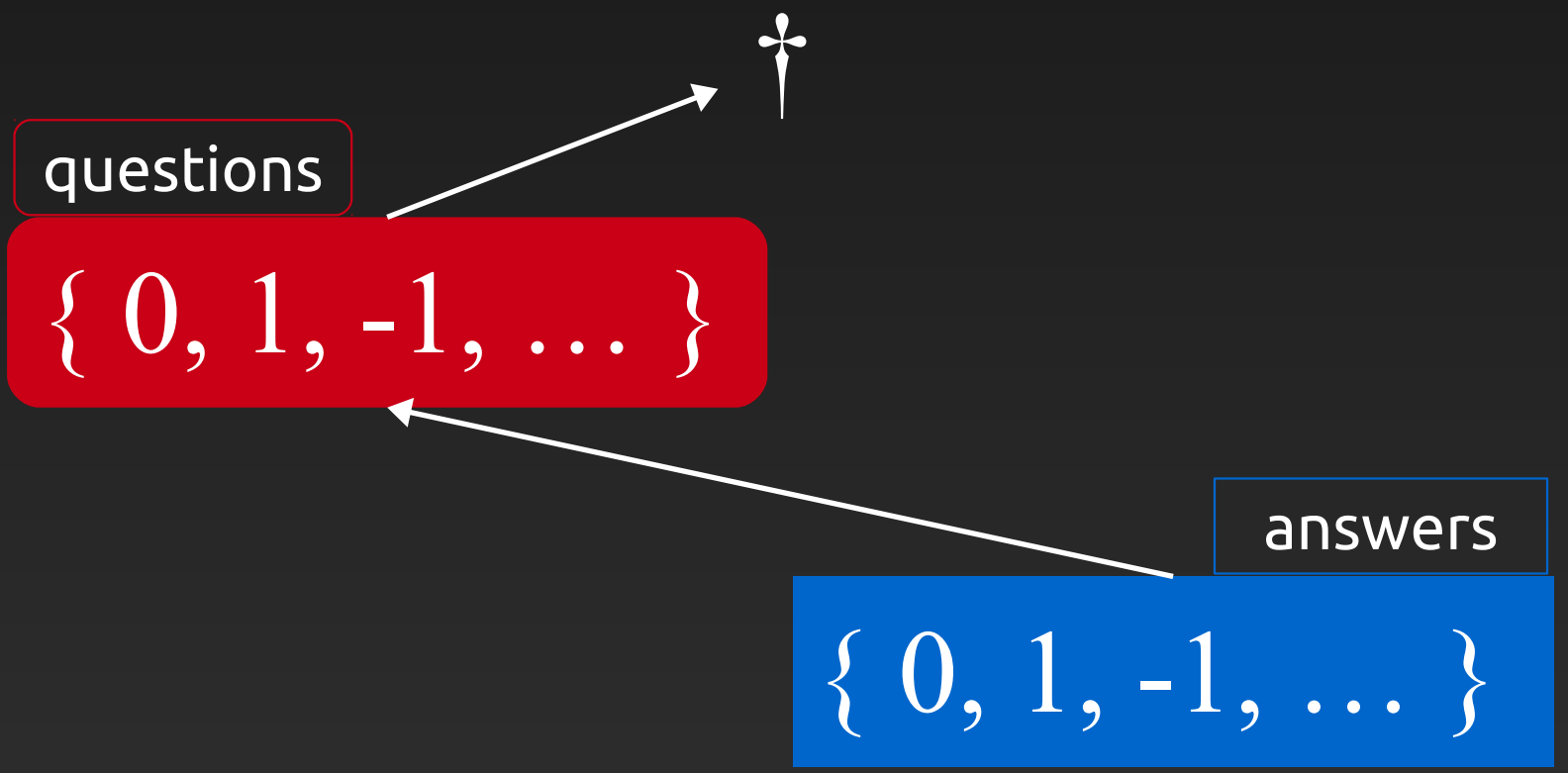

O : Opponent

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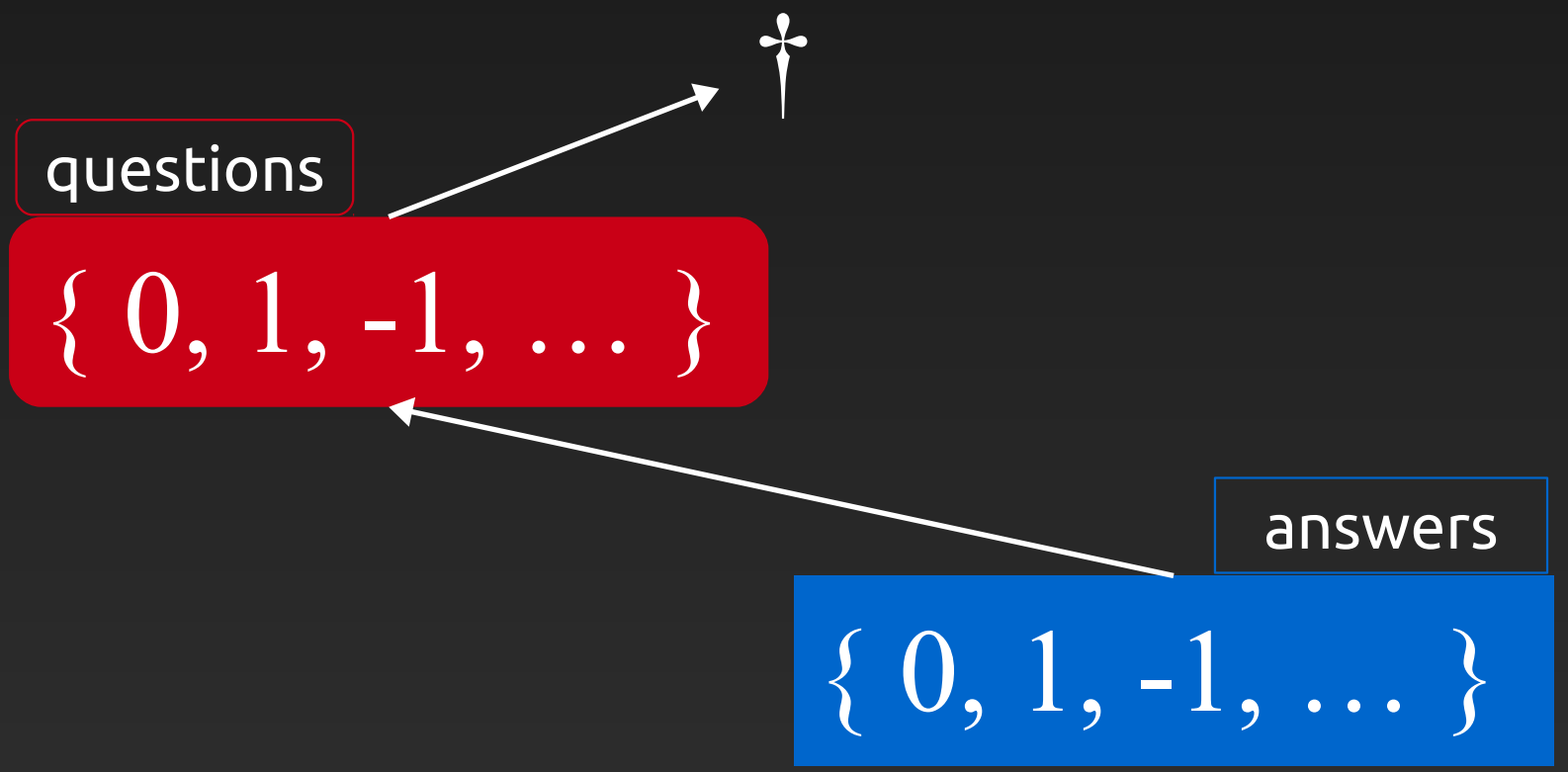
A higher-order arena

$$[\![\text{int} \to \text{int}]\!] = \mathbb{Z} \Rightarrow \mathbb{Z}$$



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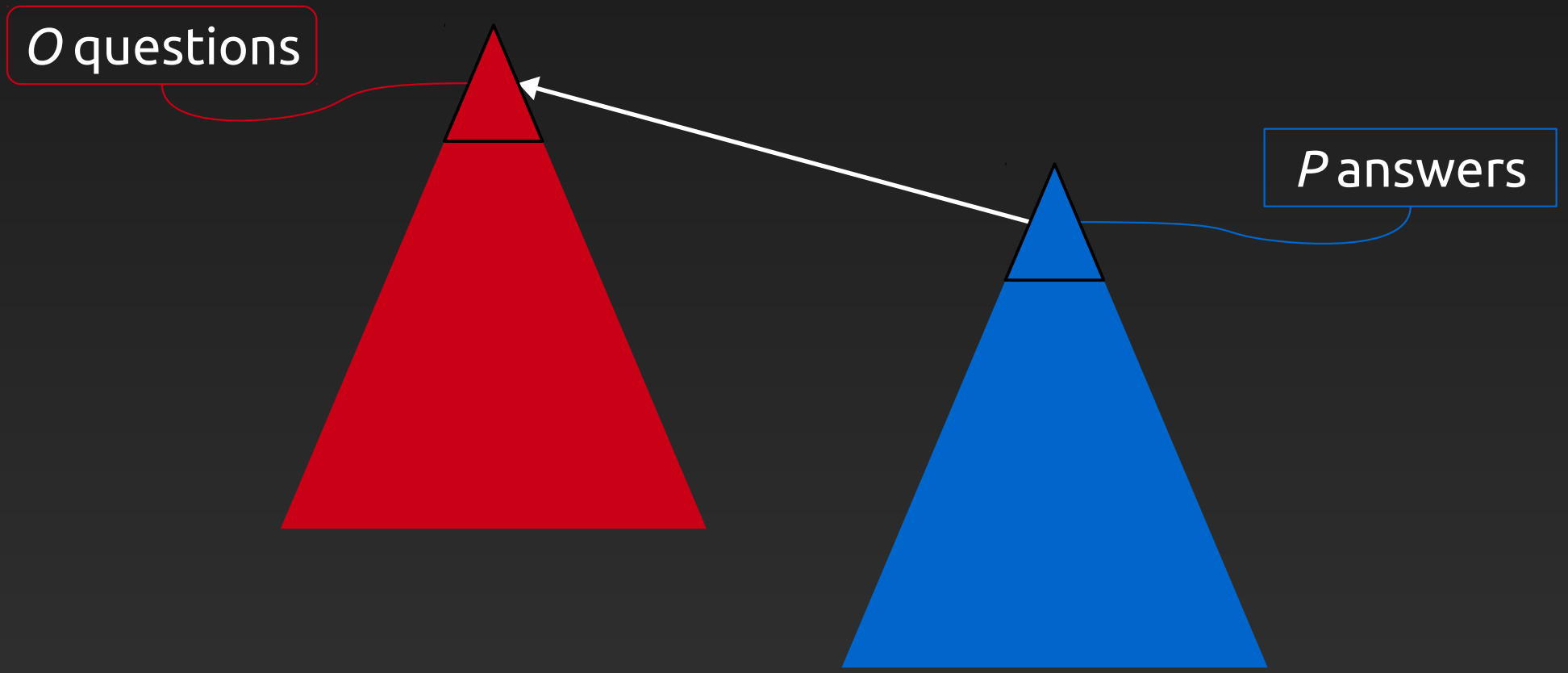
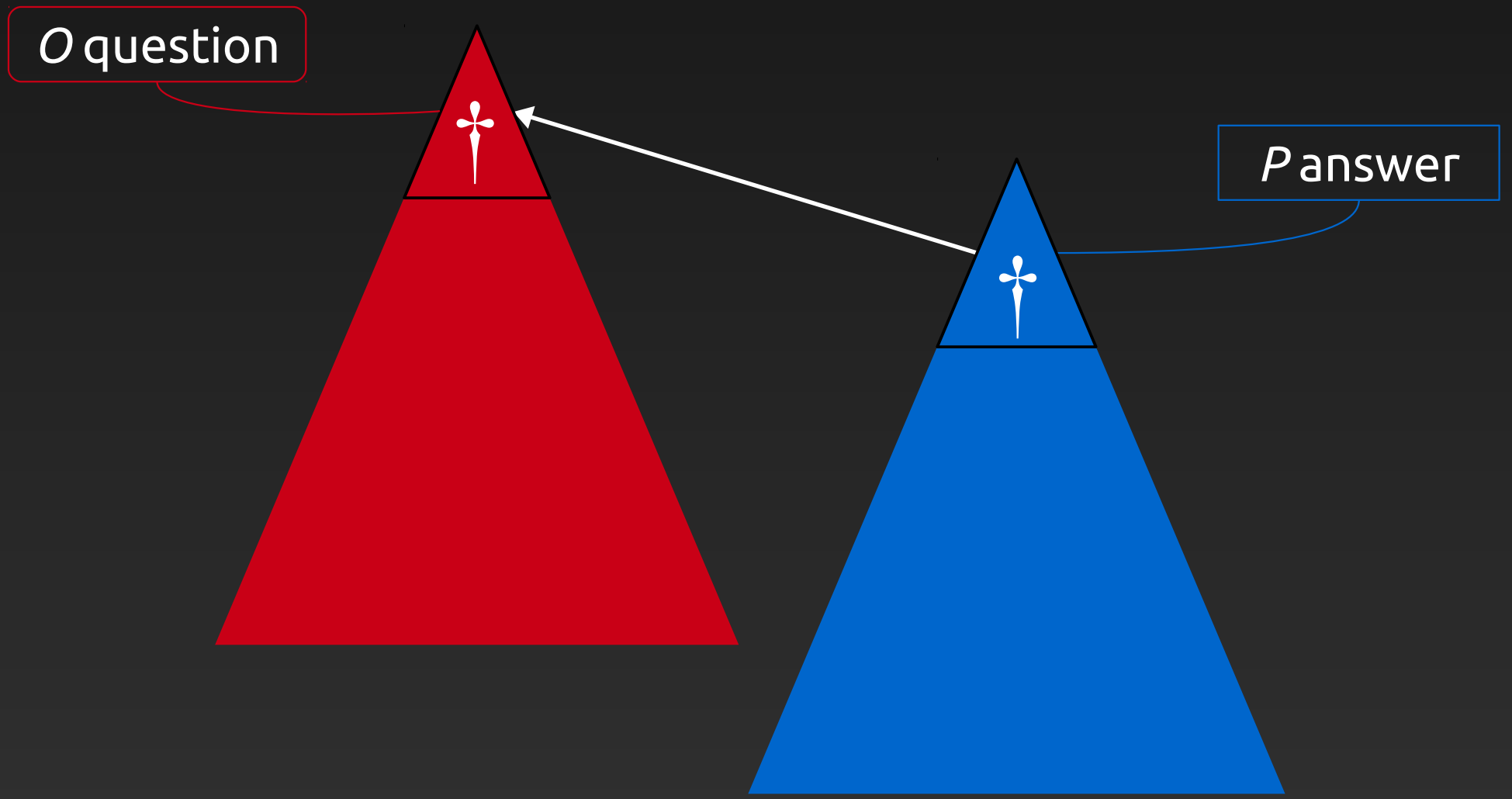
A higher-order interface

$$[\![T]\!] \longrightarrow [\![T']\!]$$



$$\mathbb{Z} \Rightarrow \mathbb{Z} \longrightarrow \mathbb{Z} \Rightarrow \mathbb{Z}$$



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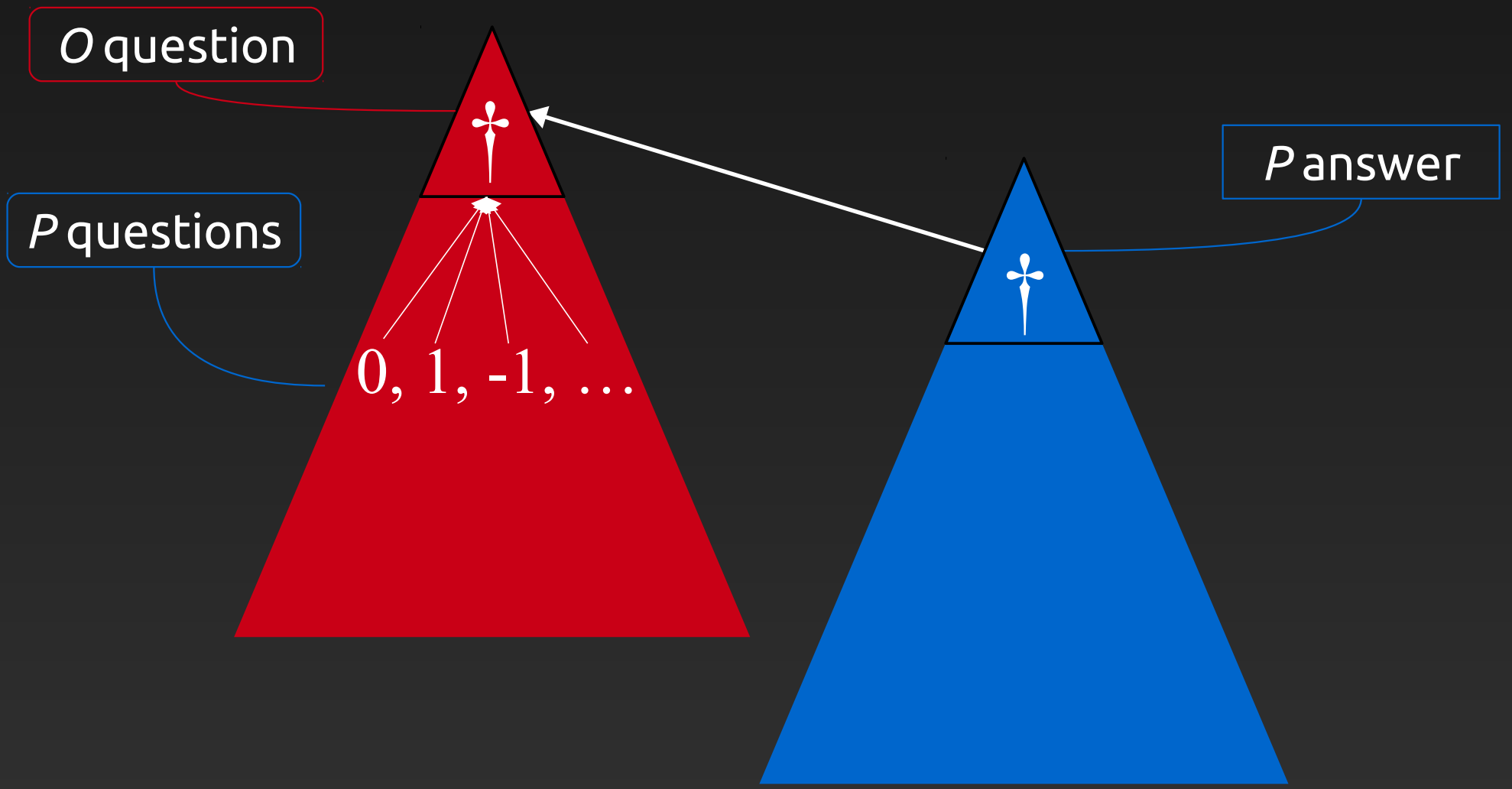

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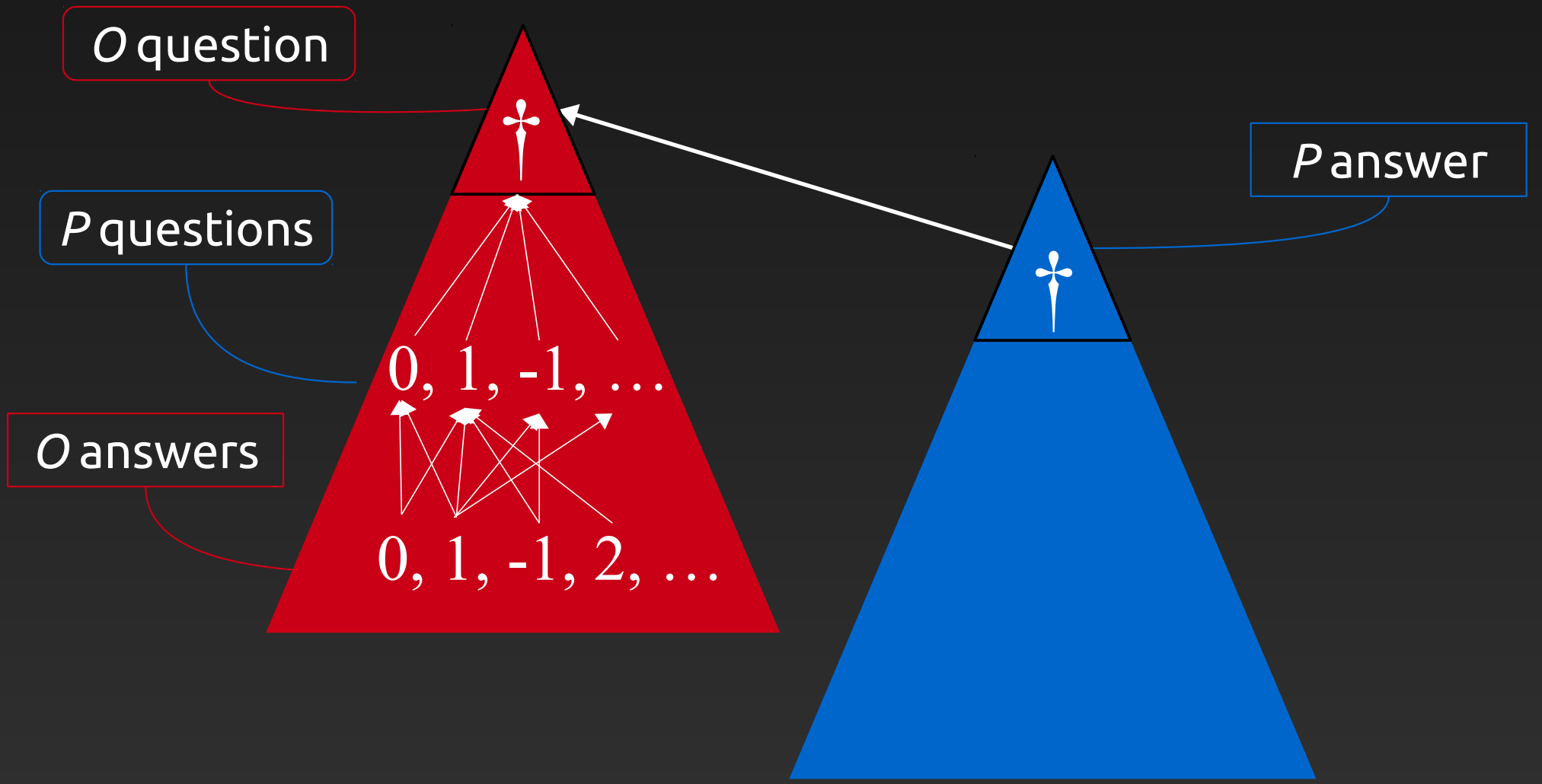

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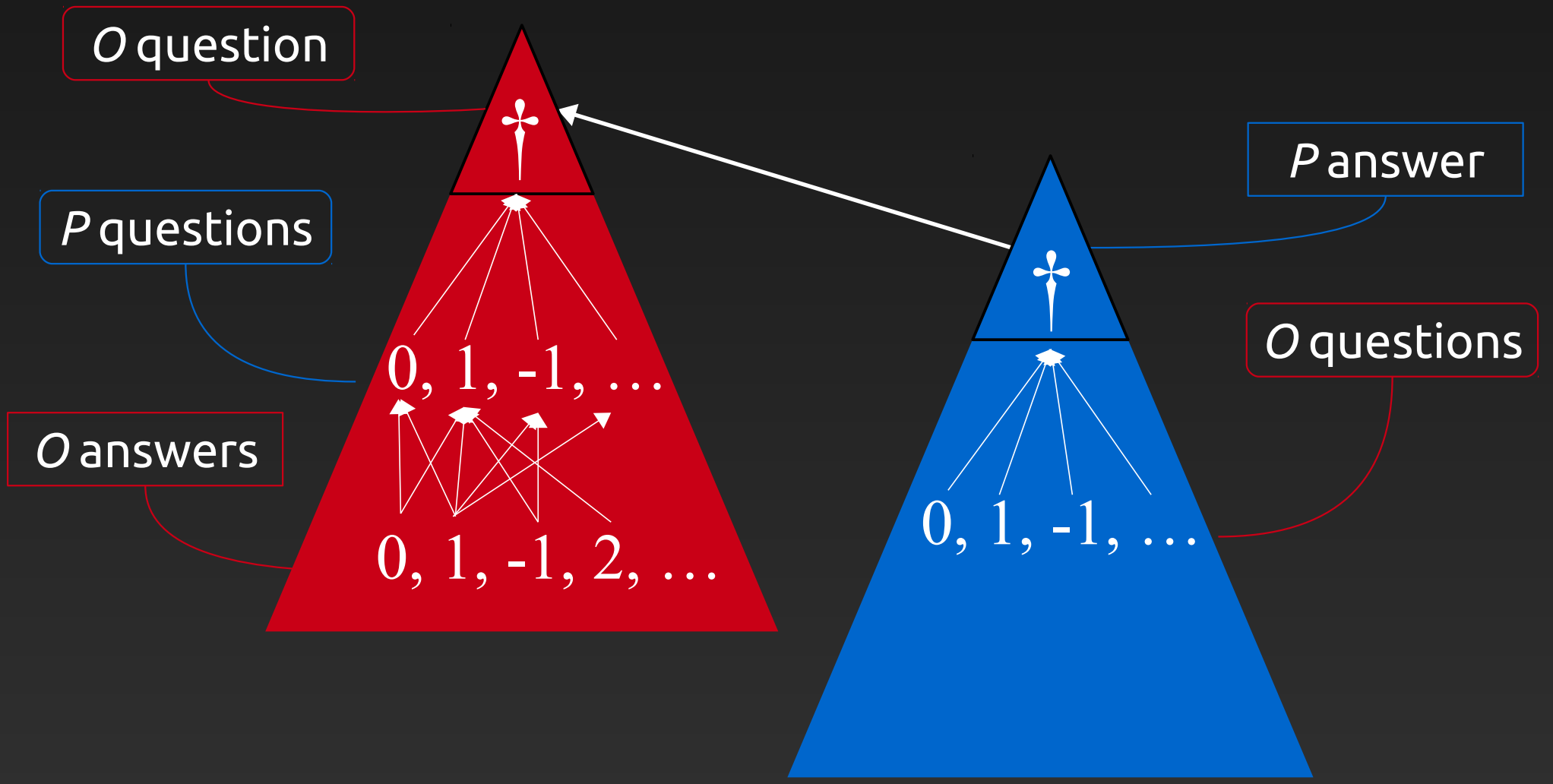

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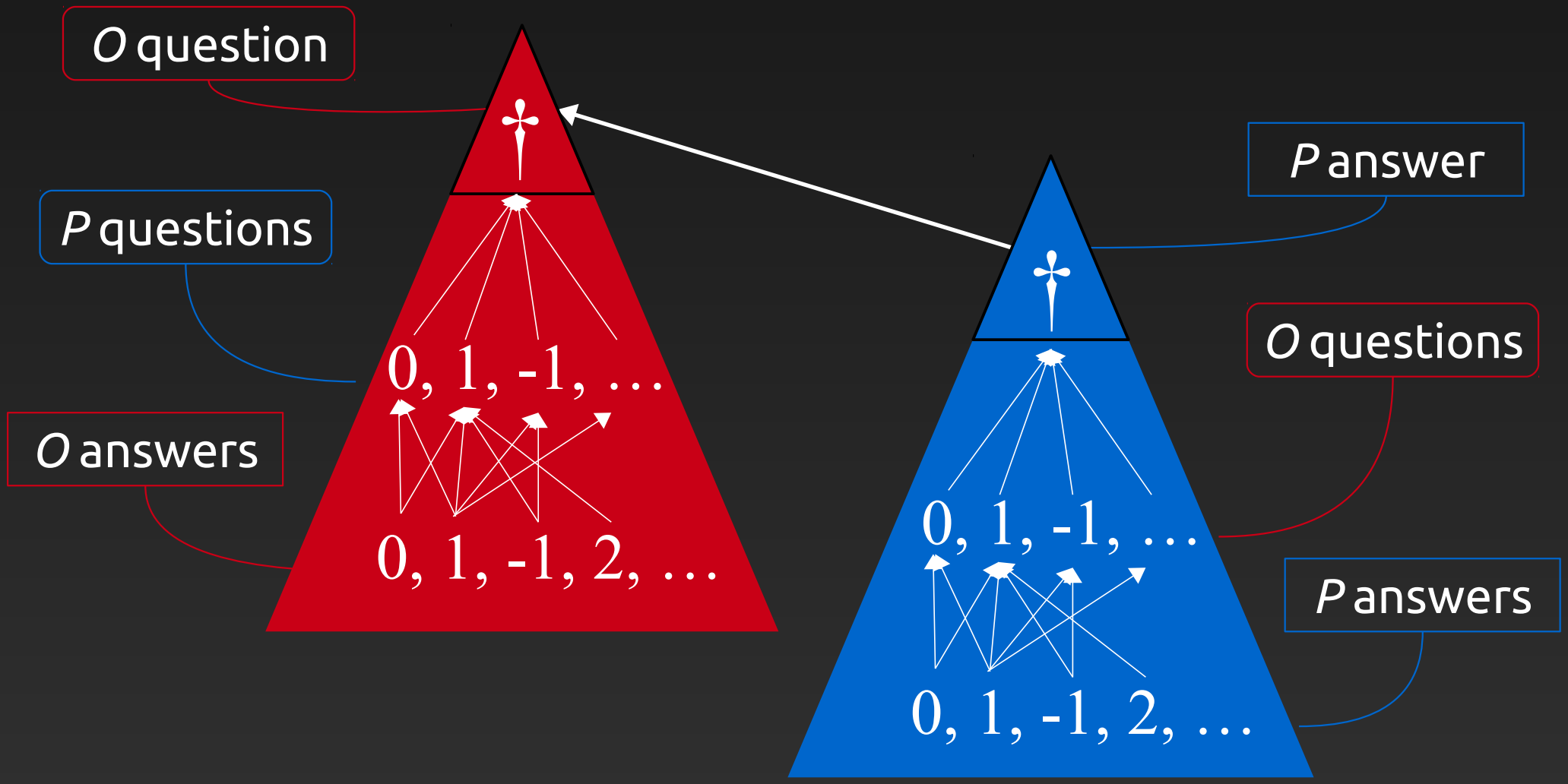

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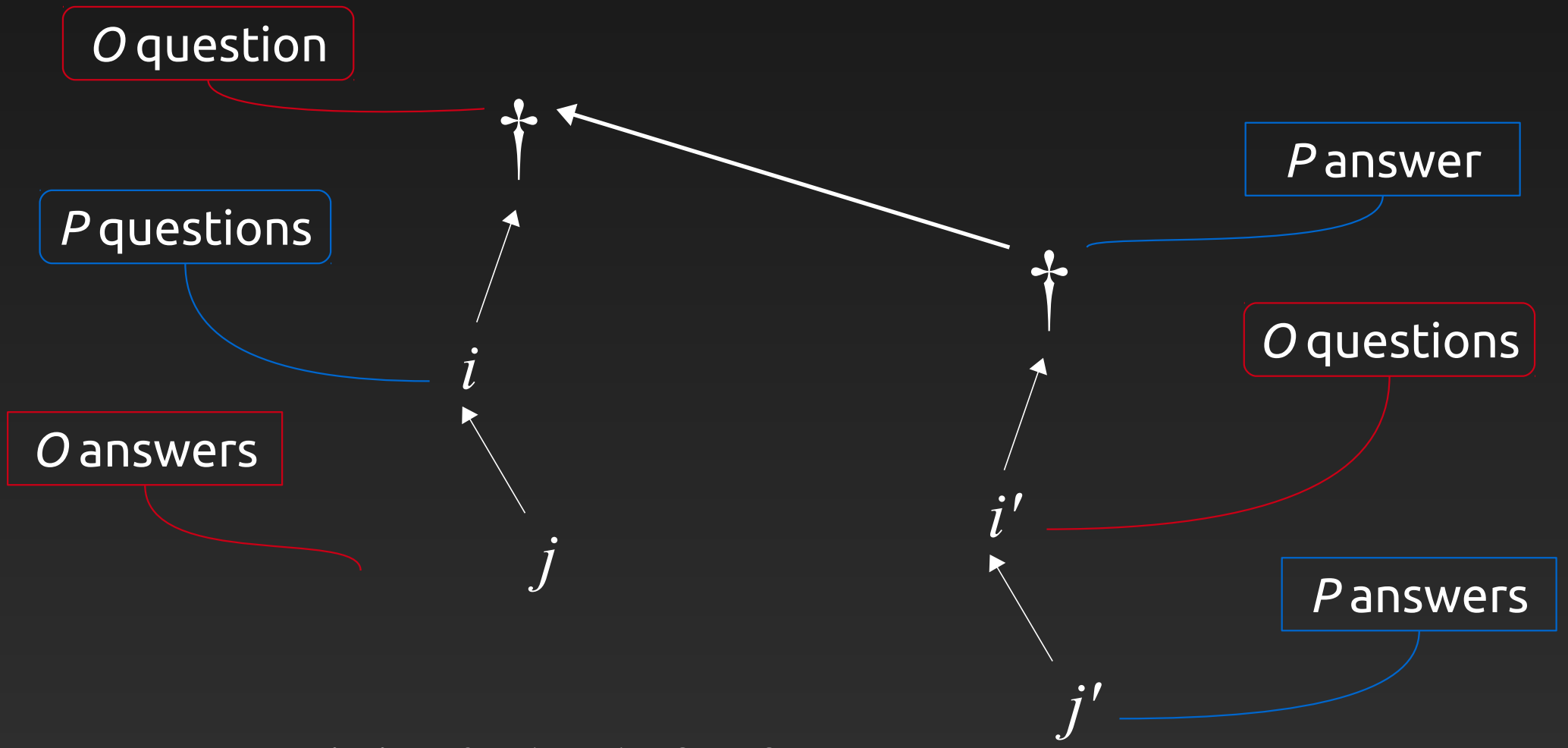
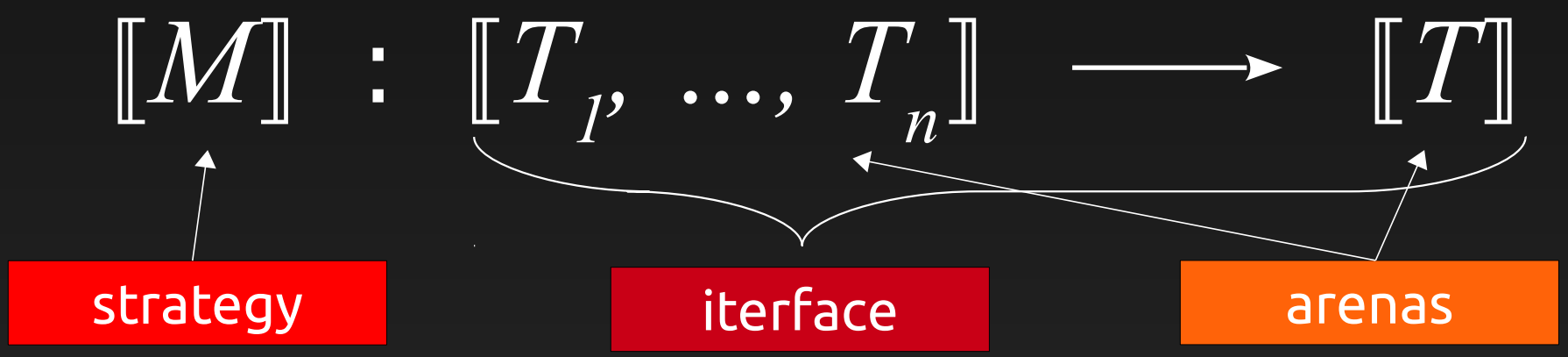


$i, j, i', j' = 0, 1, -1, 2, -2, \ldots$

Strategies



Strategies are “instructions” for *P* on how to play on a given interface

- Formally, sets of even-length plays satisfying combinatorial conditions linked to language expressivity

Example

$$f : \text{int} \to \text{int} \;\vdash\; \lambda x.\, f\, x + 1 : \text{int} \to \text{int}$$

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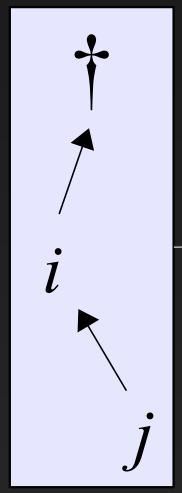

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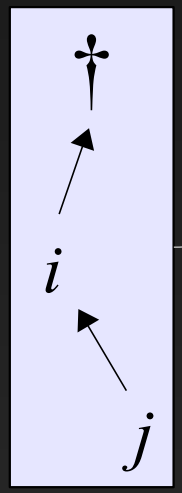

$$\frac{\mathbb{Z} \Rightarrow \mathbb{Z} \longrightarrow \mathbb{Z} \Rightarrow \mathbb{Z}}{\dagger} \; O,Q$$

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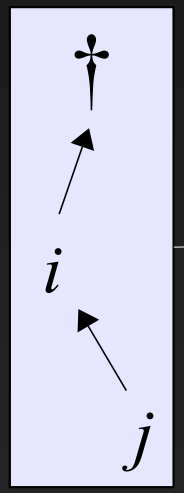

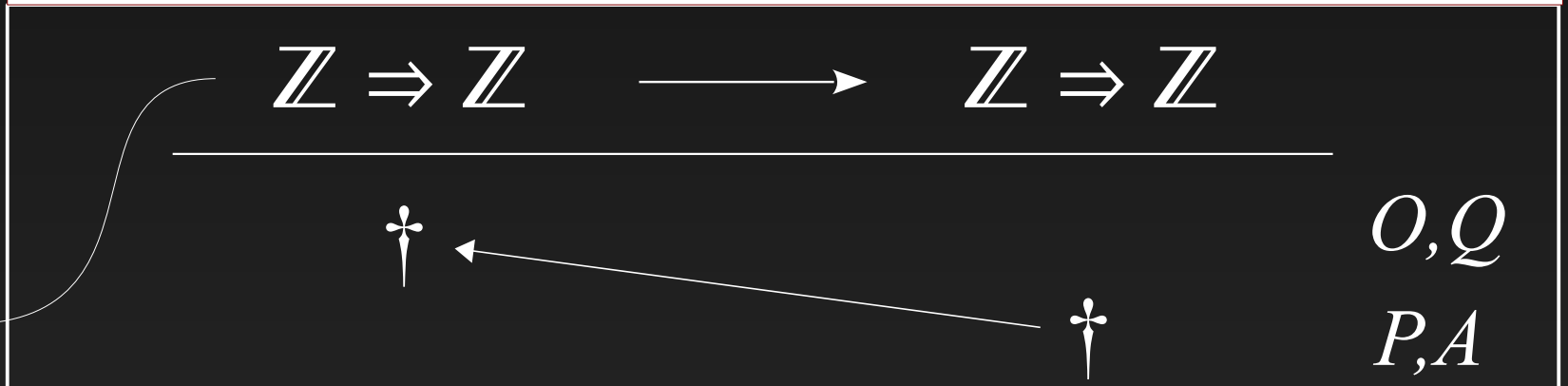

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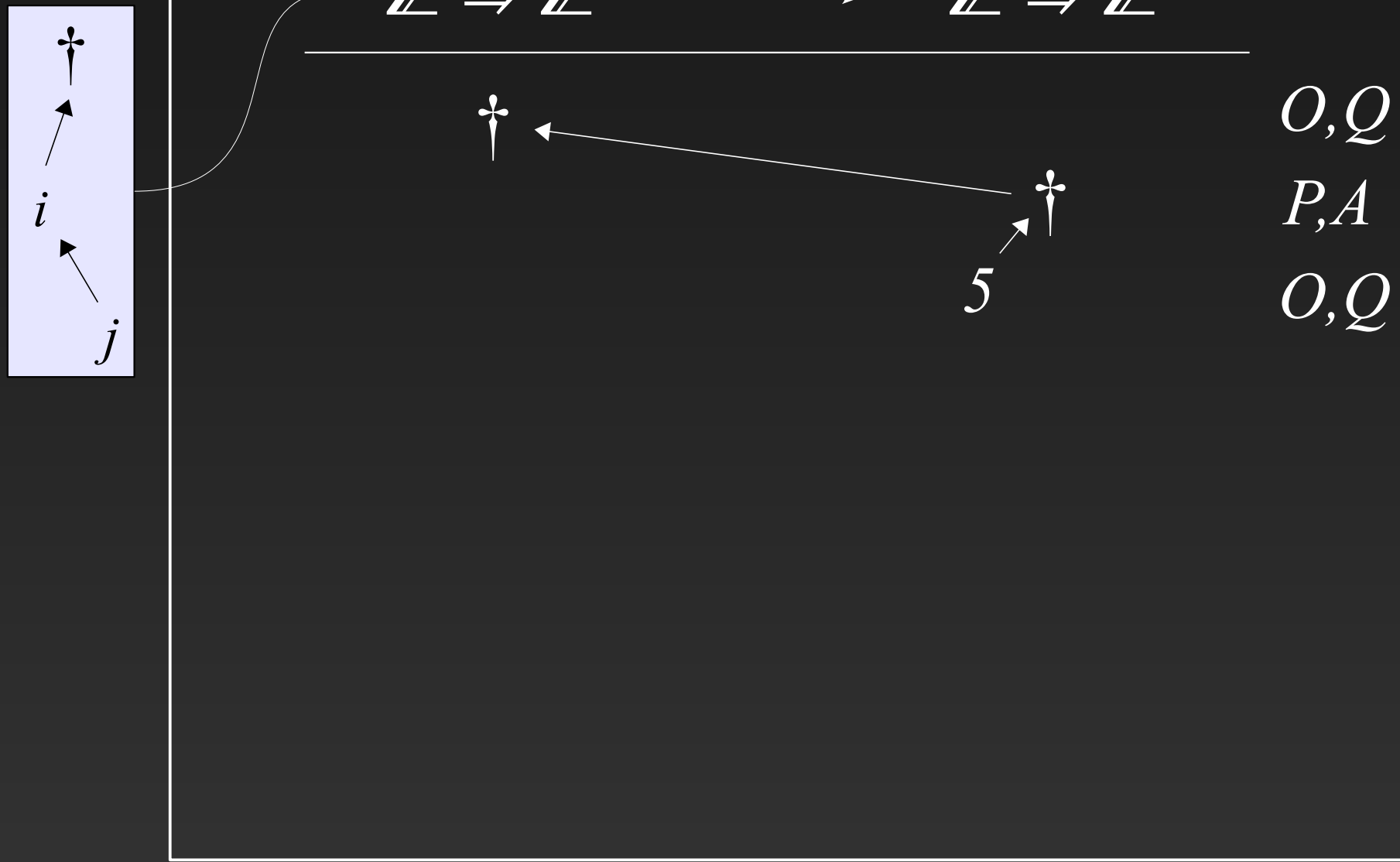

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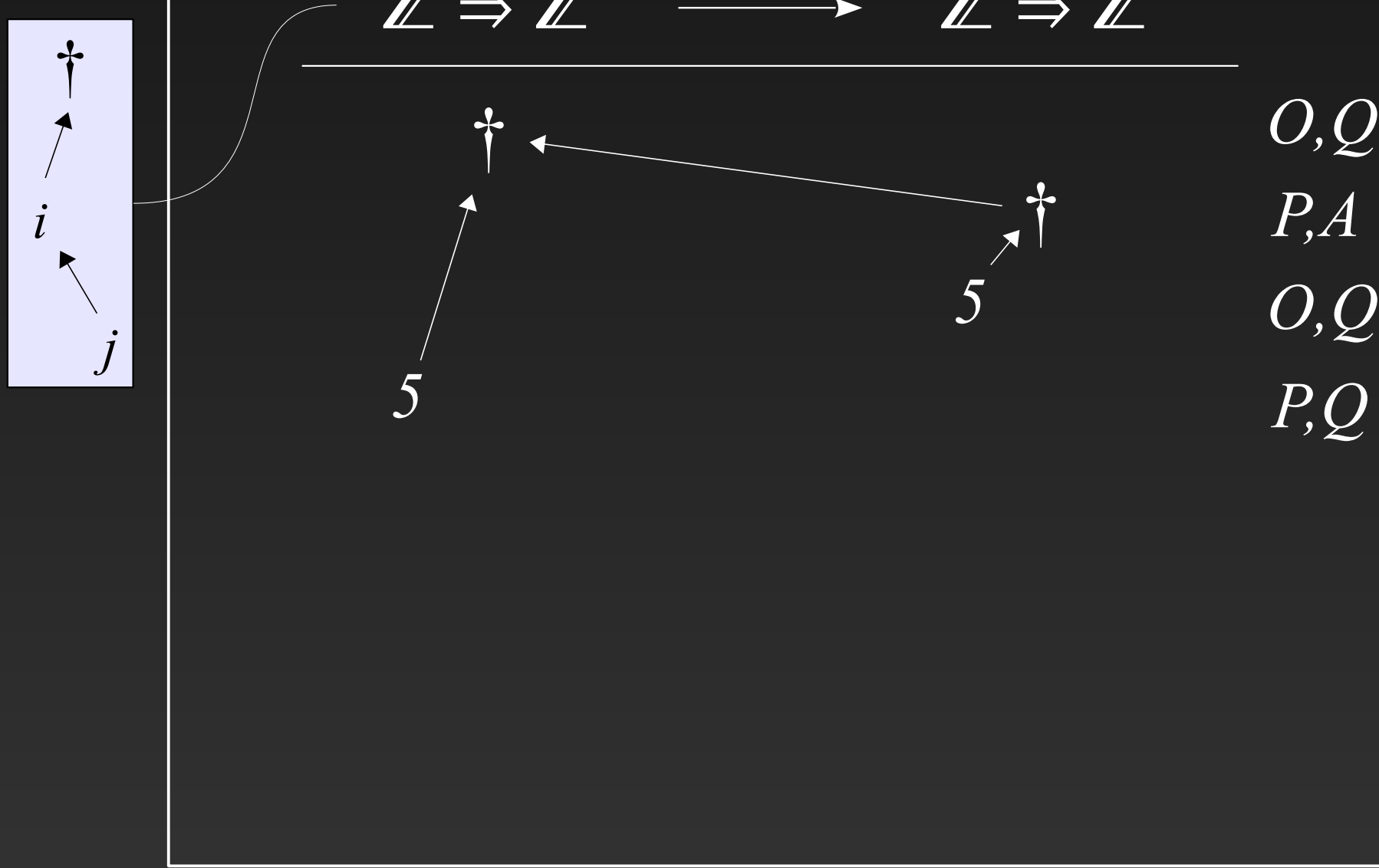

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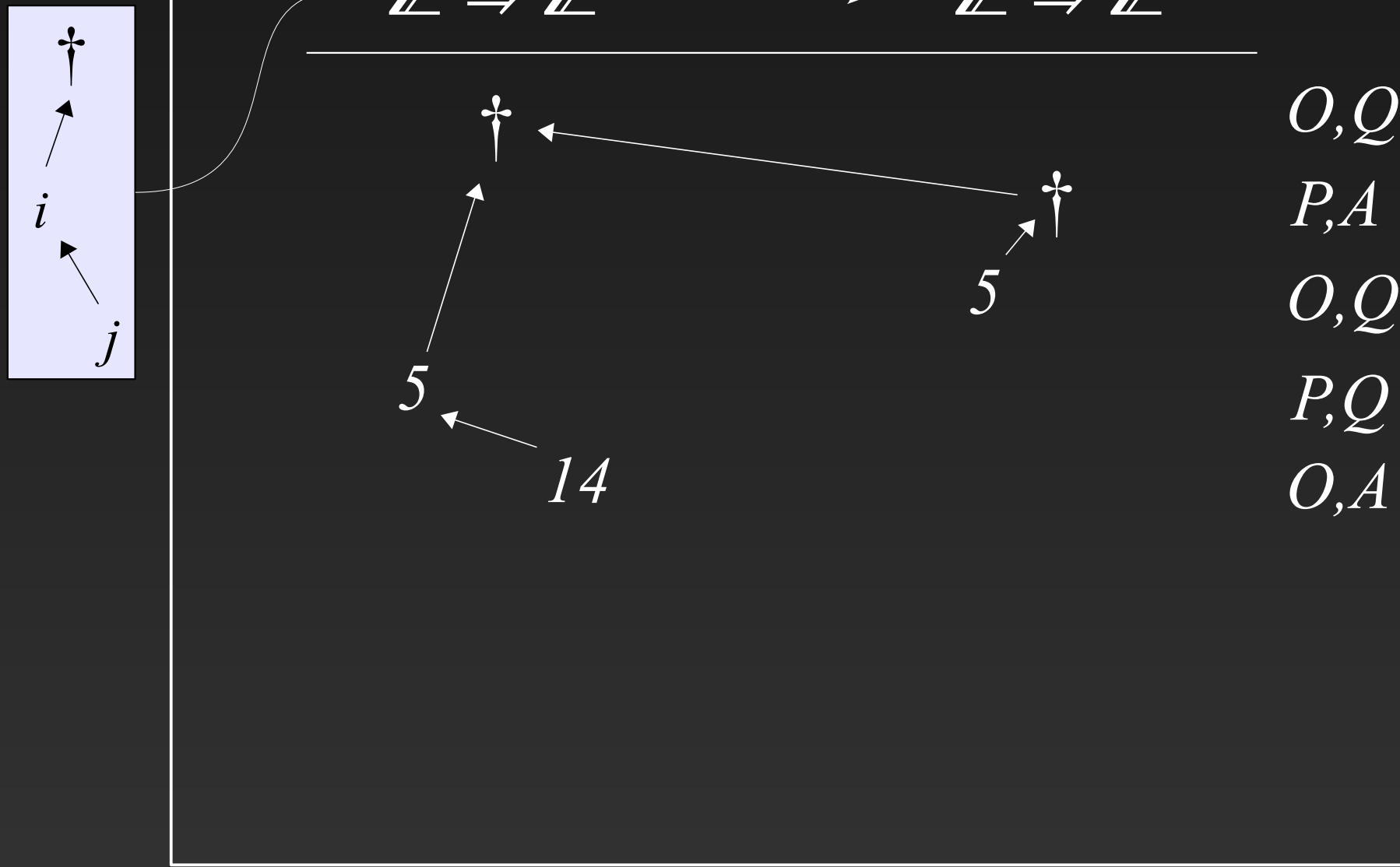

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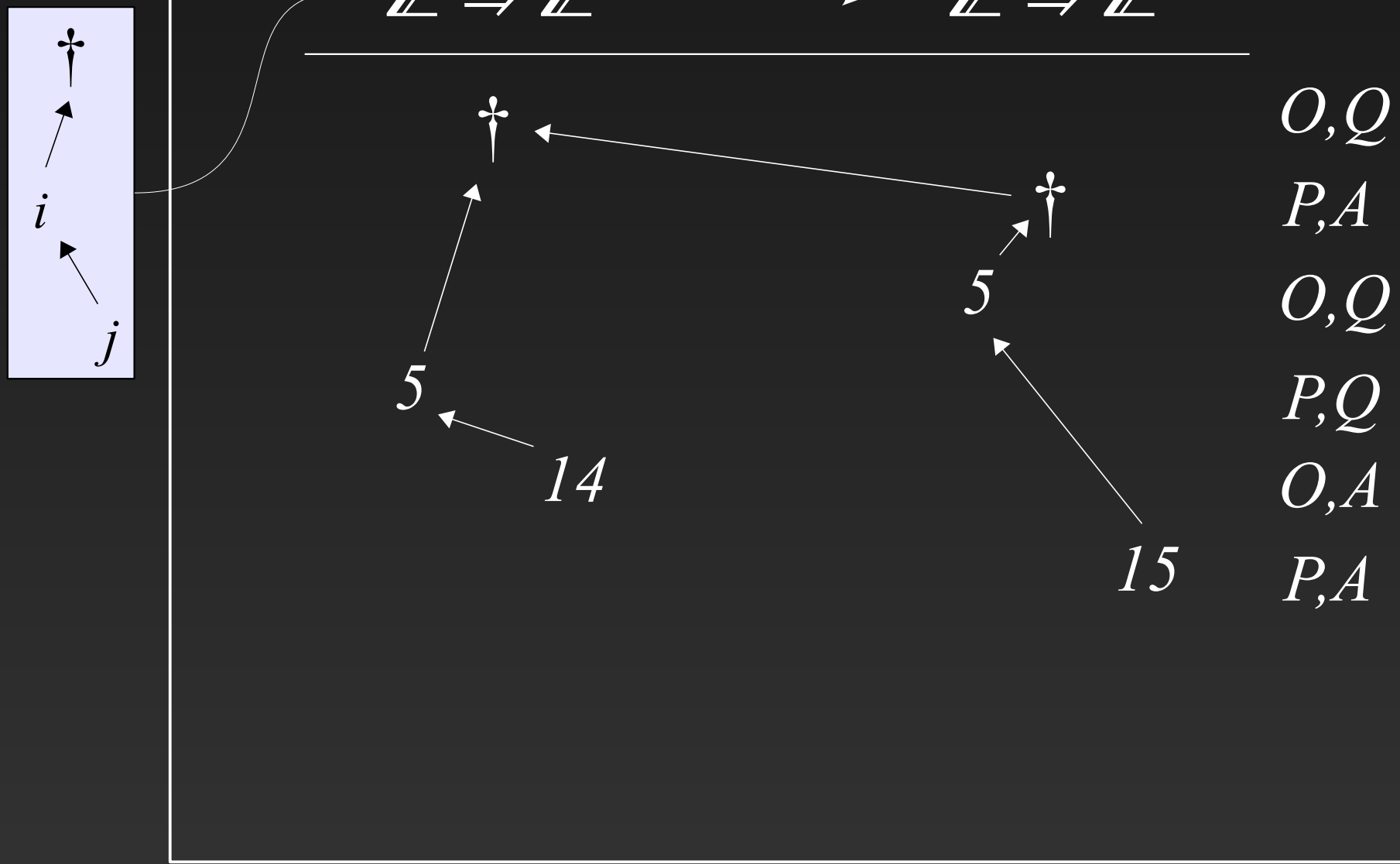

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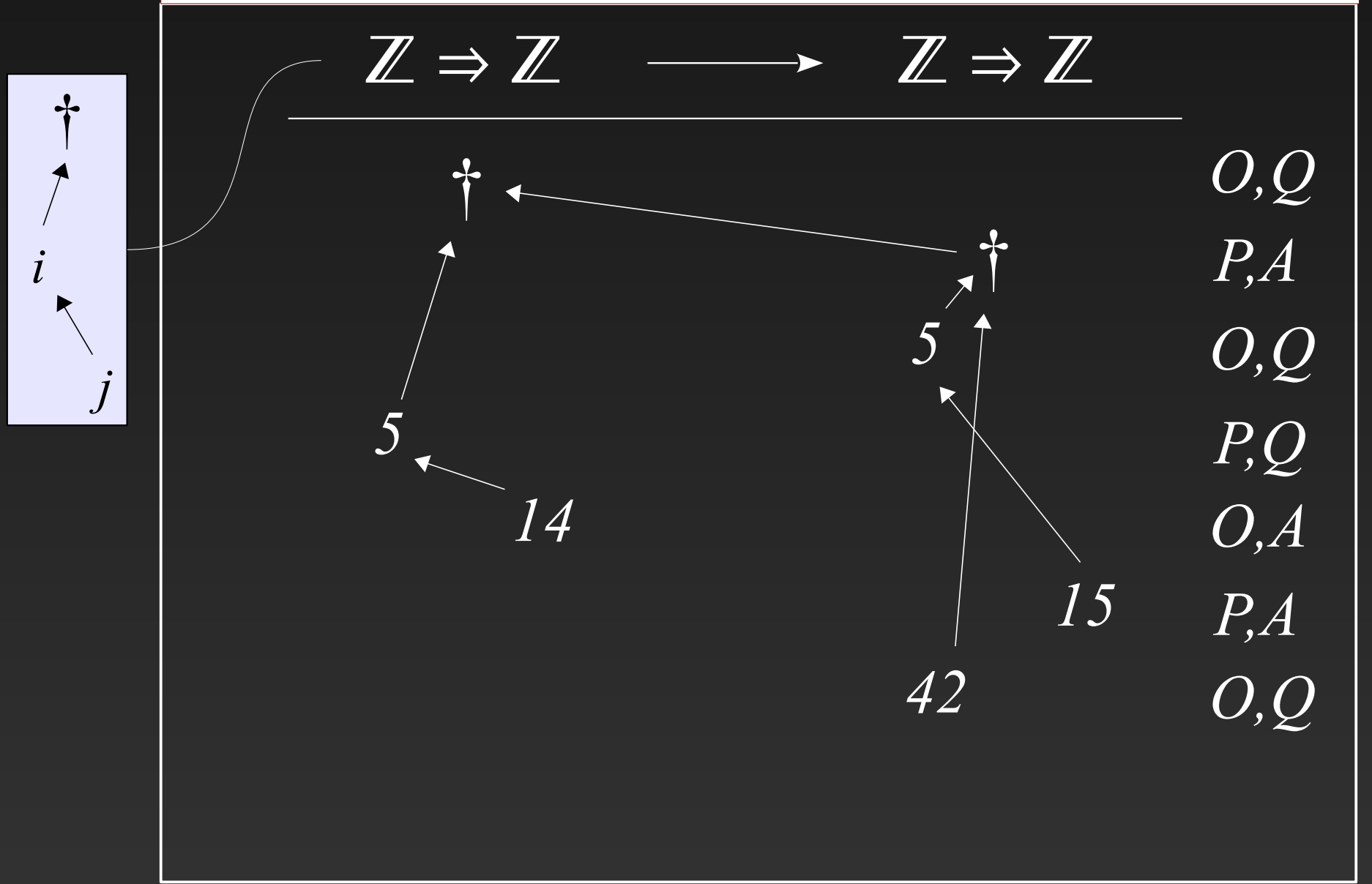

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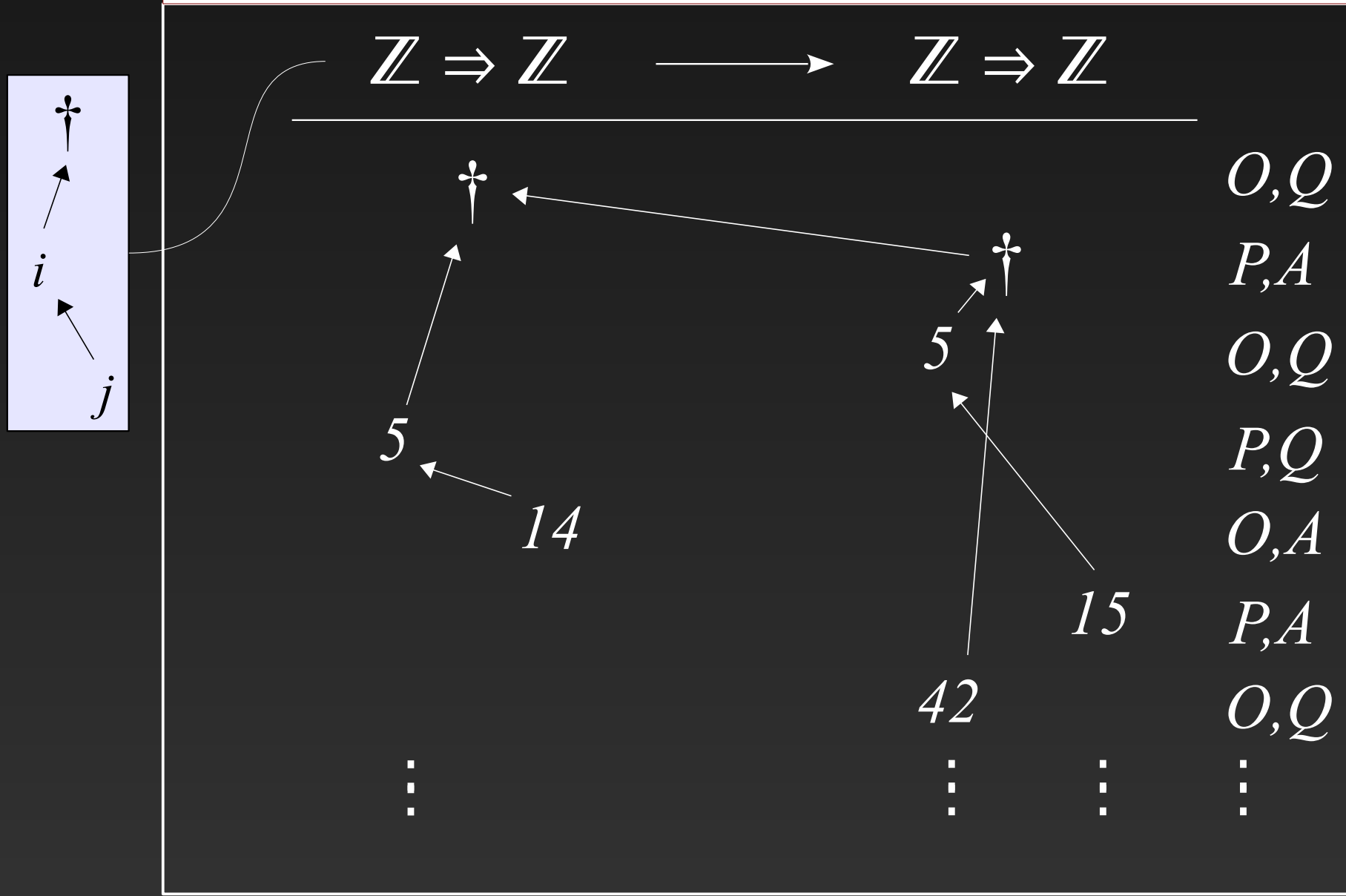

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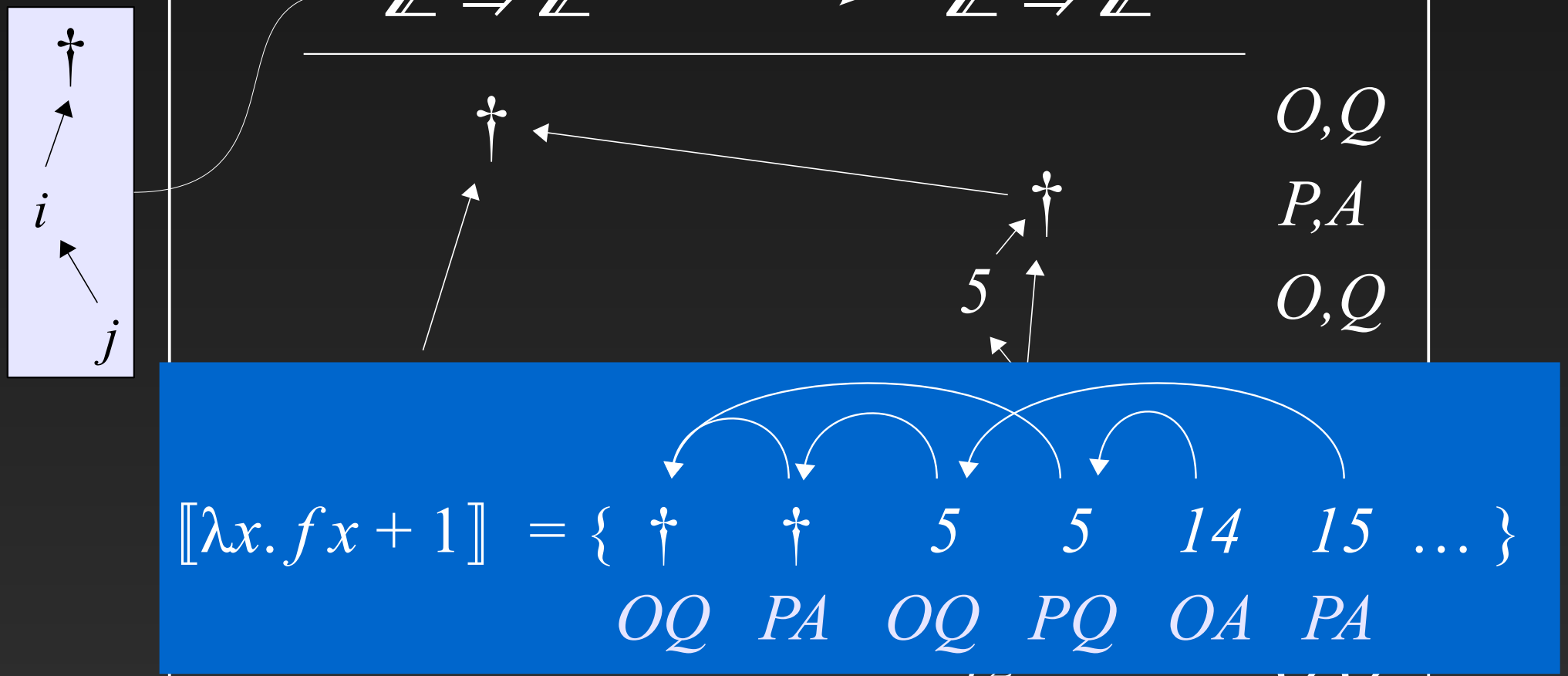
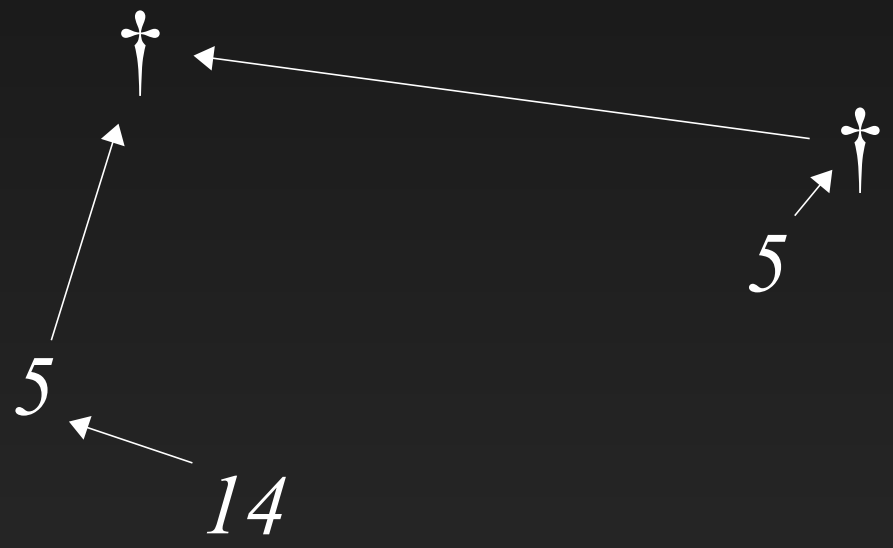


Another example

$\mathbb{Z} \Rightarrow \mathbb{Z} \longrightarrow \mathbb{Z} \Rightarrow \mathbb{Z}$



O,Q

P,A

O,Q

P,Q

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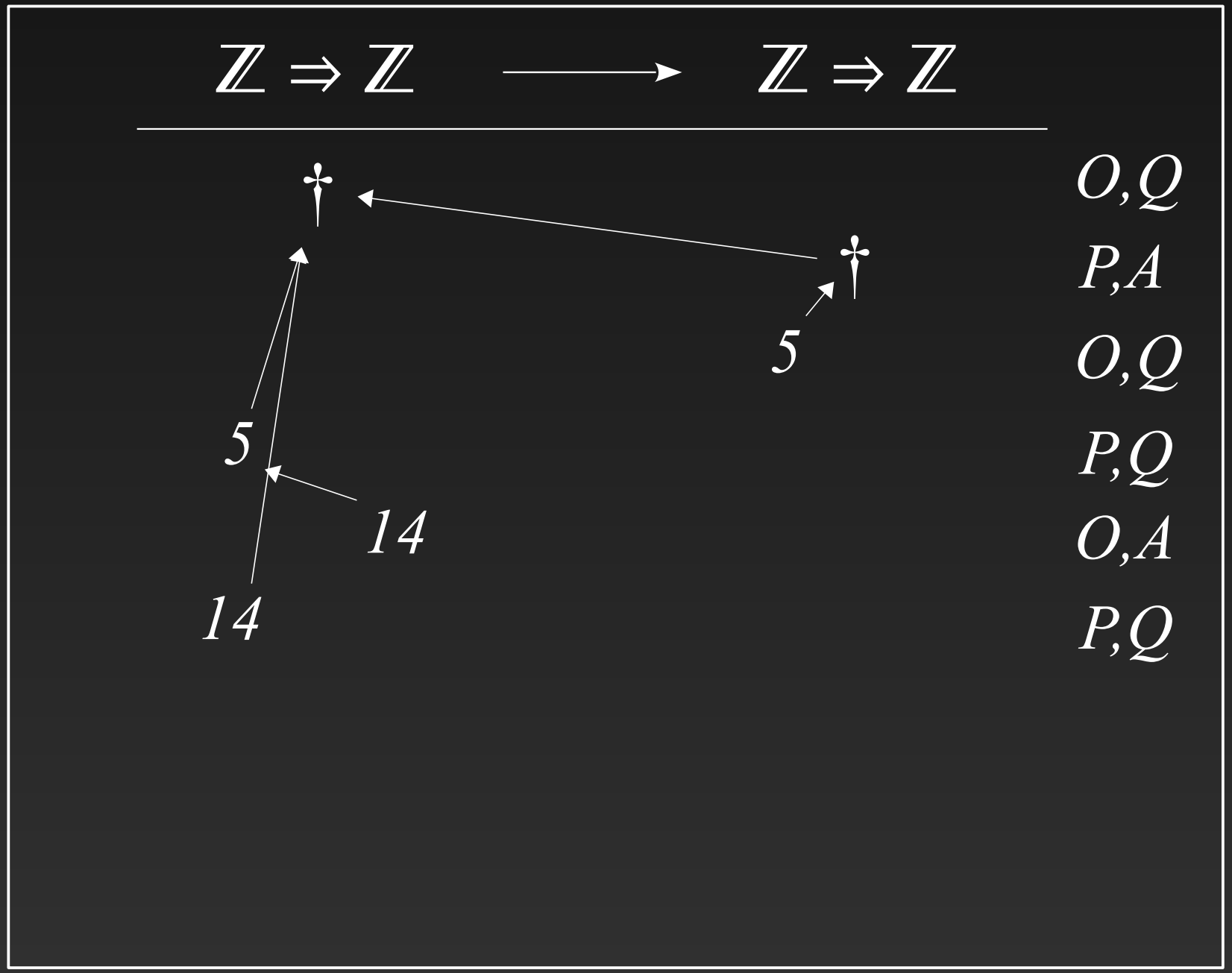

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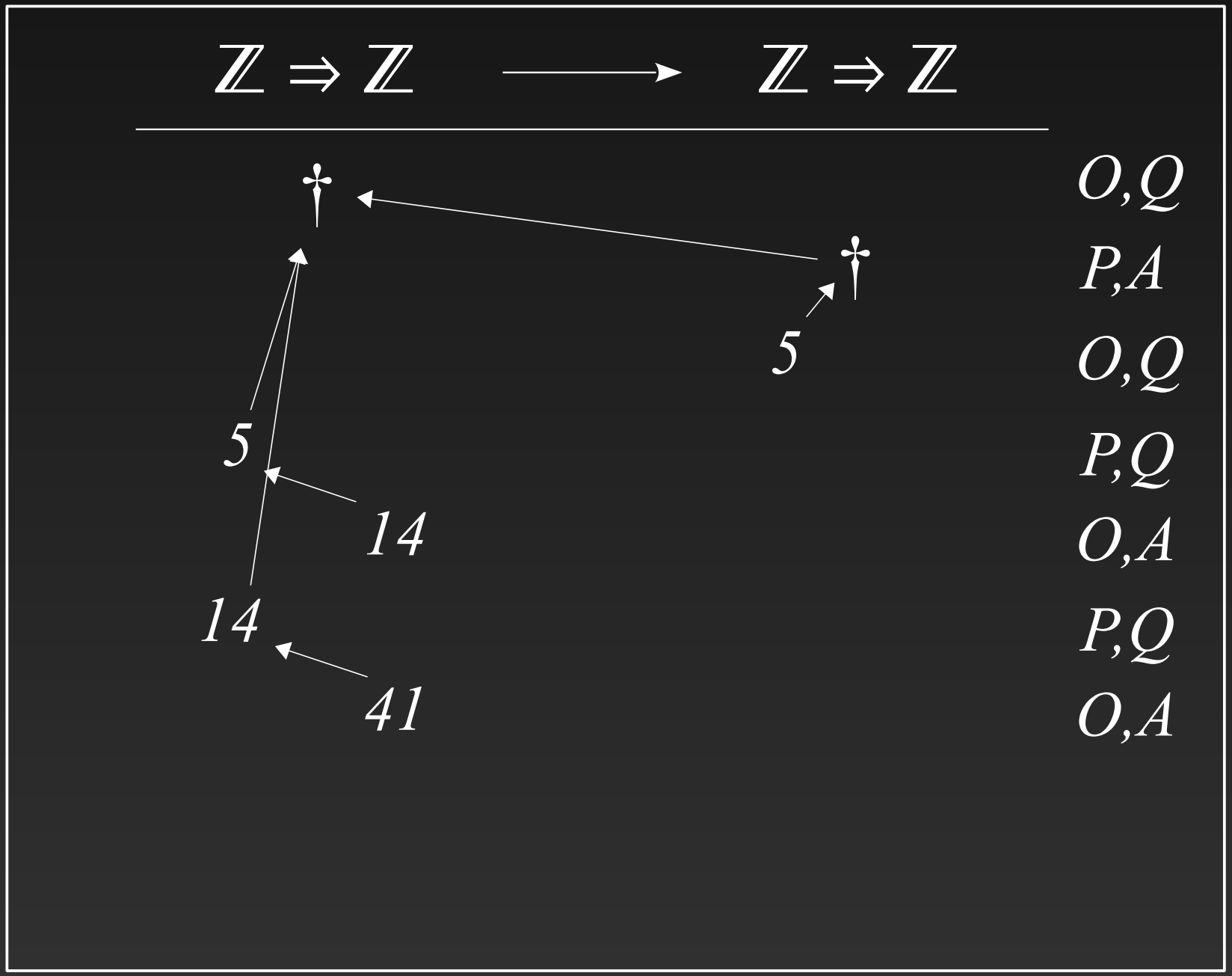

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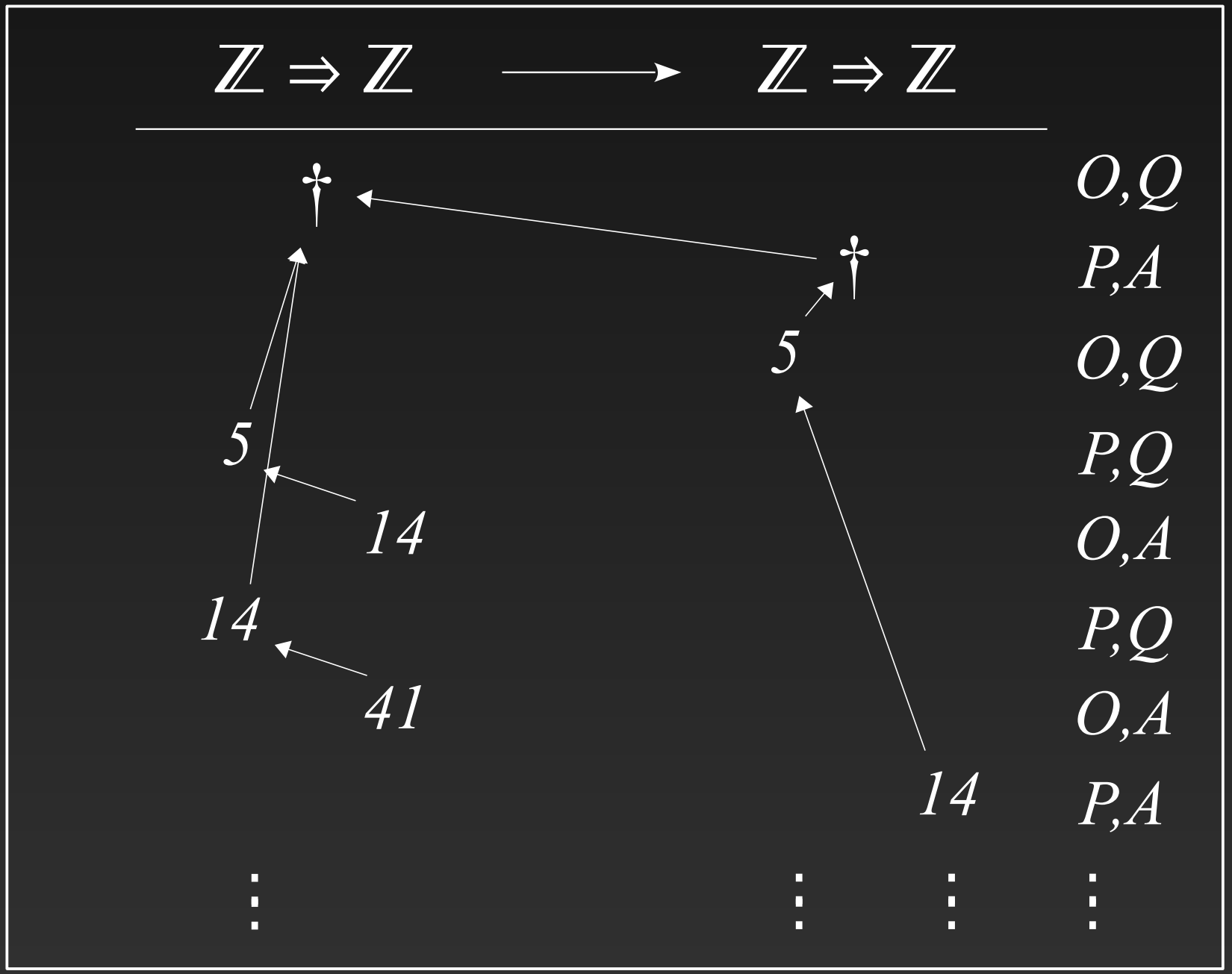

$$f : \mathrm{int} \to \mathrm{int} \;\vdash\; \lambda x.\, f\,(f\,(x)) - 27 : \mathrm{int} \to \mathrm{int}$$

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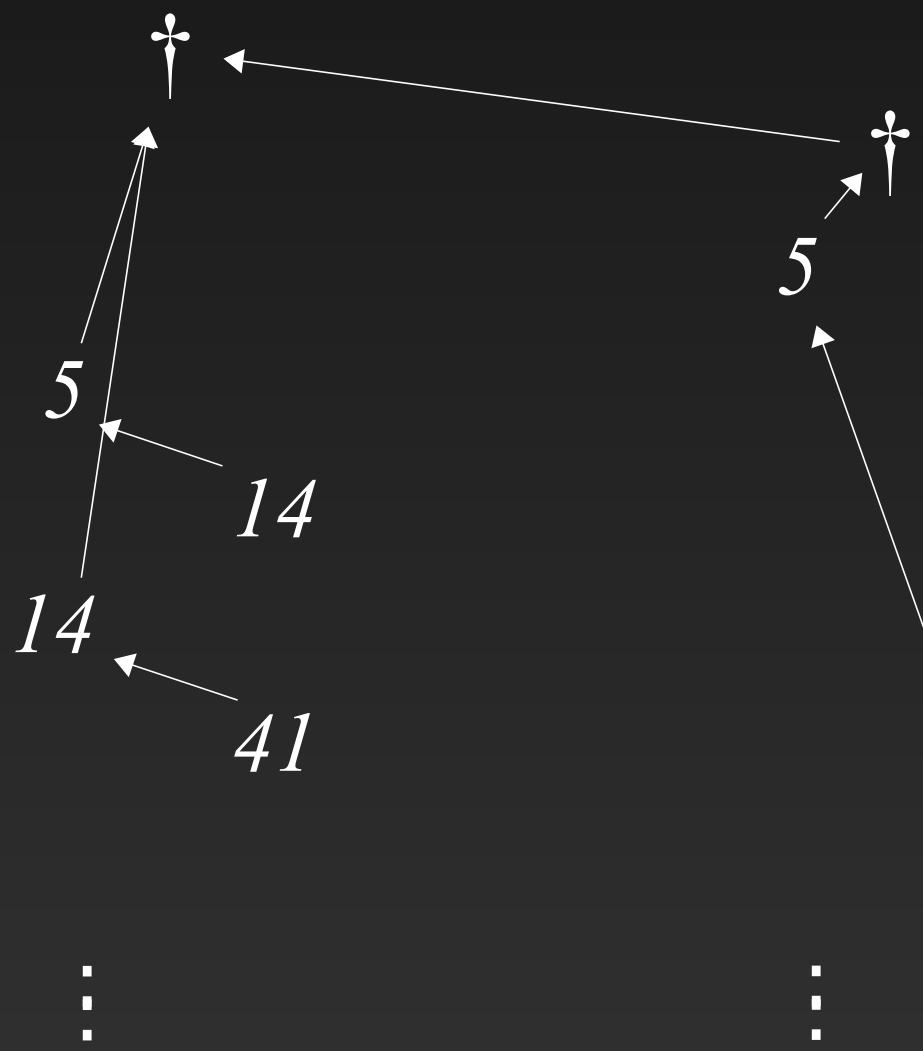
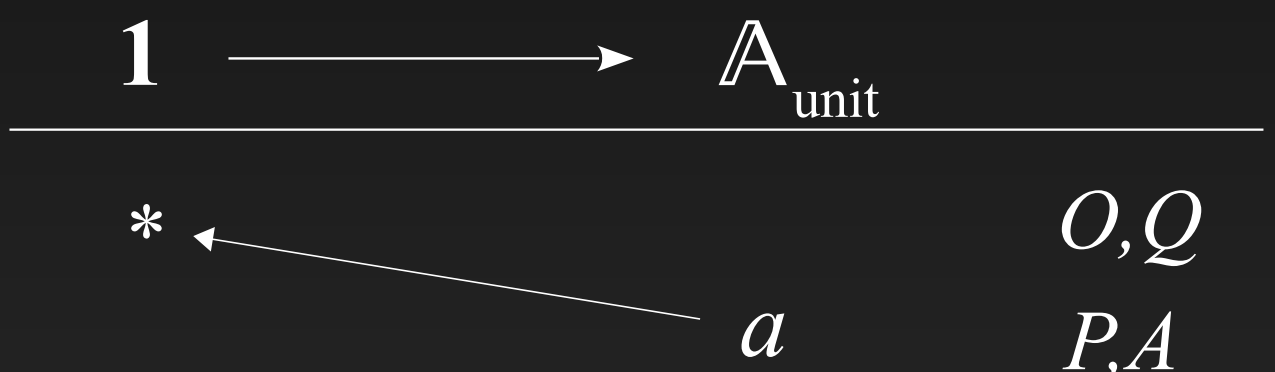


O,Q
P,A
O,Q
P,Q
O,A
P,Q
O,A
P,A
⋮ ⋮ ⋮ ⋮

Nominal games

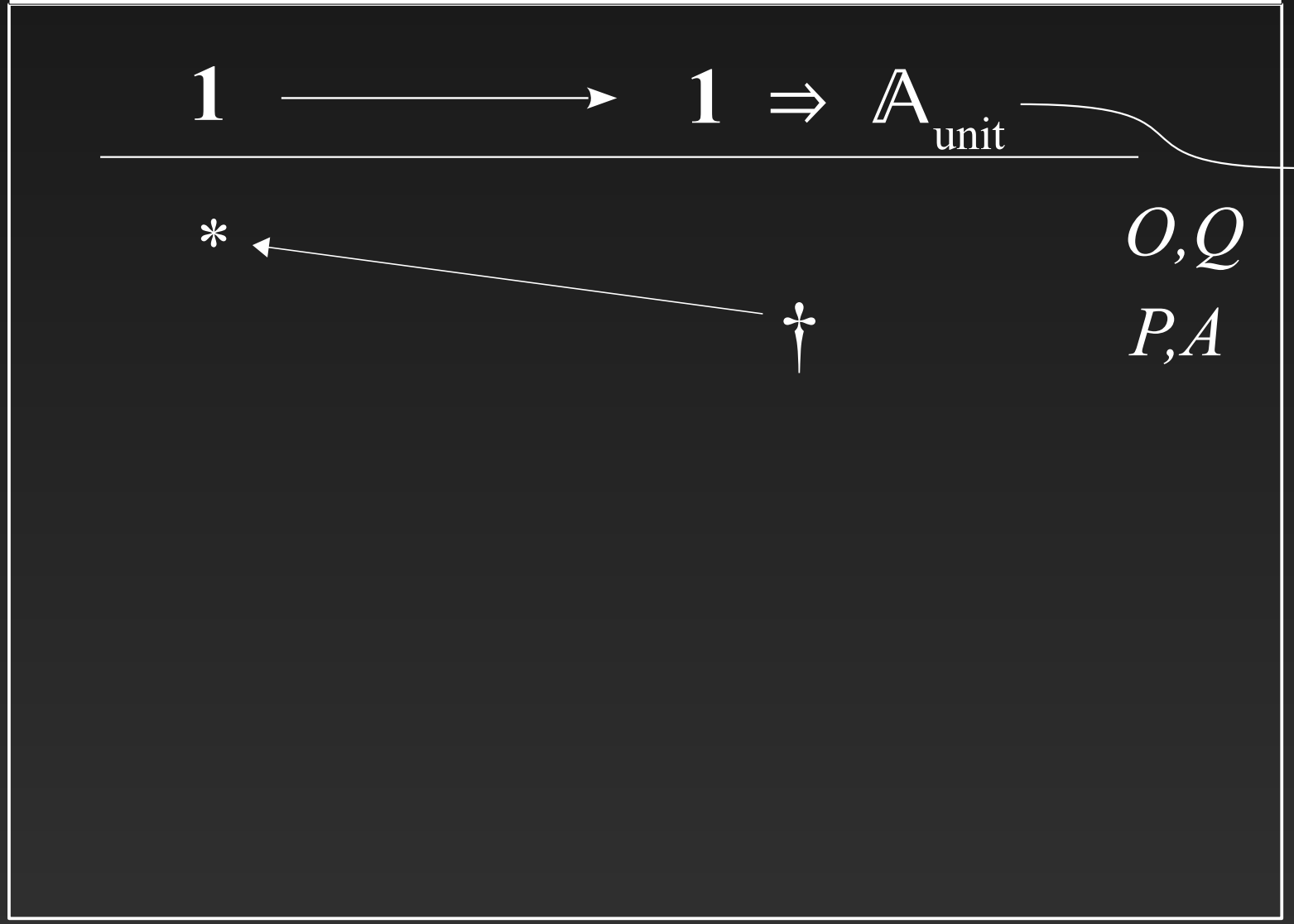
$\vdash \mathrm{ref}() : \mathrm{ref\ unit}$



$a \in \mathcal{N}_{\mathrm{unit}}$

Nominal games (2)

$\vdash \lambda x.\ \mathrm{ref}() : \mathrm{unit} \to \mathrm{ref\ unit}$



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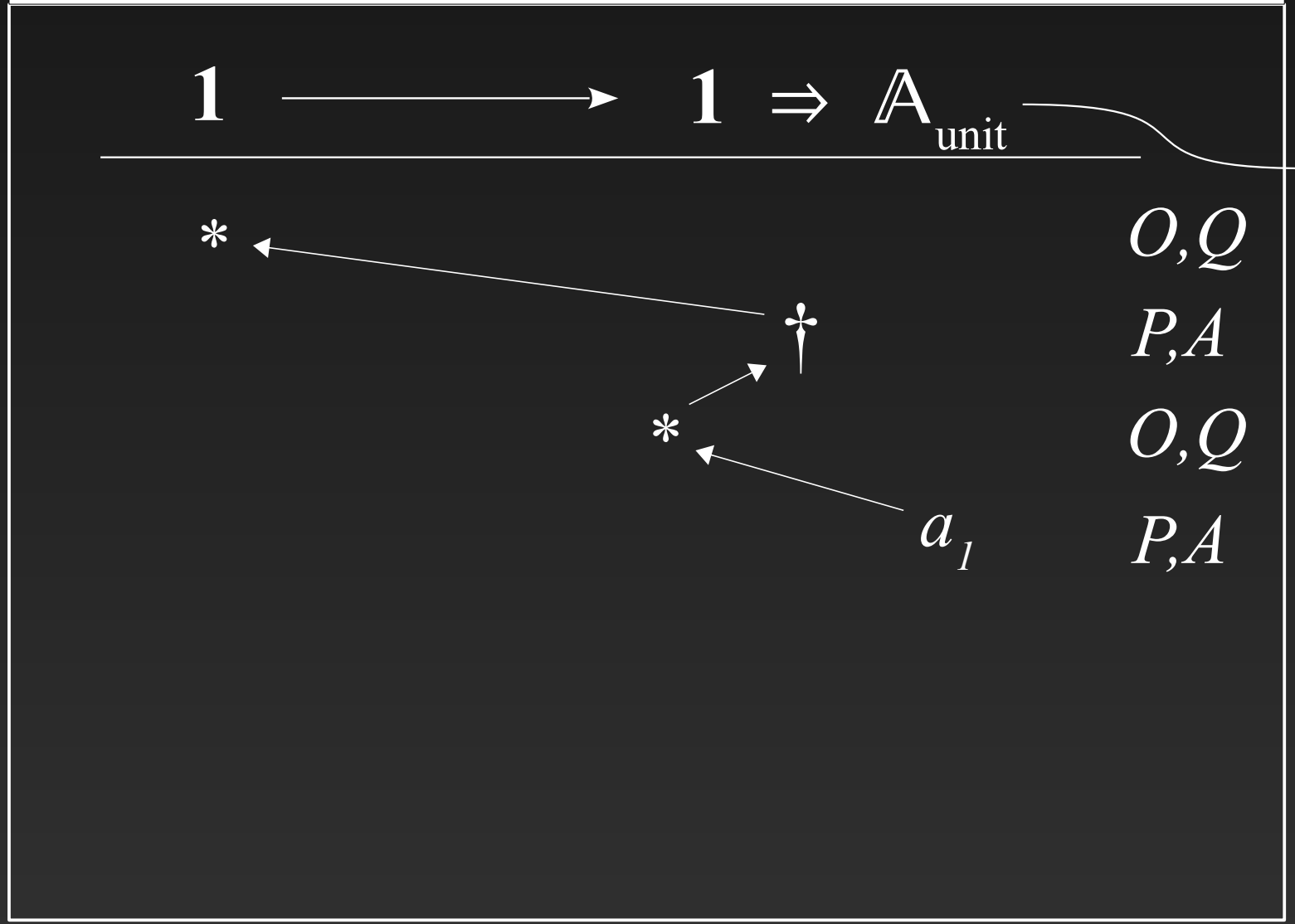

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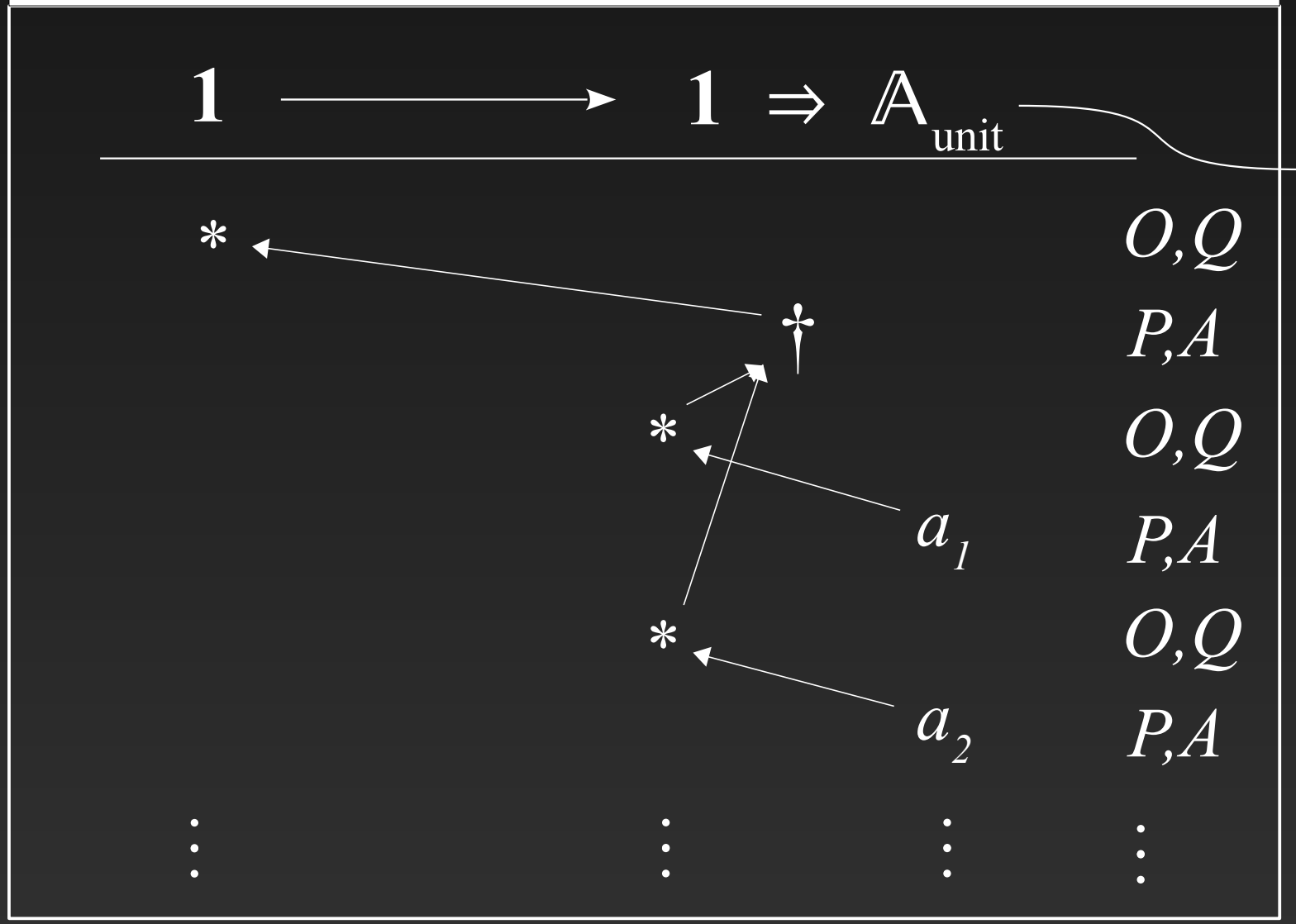
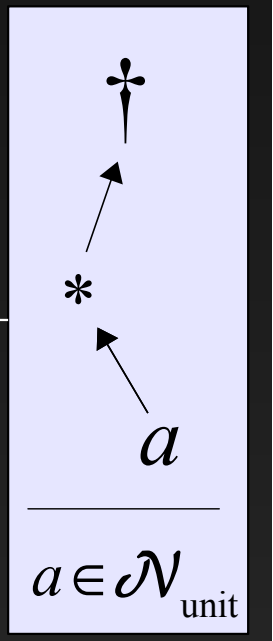
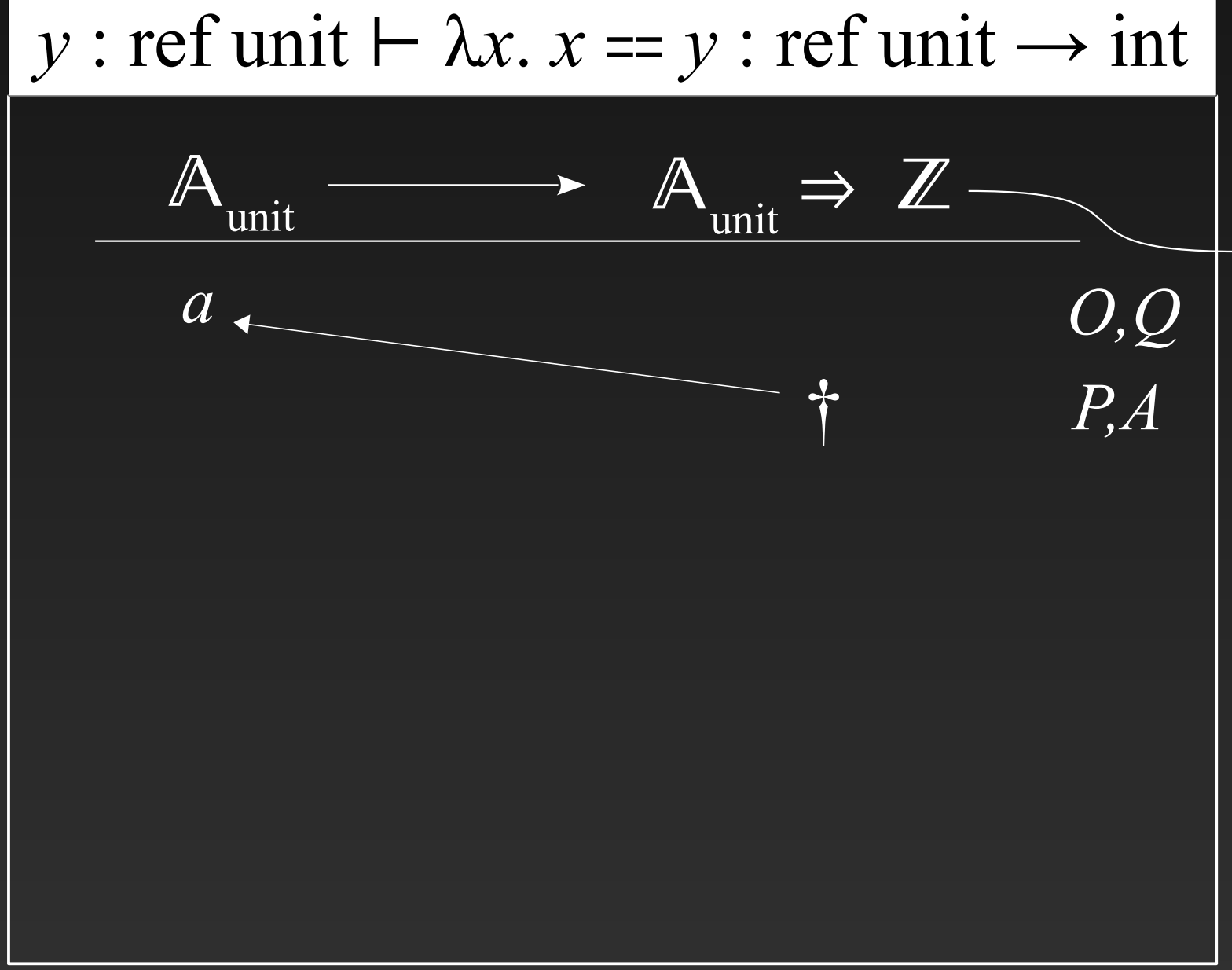




Nominal games (3)

$y : \mathrm{ref\ unit} \vdash \lambda x.\ x == y : \mathrm{ref\ unit} \longrightarrow \mathrm{int}$



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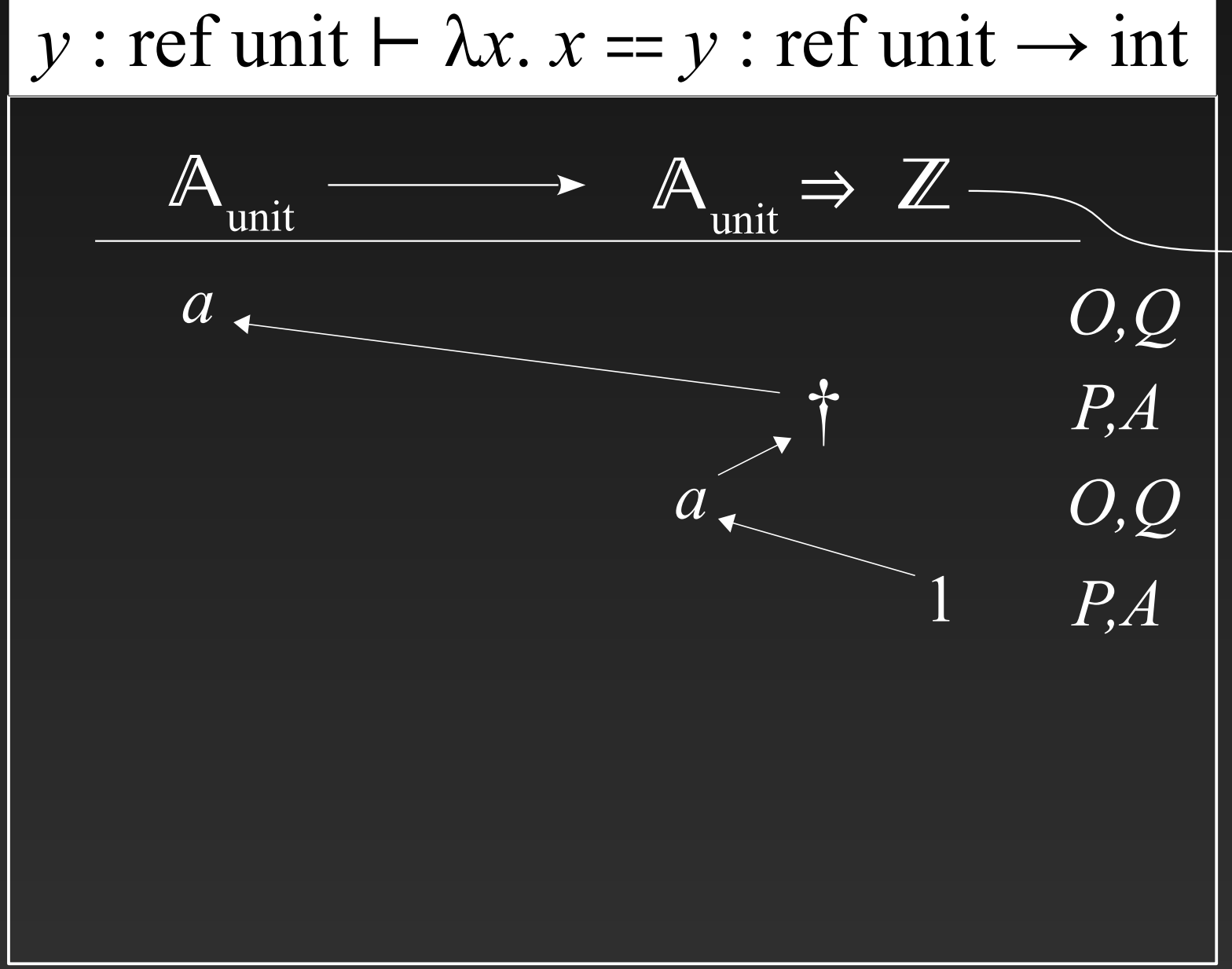

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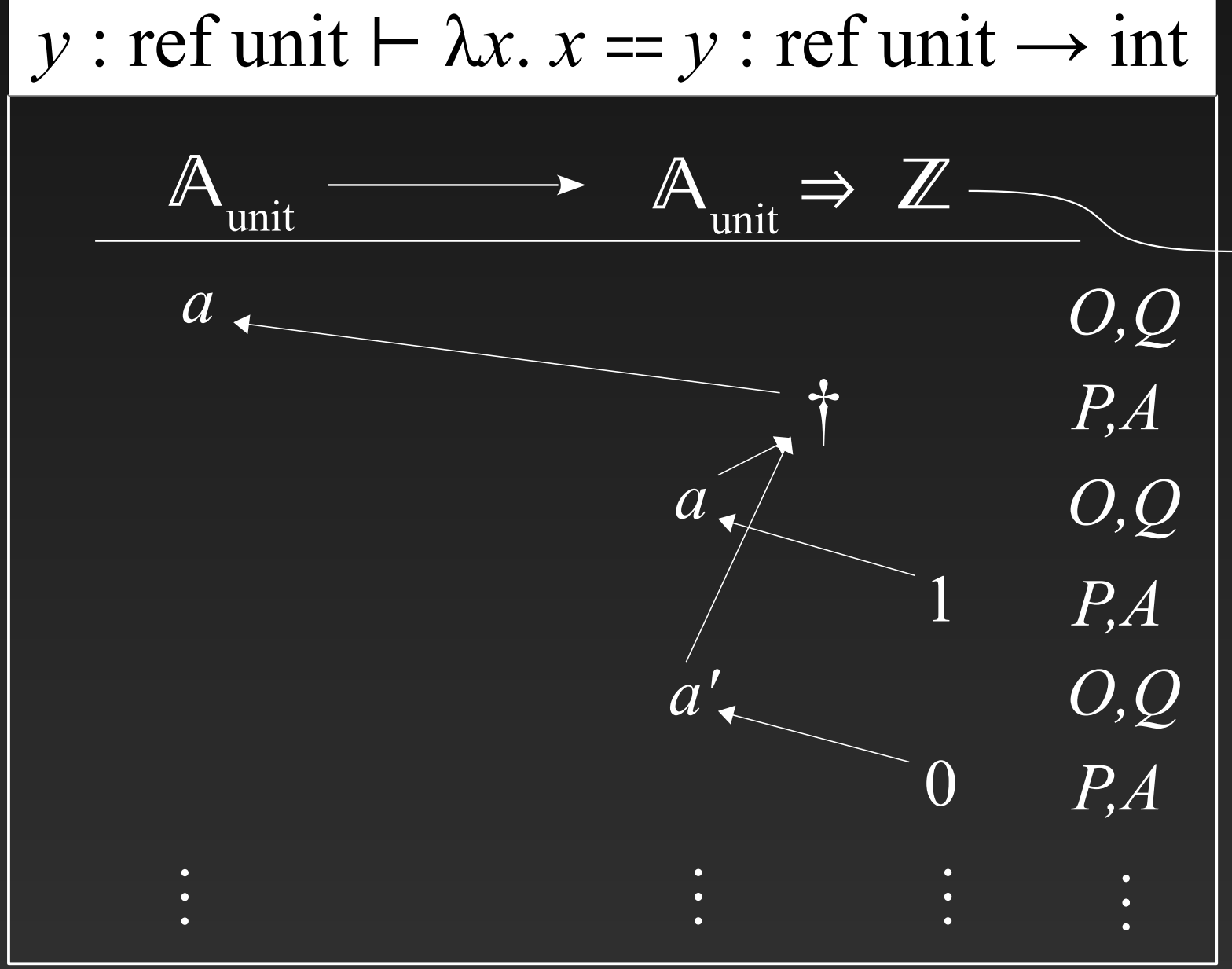
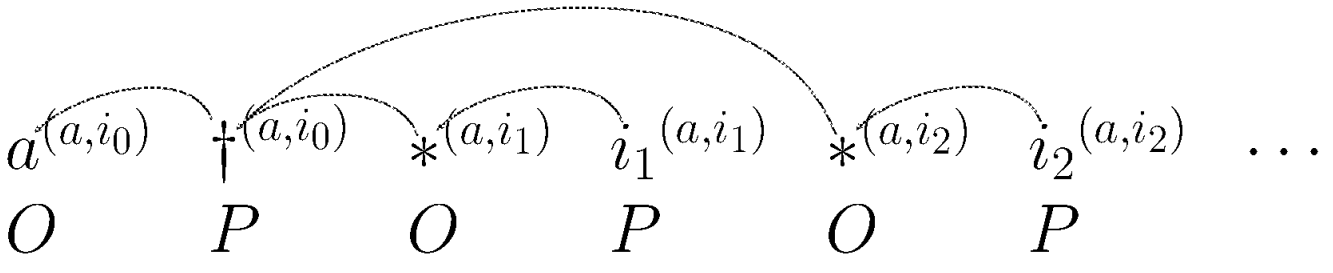
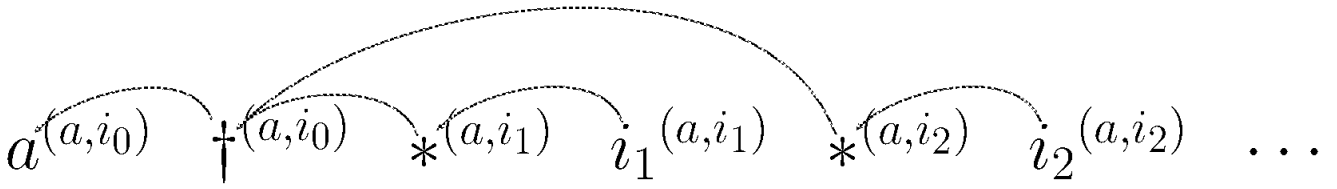


Nominal games: explicit store

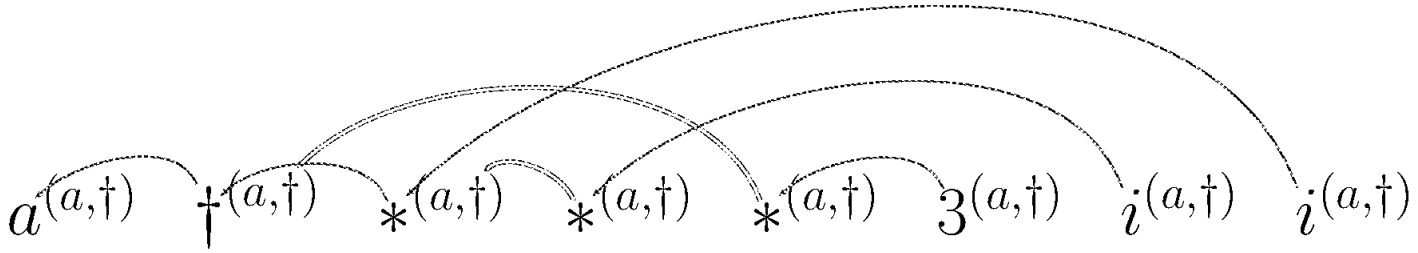


$$= \llbracket x : \mathsf{int\ ref} \vdash \lambda y.\, !x \ : \mathsf{unit} \to \mathsf{int} \rrbracket : \mathbb{A}_{\mathsf{int}} \to (1 \Rightarrow \mathbb{Z})$$

Nominal games: explicit store



$= \ [\![x : \mathsf{int\ ref} \vdash \lambda y.\, !x \ : \mathsf{unit} \to \mathsf{int}]\!] : \mathbb{A}_{\mathsf{int}} \to (1 \Rightarrow \mathbb{Z})$



$= \ [\![x : (\mathsf{unit} \to \mathsf{int})\ \mathsf{ref} \vdash x := \lambda y.3;\ \lambda y.(!x)()]\!] : \mathbb{A}_{\mathsf{unit}\to\mathsf{int}} \to (1 \Rightarrow \mathbb{Z})$

In LaTeX

Games live in a category of *nominal sets* [Gabbay&Pitts]

Definition An ***arena*** $A = (M_A, I_A, \vdash_A, \lambda_A)$ is given by:

- a strong nominal set M_A of *moves*,
- a nominal subset $I_A \subseteq M_A$ of *initial moves*,
- a nominal *labelling* function $\lambda_A : M_A \to \{O, P\} \times \{Q, A\}$,
- a nominal *justification* relation $\vdash_A \subseteq M_A \times (M_A \setminus I_A)$;

satisfying the conditions:

- $m \in I_A \implies \lambda_A(m) = (P, A)$
- $m \vdash_A n \implies \lambda_A^{O,P}(m) \neq \lambda_A^{O,P}(n) \wedge (\lambda_A^{Q,A}(m) = A \implies \lambda_A^{Q,A}(n) = Q)$

In LaTeX

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Given arenas A, B, the ***interface*** $A \to B = (M_{A\to B}, I_{A\to B}, \vdash_{A\to B}, \lambda_{A\to B})$ is given by:

- $M_{A\to B} = M_A \uplus M_B$ and $I_{A\to B} = I_A$
- $\lambda_{A\to B} = [\lambda_A, \lambda_B]$
- $\vdash_{A\to B} = (I_A \times I_B) \cup \vdash_A \cup \vdash_B$

In LaTeX

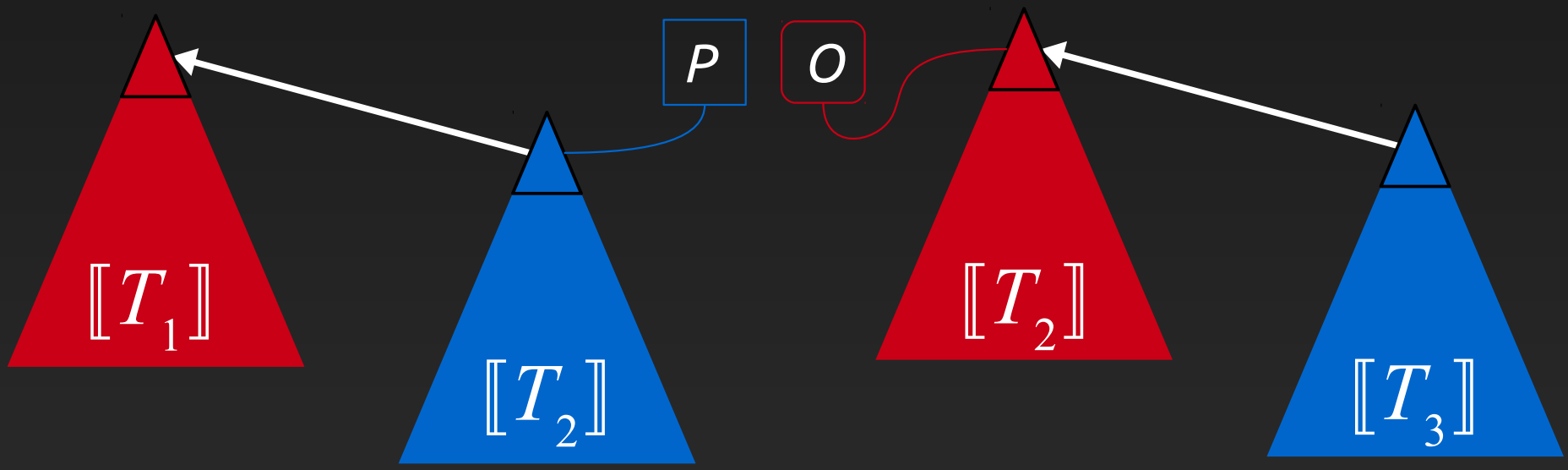
Definition A ***play*** in $A \to B$ is a sequence of *moves-with-store*, written m^{Σ}, where $m \in M_{A \to B}$ and Σ a finite *store*, satisfying the conditions:

- O/P alternation,
- justification,
- well-bracketing,
- frugality (stores contain only visible/reachable names),
- ...

Definition Given an interface $A \to B$, a ***strategy*** $\sigma : A \to B$ is a nominal set of even-length plays satisfying the conditions: *[. . .]*

Composition

$$[\![f_1 : T_1 \vdash M_1 : T_2]\!] \qquad [\![f_2 : T_2 \vdash M_2 : T_3]\!]$$



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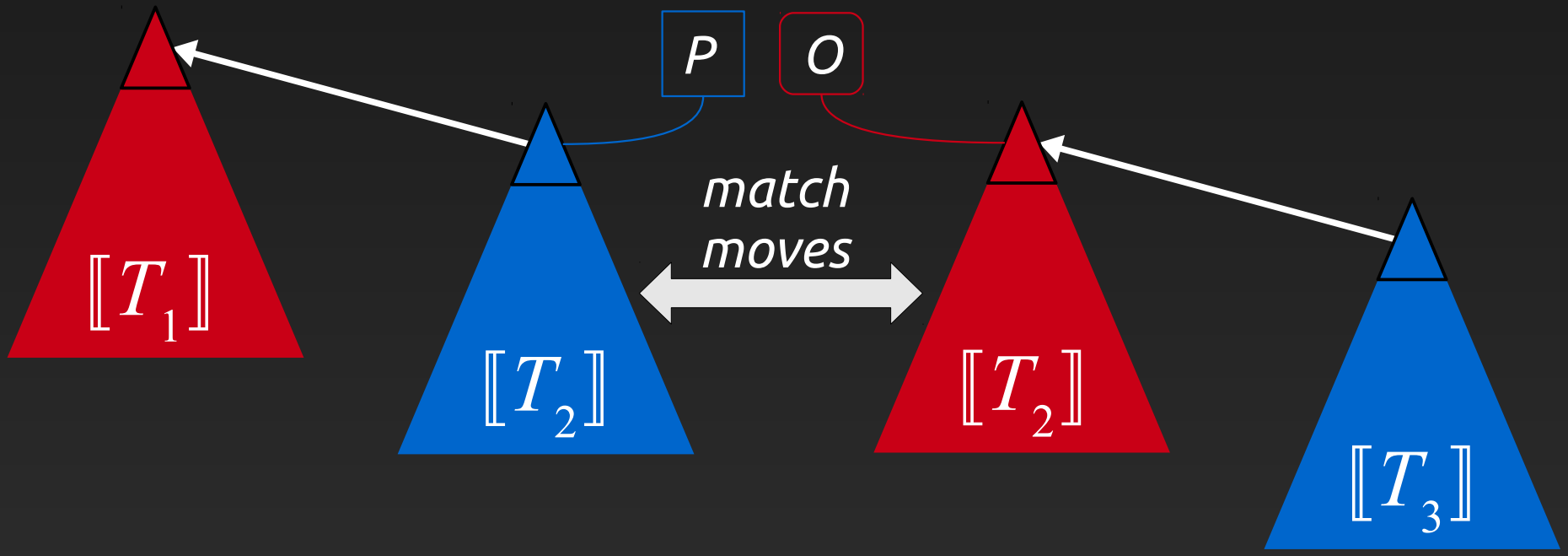

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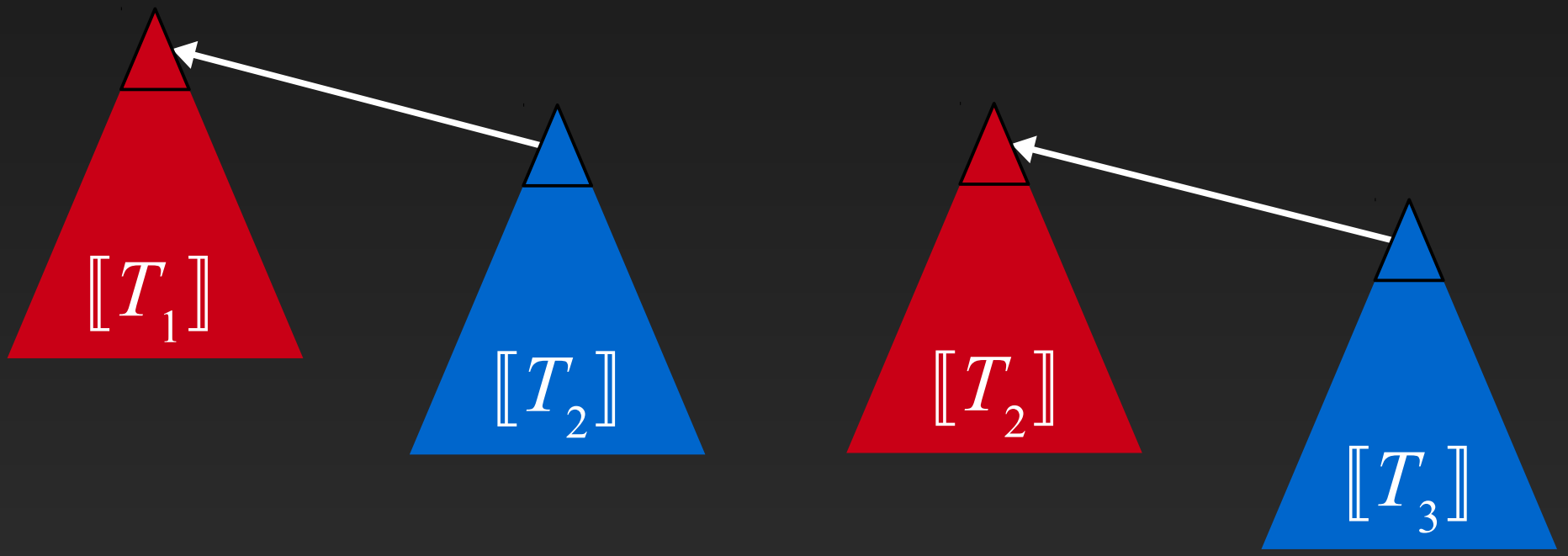

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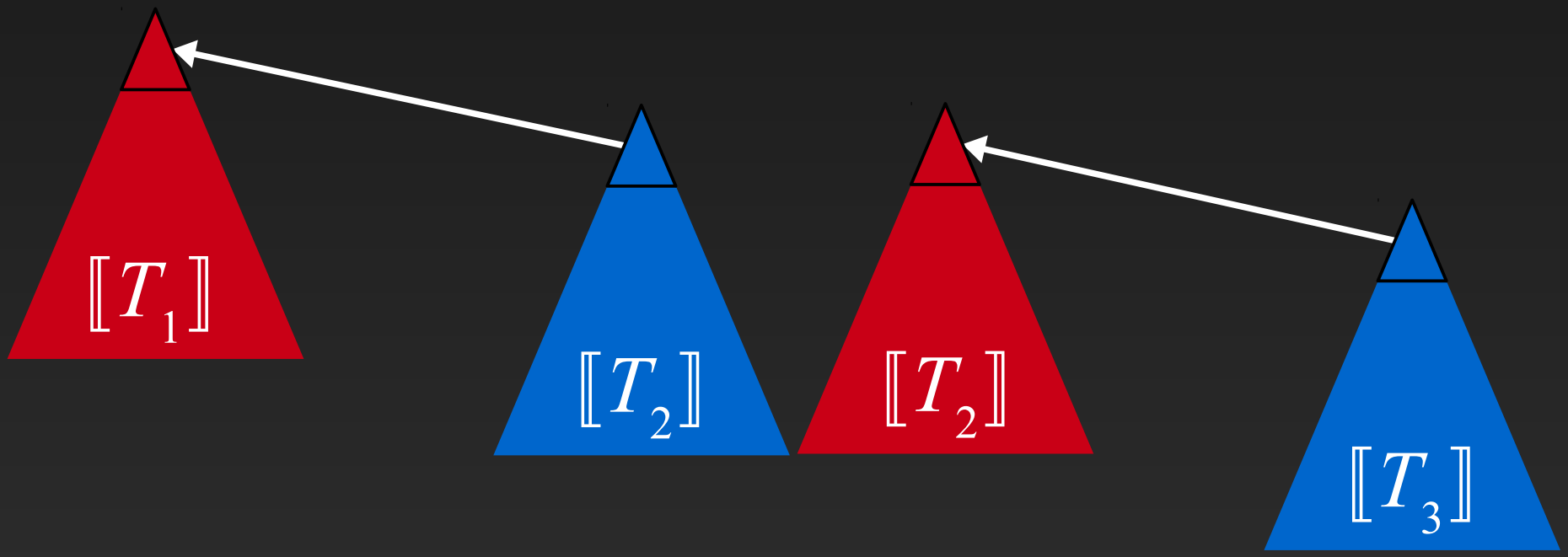

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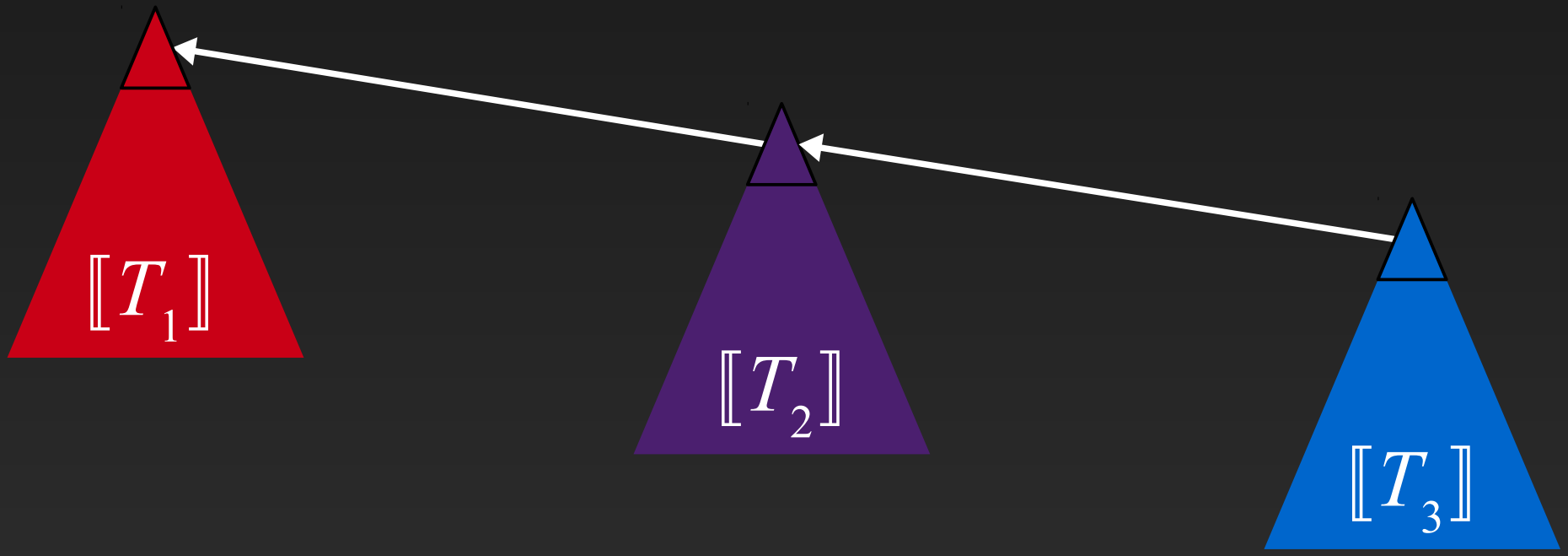

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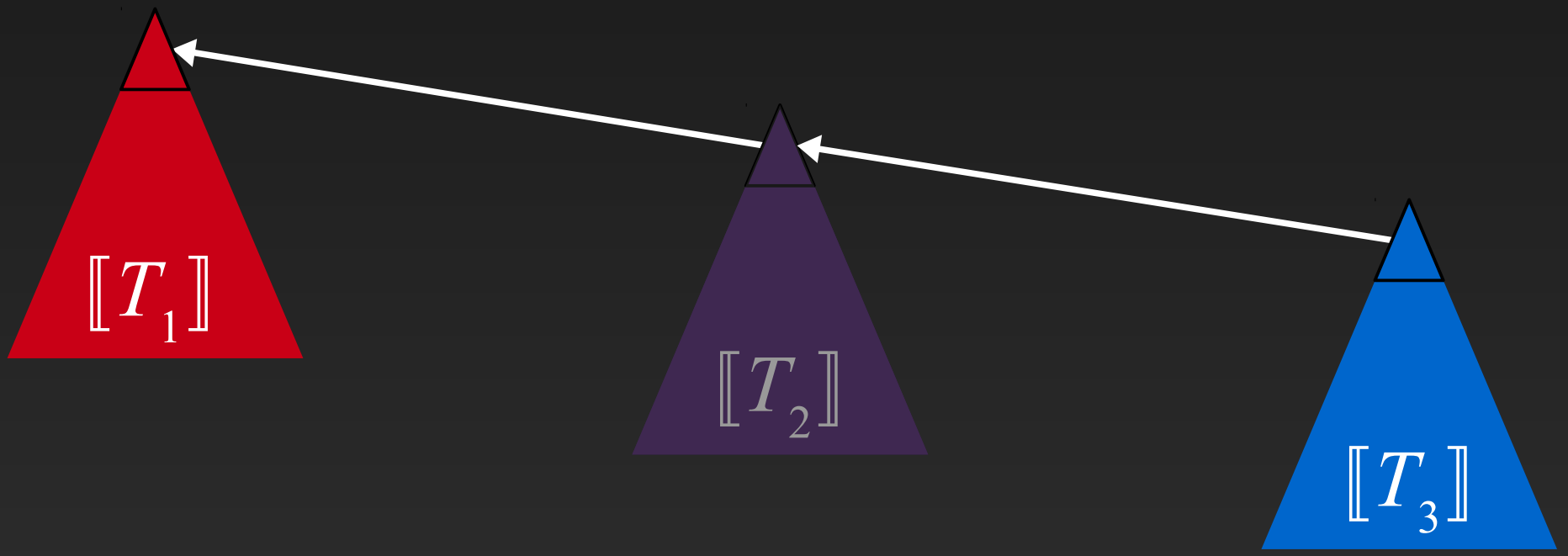

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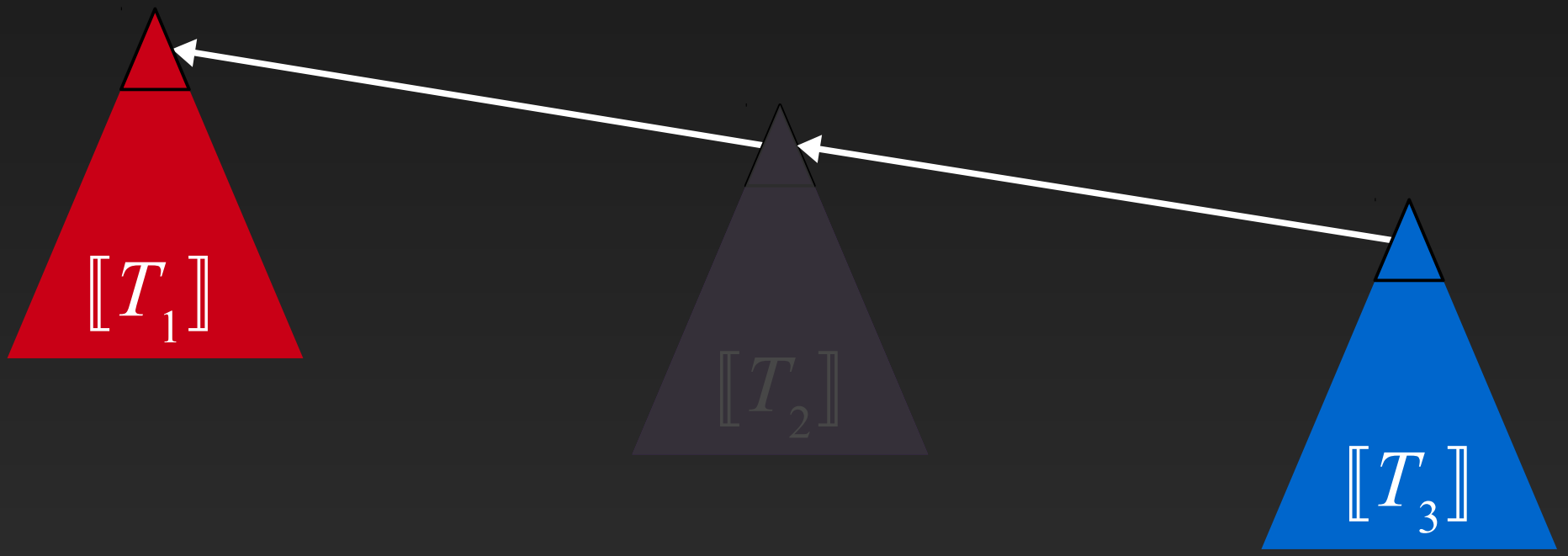
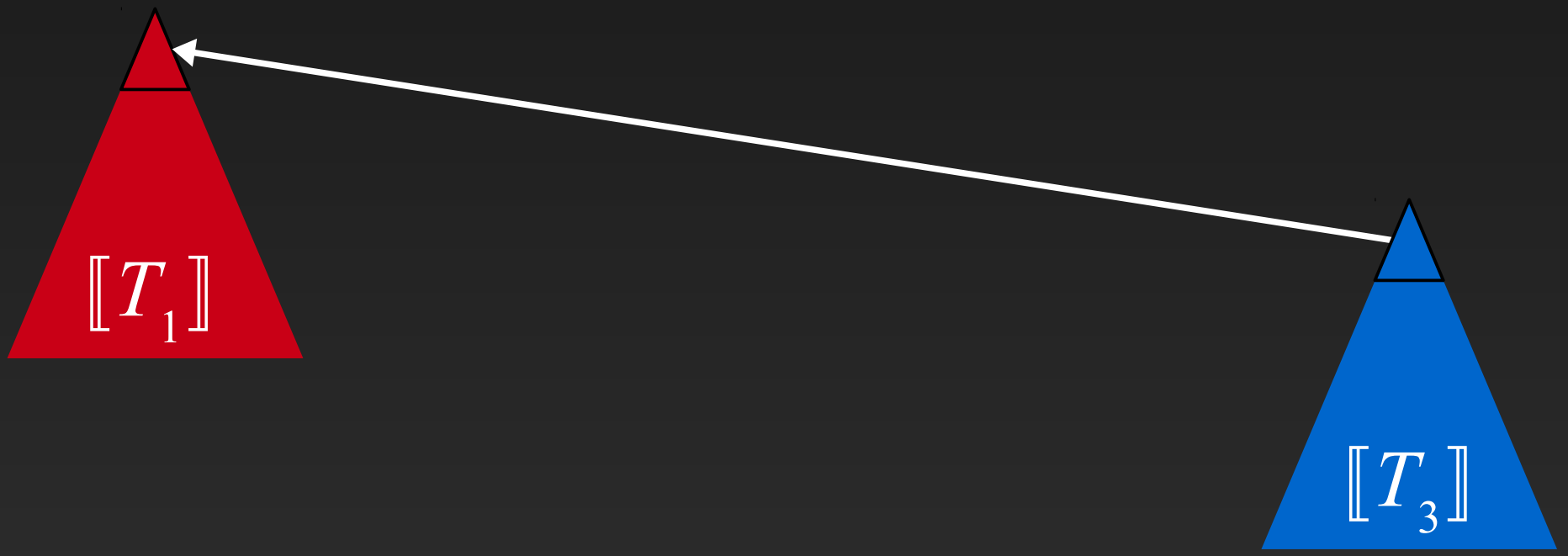


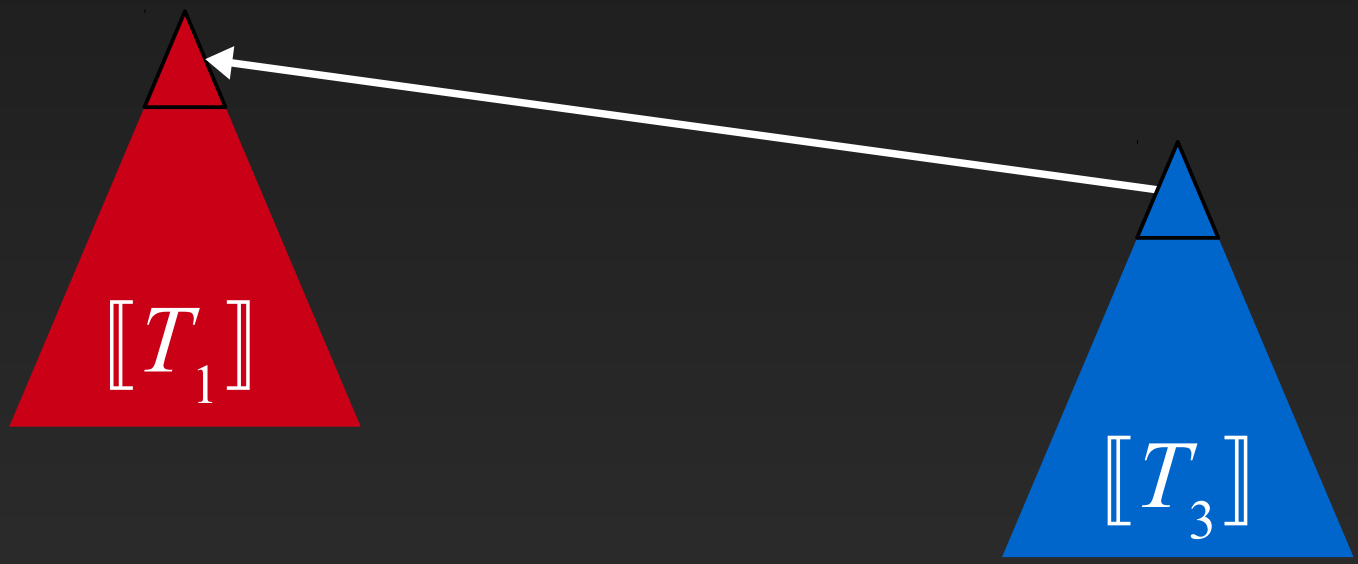
Composition

$$[\![f_1 : T_1 \vdash M_1 : T_2]\!] \qquad [\![f_2 : T_2 \vdash M_2 : T_3]\!]$$



Composition

$$[\![f_1{:}T_1 \vdash M_1{:}T_2]\!] \;;\; [\![f_2{:}T_2 \vdash M_2{:}T_3]\!]$$
$$= [\![f_1{:}T_1 \vdash \mathrm{let}\, f_2 = M_1 \,\mathrm{in}\, M_2{:}T_3]\!]$$



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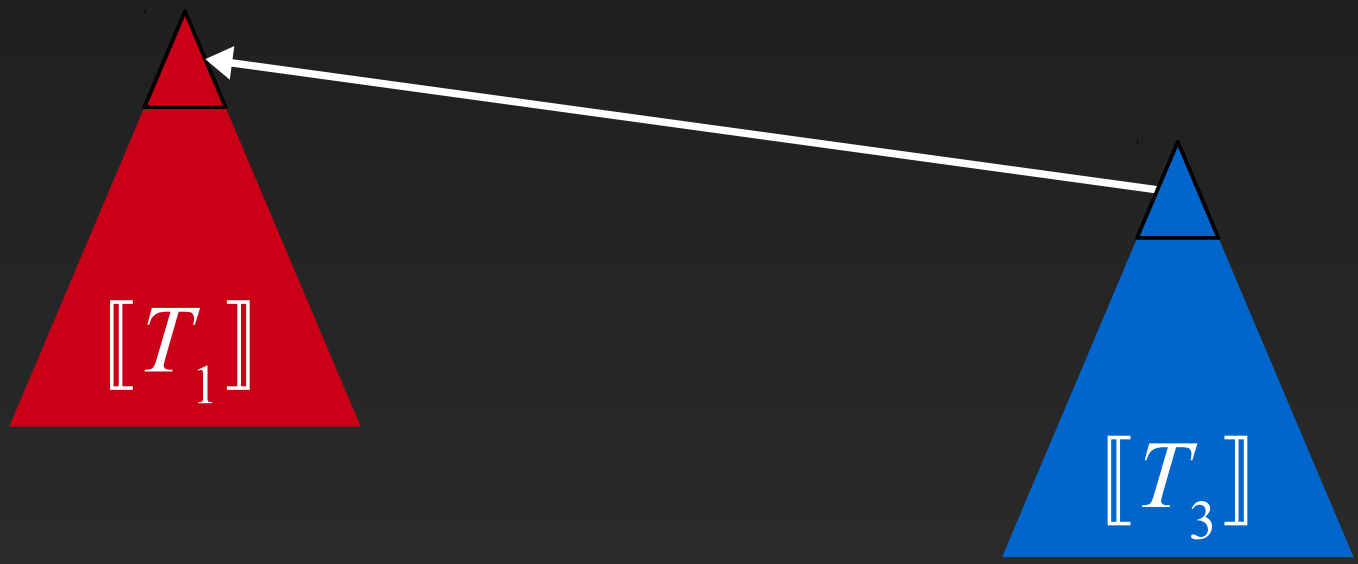

$$A \xrightarrow{\sigma} B \xrightarrow{\tau} C \quad = \quad A \xrightarrow{\sigma;\tau} C$$

Composition

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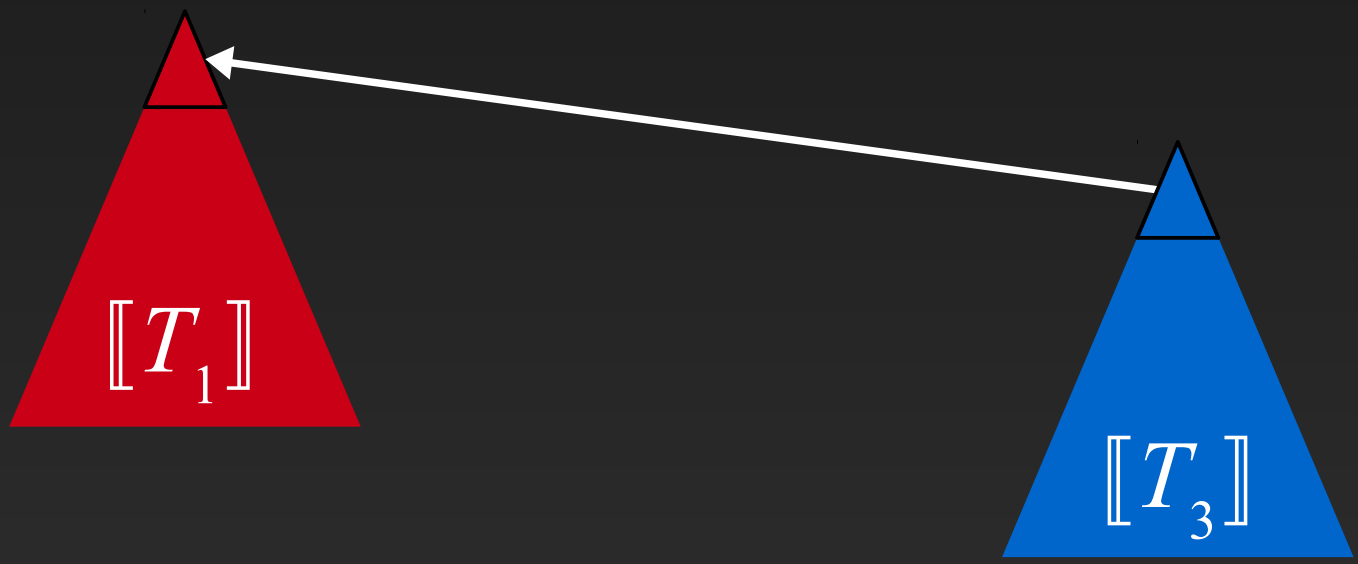
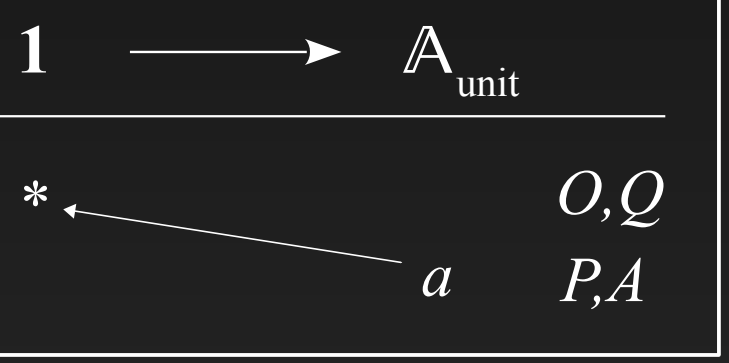


Nominal composition requires *Laird conditions* :
these ensure name and store privacy during composition

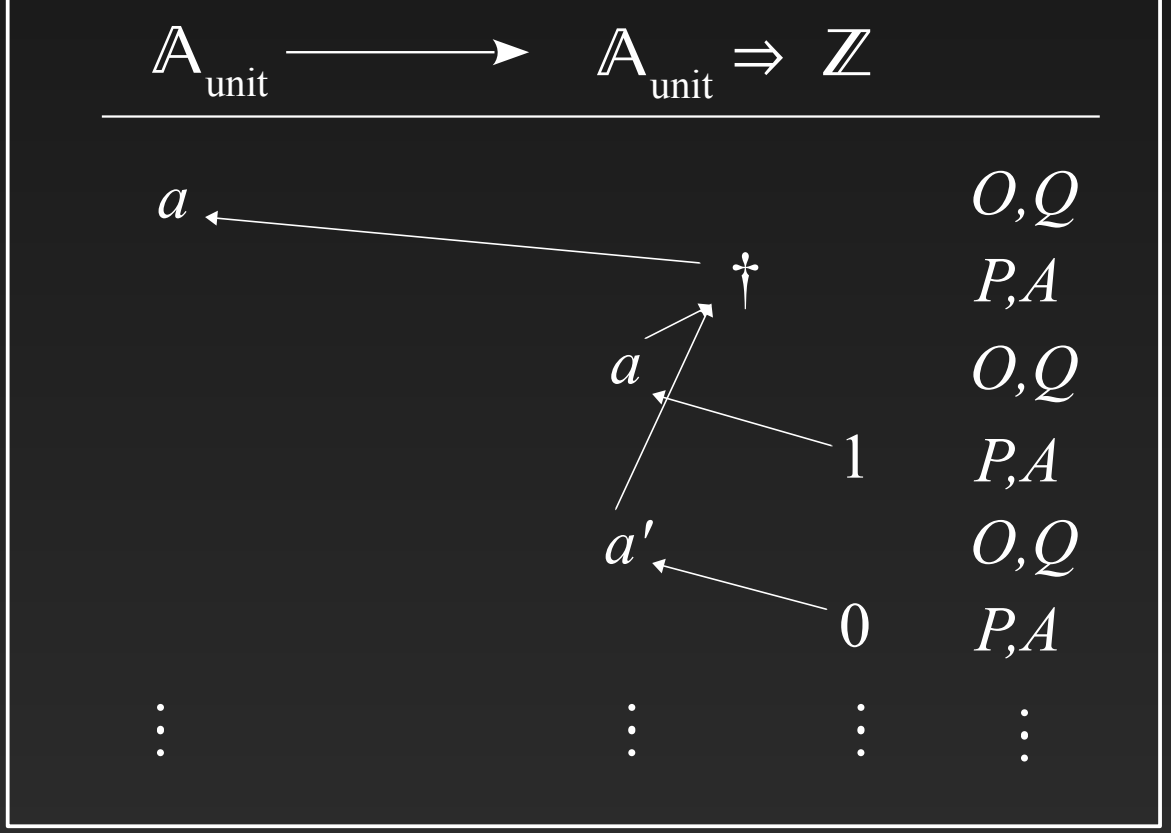
Nominal composition example

$\vdash \mathrm{ref}() : \mathrm{ref\ unit}$



;

$y : \mathrm{ref\ unit} \vdash \lambda x.\ x == y : \mathrm{ref\ unit} \to \mathrm{int}$



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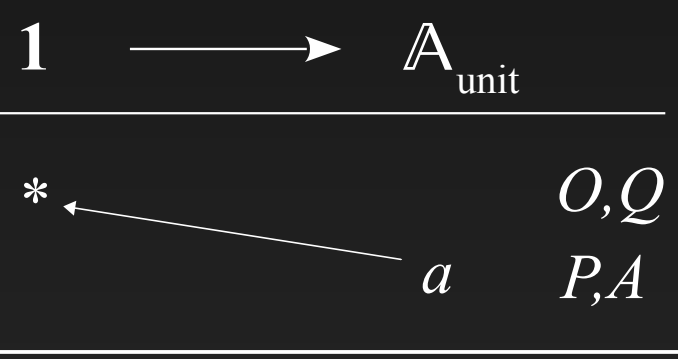

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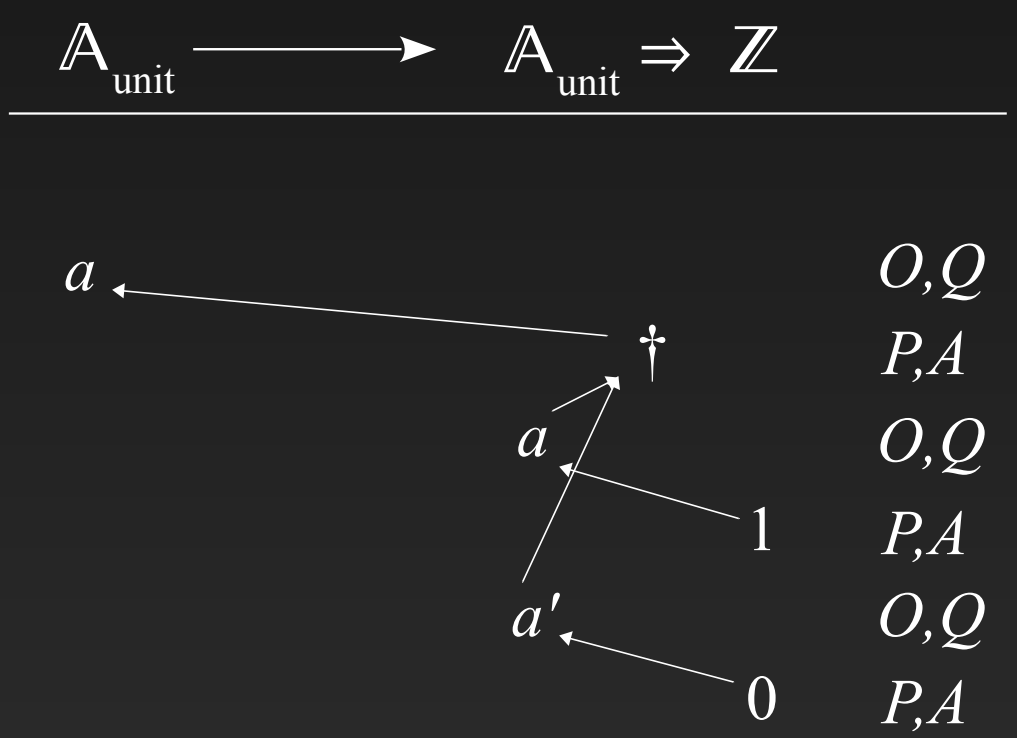

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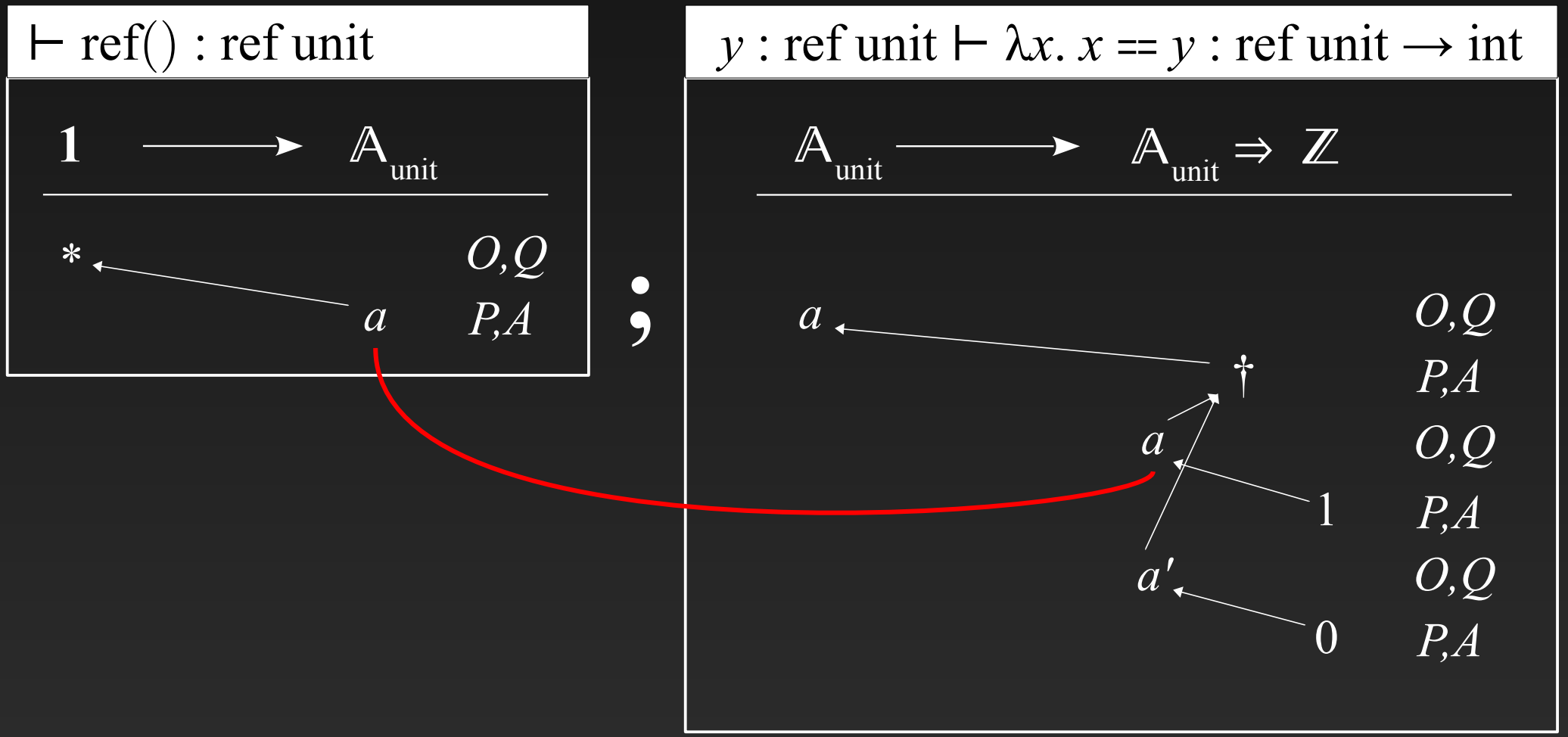

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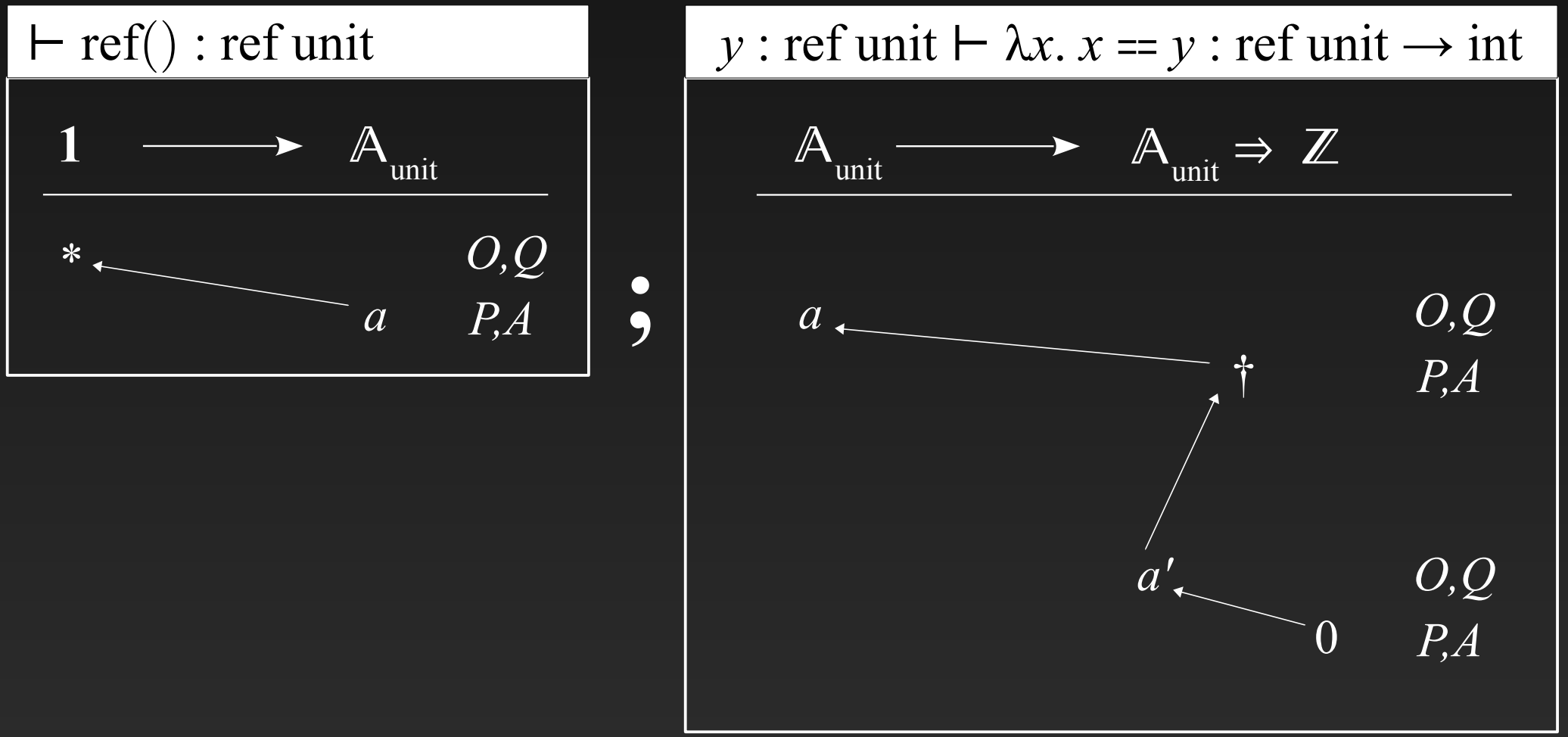


Nominal composition example

$\vdash \mathrm{ref}() : \mathrm{ref\ unit}$

$\mathbb{1} \longrightarrow \mathbb{A}_{\mathrm{unit}}$

$*$ ← a $\quad O,Q\ P,A$

;

$y : \mathrm{ref\ unit} \vdash \lambda x.\ x == y : \mathrm{ref\ unit} \to \mathrm{int}$

$\mathbb{A}_{\mathrm{unit}} \longrightarrow \mathbb{A}_{\mathrm{unit}} \Rightarrow \mathbb{Z}$

a ← $\dagger$ $\quad O,Q\ P,A$

a' ← 0 $\quad O,Q\ P,A$

$= [\![\ \lambda x.0 : \mathrm{ref\ unit} \to \mathrm{int}\ ]\!]$

Taking stock

- Freyd categories for several different effects
- Soundness – Definability – Full Abstraction

Effects = – conditions + name generators

- Ground store:

  – innocence + ground reference names

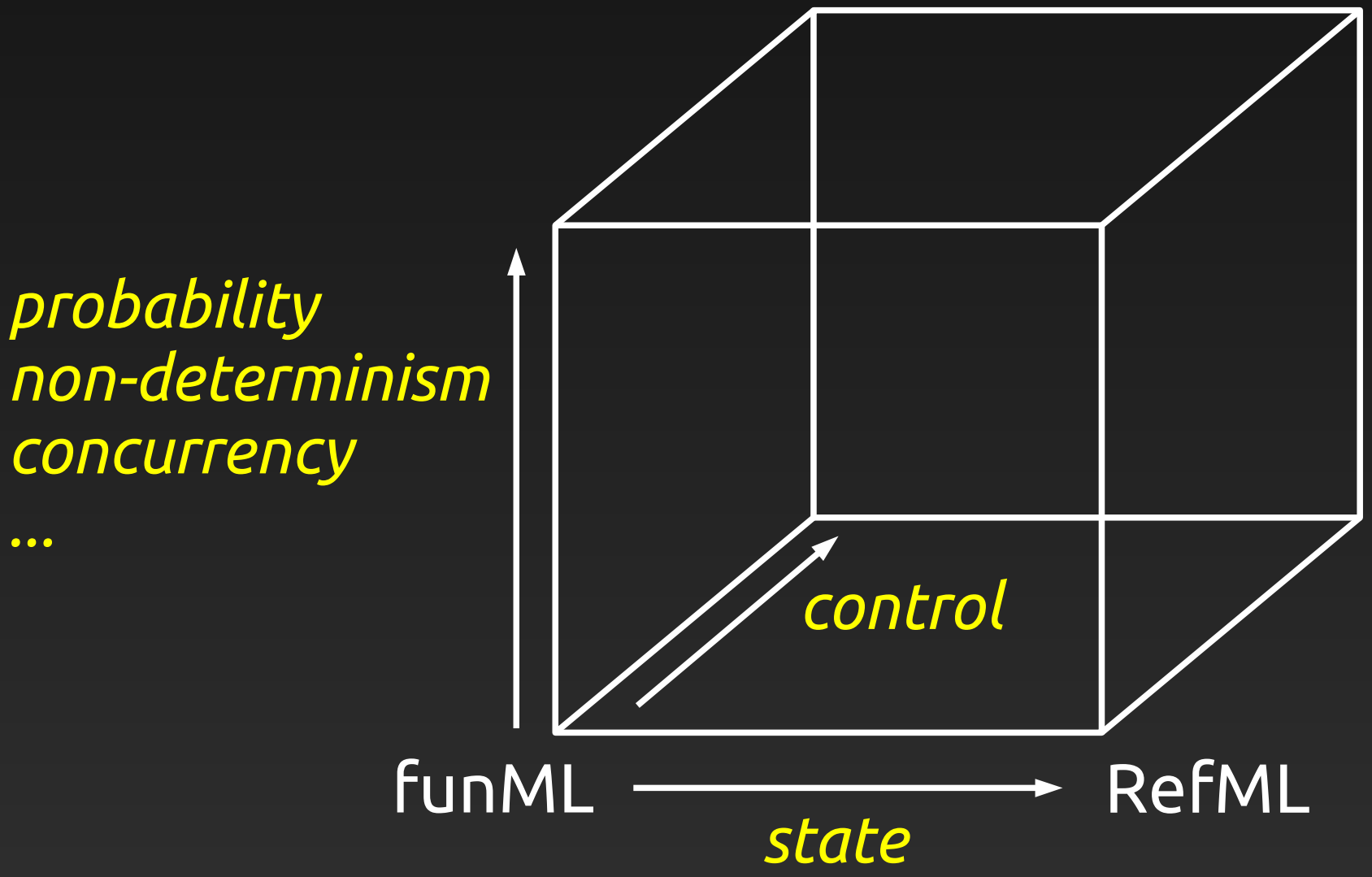
- Higher-order store:

  – visibility + HO reference names

- Exceptions:

  – well-bracketing + exception names

Nominal Abramsky cube



Nominal games so far

- Full abstraction for languages with:
  - References (ground & higher-order)
  - Concurrency (higher-order channels)
  - Exceptions (local, private)
  - Objects (Java)

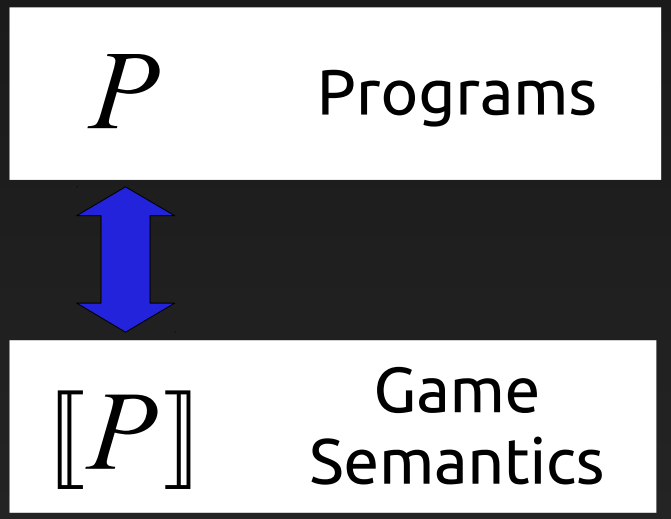
`dblp:` AGMOS, Laird, Murawski, T.

- Trace models

`dblp:` Laird, Ghica, Jaber, T.

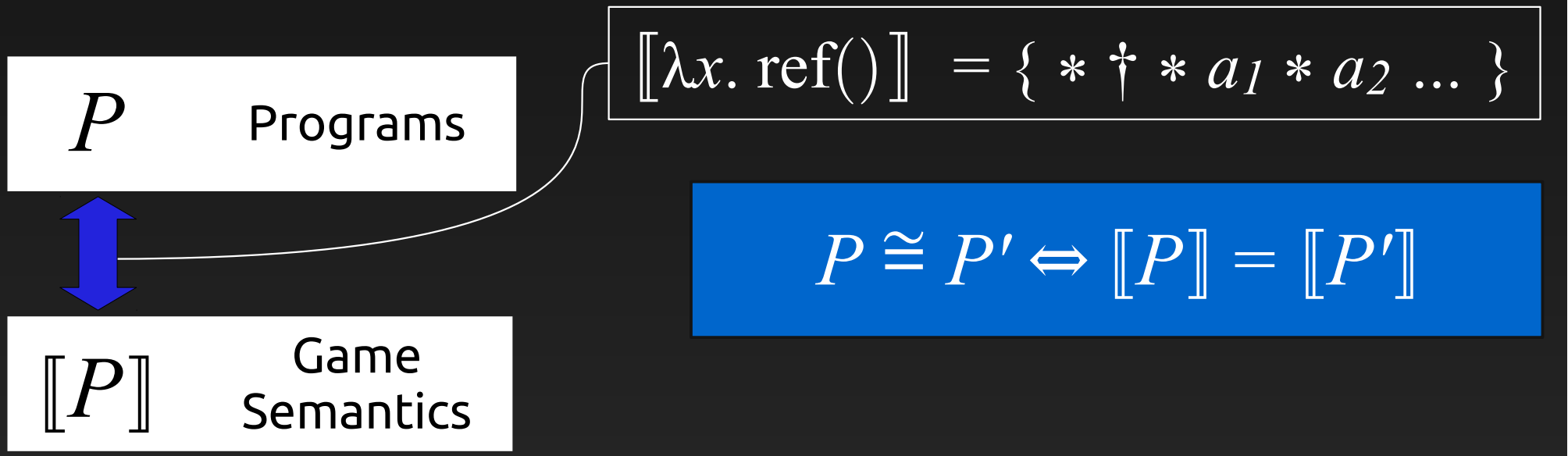
- Algorithmic games

Algorithmic game semantics

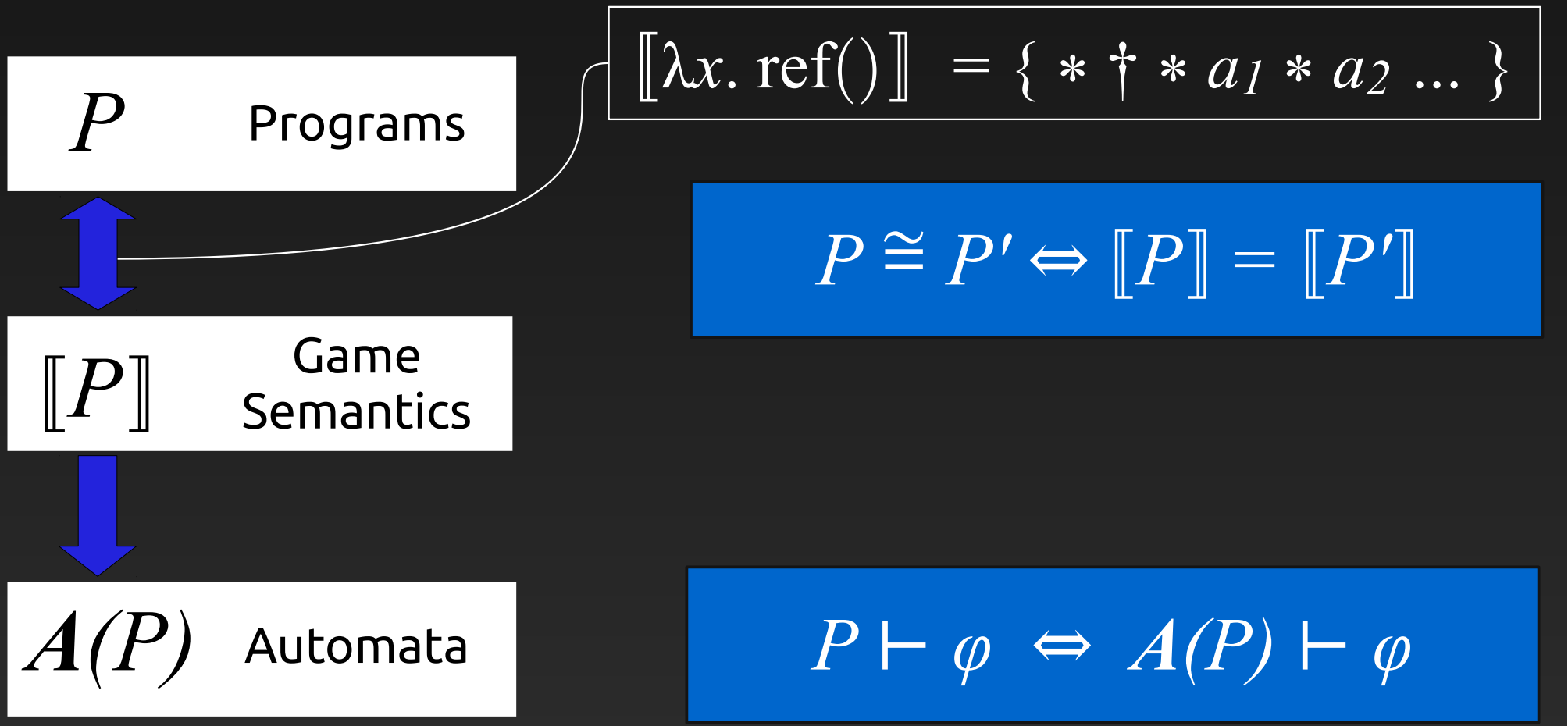


$$P \cong P' \Leftrightarrow [\![P]\!] = [\![P']\!]$$

Algorithmic game semantics



Algorithmic game semantics




Nominal Automata

The need for nominal automata

- Nice (fully abstract) game models
- Represented as sets of strings (with structure)

- *How to make them algorithmic/automated?*
- Use automata

- *But, the alphabet is infinite...*
- Use nominal automata

Fresh-register automata

$$[\![\lambda x.\ \mathrm{ref}()]\!] = \{ * \dagger * a_1 * a_2 \ldots \mid a_i\text{'s distinct} \}$$

Automata with names

- Infinite alphabet $\mathcal{N}$
- Freshness recognition

Fresh-register automata

$$[\![\lambda x.\ \mathrm{ref}()]\!] = \{ * \dagger * a_1 * a_2 \ldots \mid a_i\text{'s distinct} \}$$

Automata with names

- Infinite alphabet $\mathcal{N}$
- Freshness recognition

Part of a huge body of work on automata over infinite alphabets!

Finite-state machines with registers

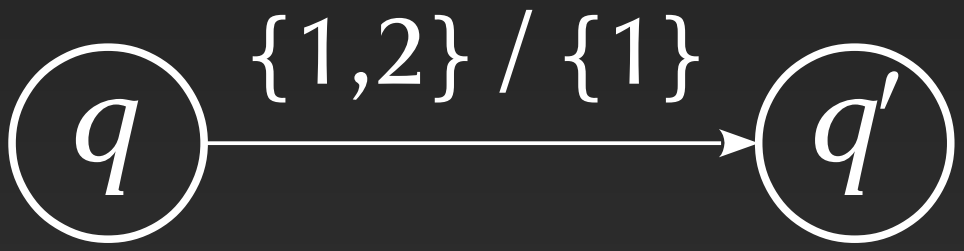
- *Register Automata (Kaminski & Francez, 1994)*

Register automata

X,Y: *sets of register indices*

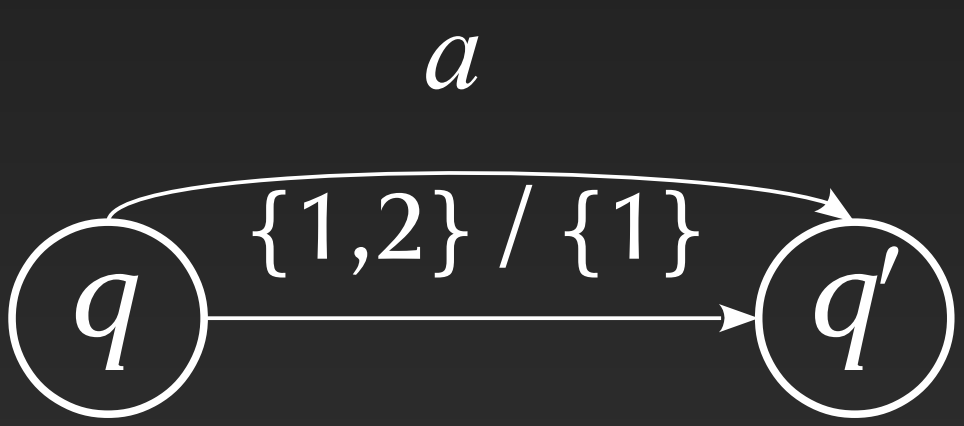
Register automata

X,Y: *sets of register indices*



Register automata

X,Y: *sets of register indices*



Fresh-register automata: add “fresh”

Y: *set of register indices*

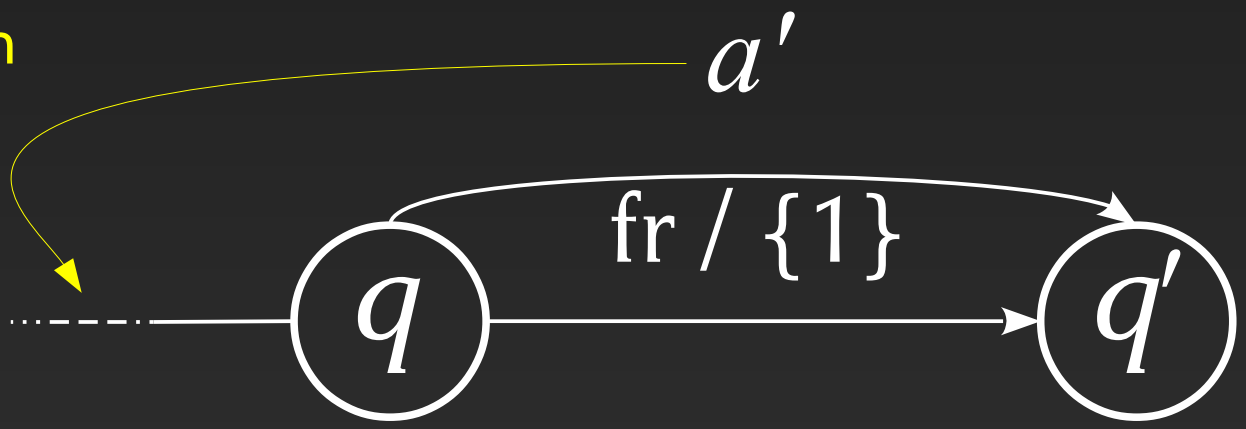
Fresh-register automata: add "fresh"

Y: *set of register indices*

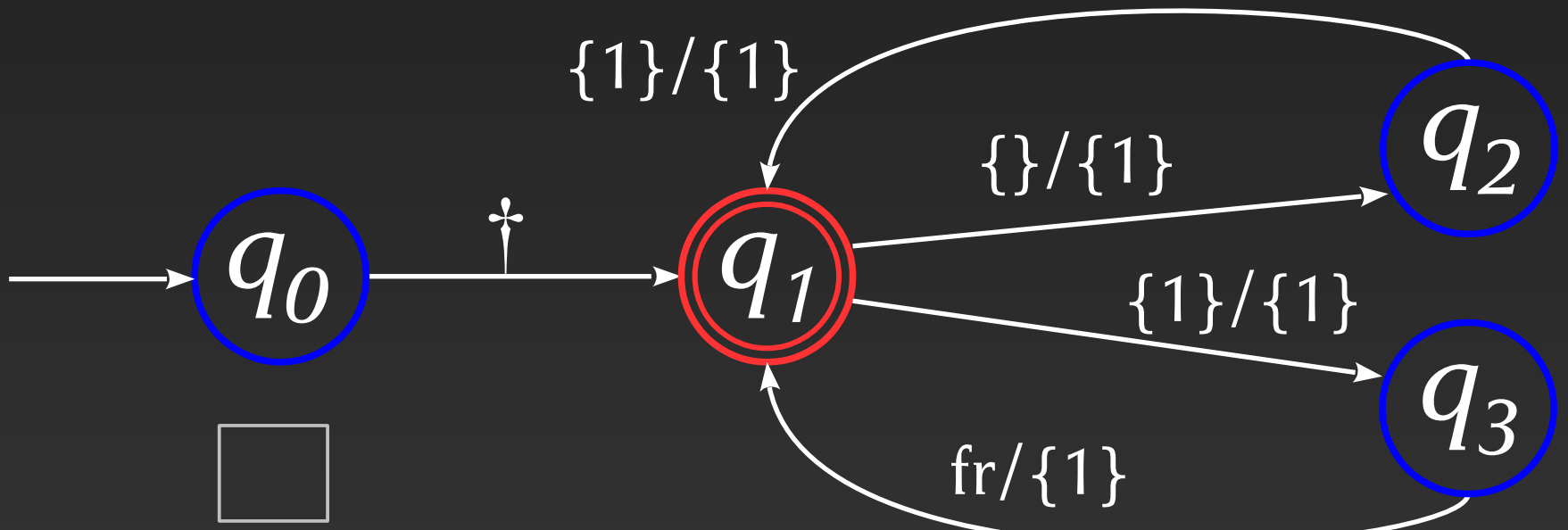
Fresh-register automata: add "fresh"

Y: *set of register indices*

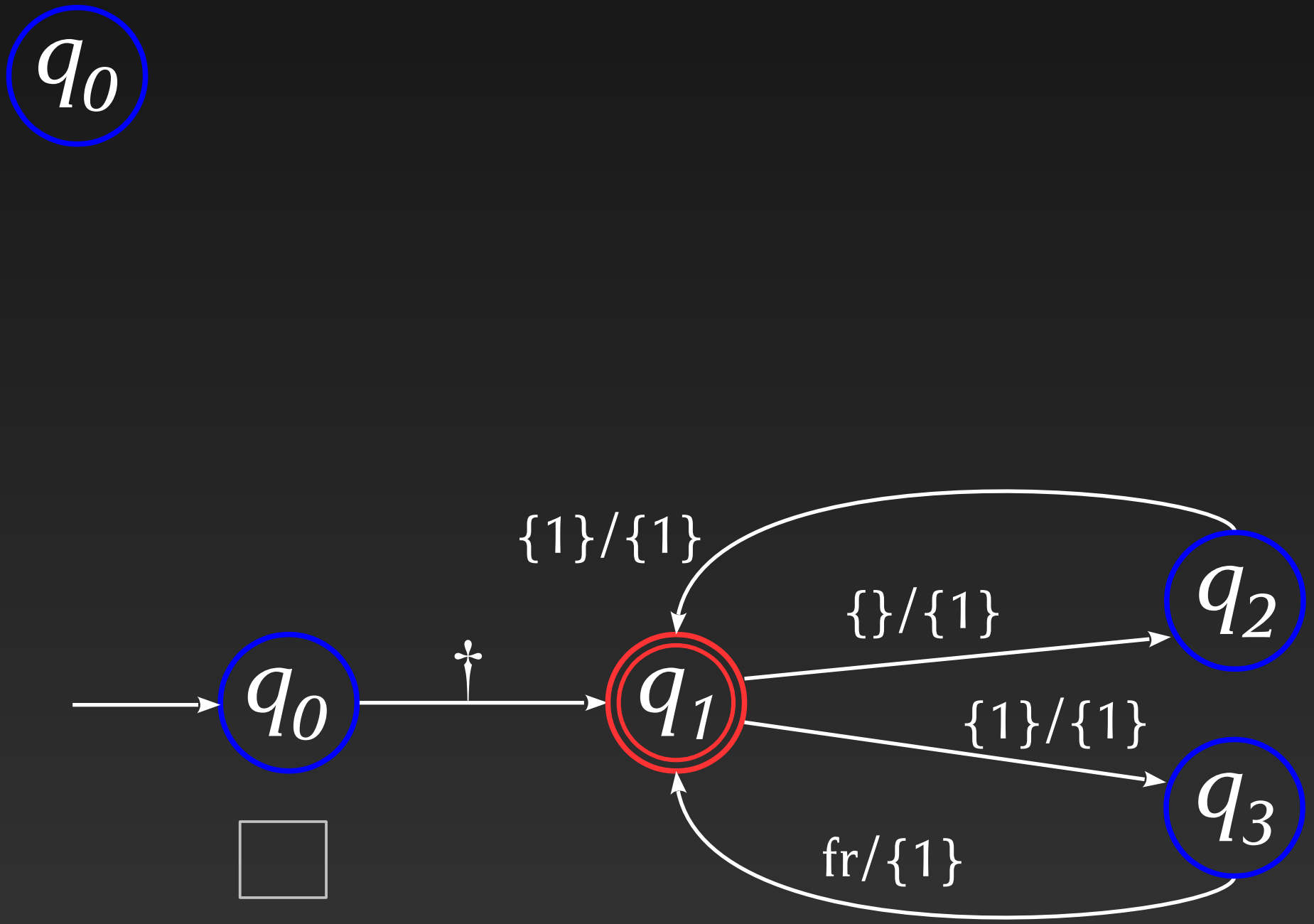
fresh here



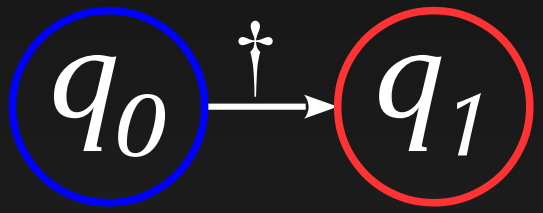
Fresh-register automata

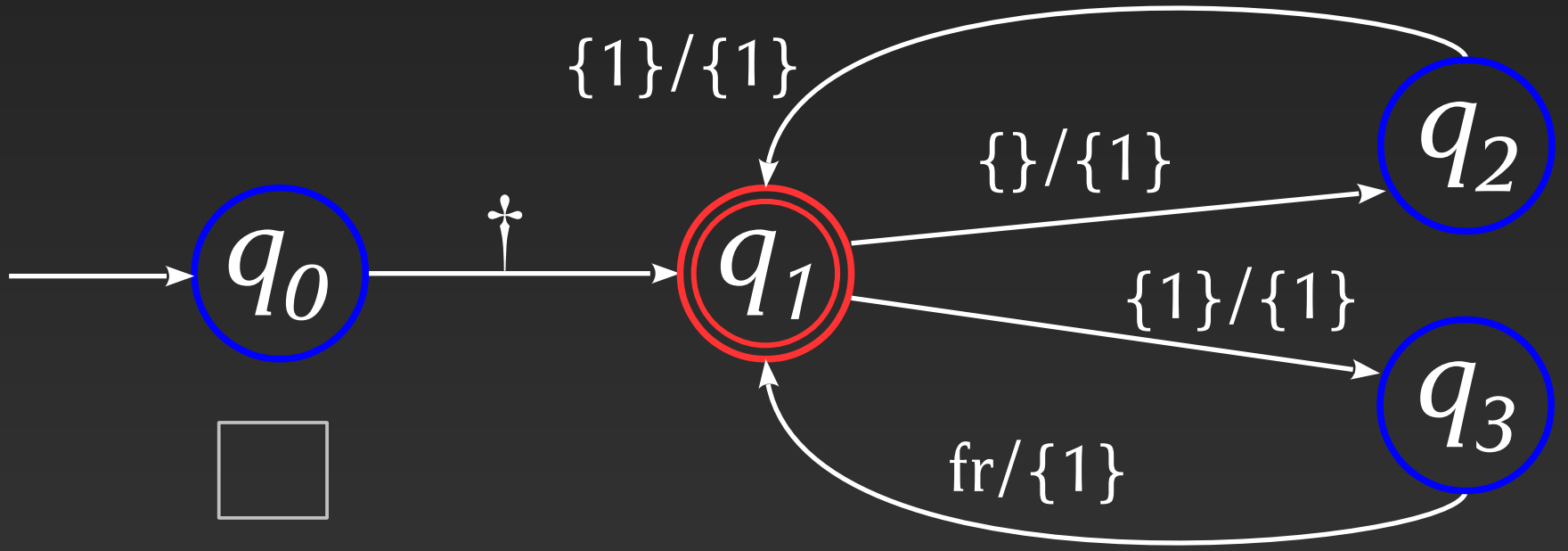


Fresh-register automata

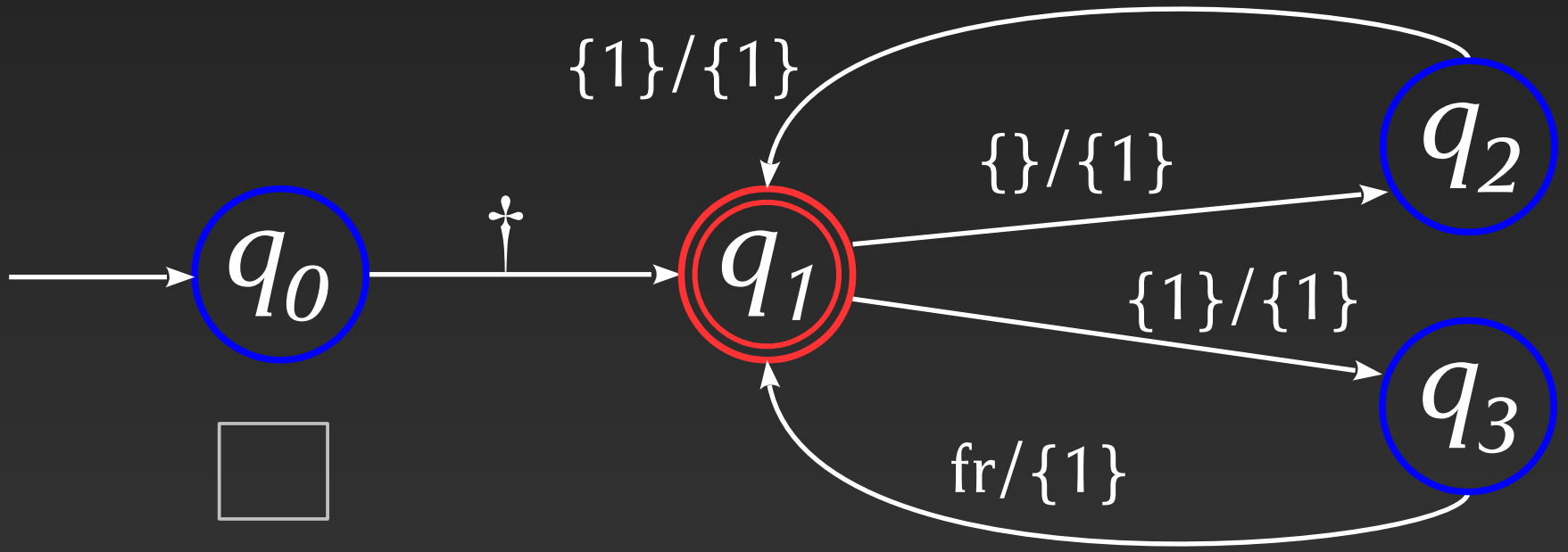


Fresh-register automata

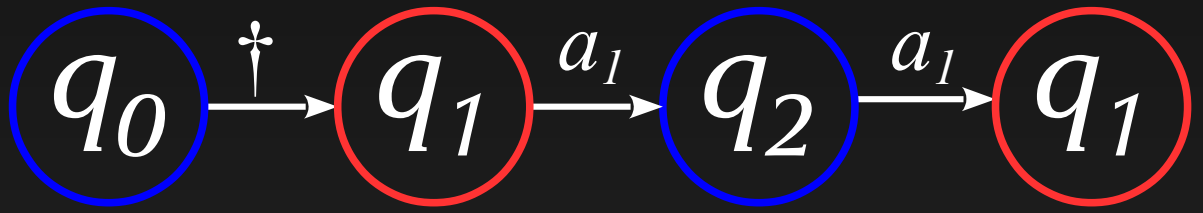


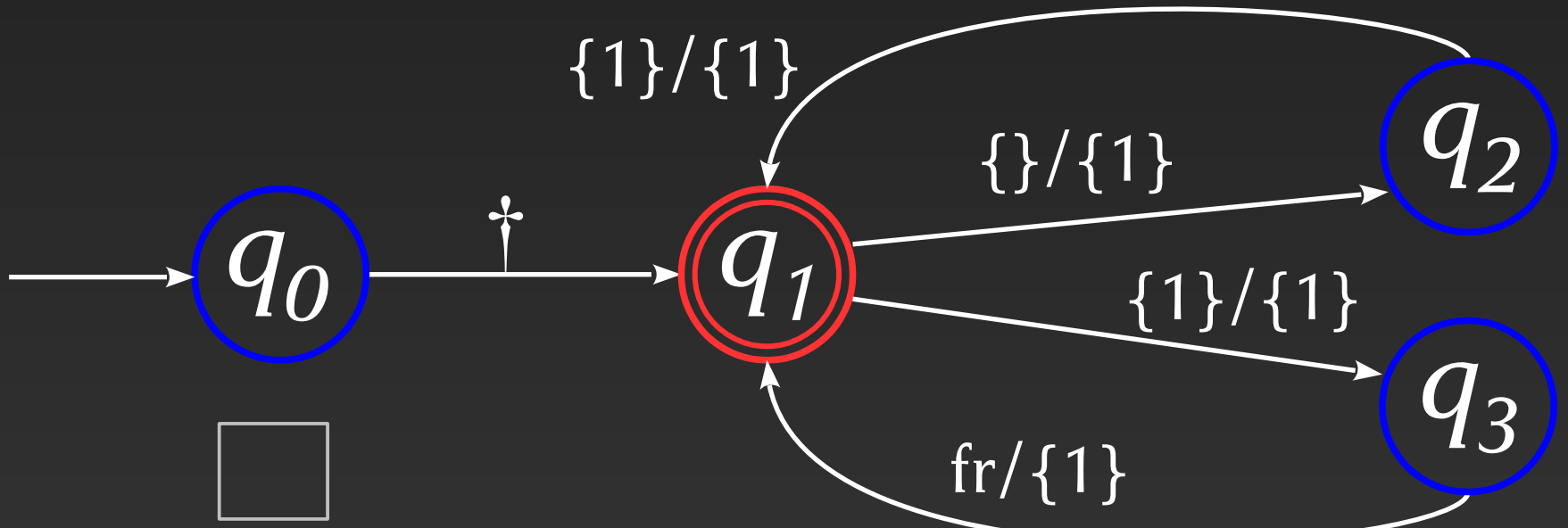


Fresh-register automata

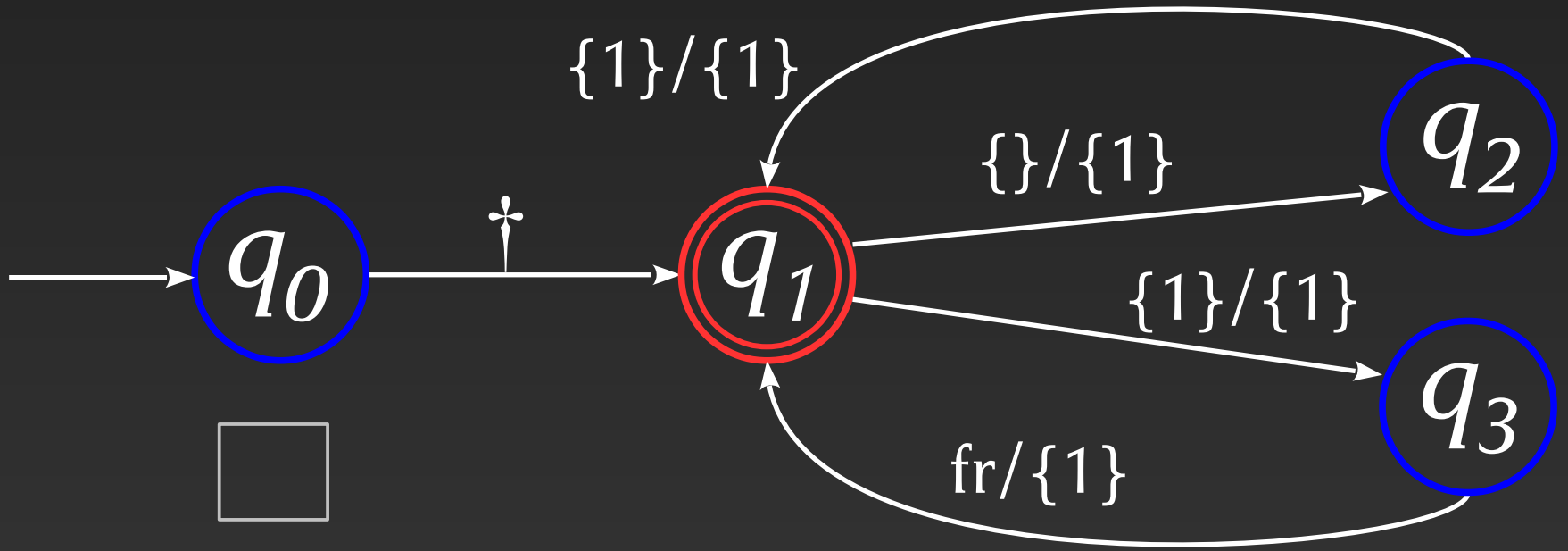


Fresh-register automata

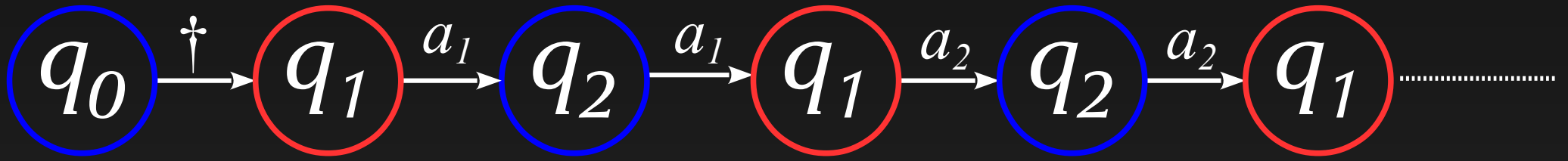


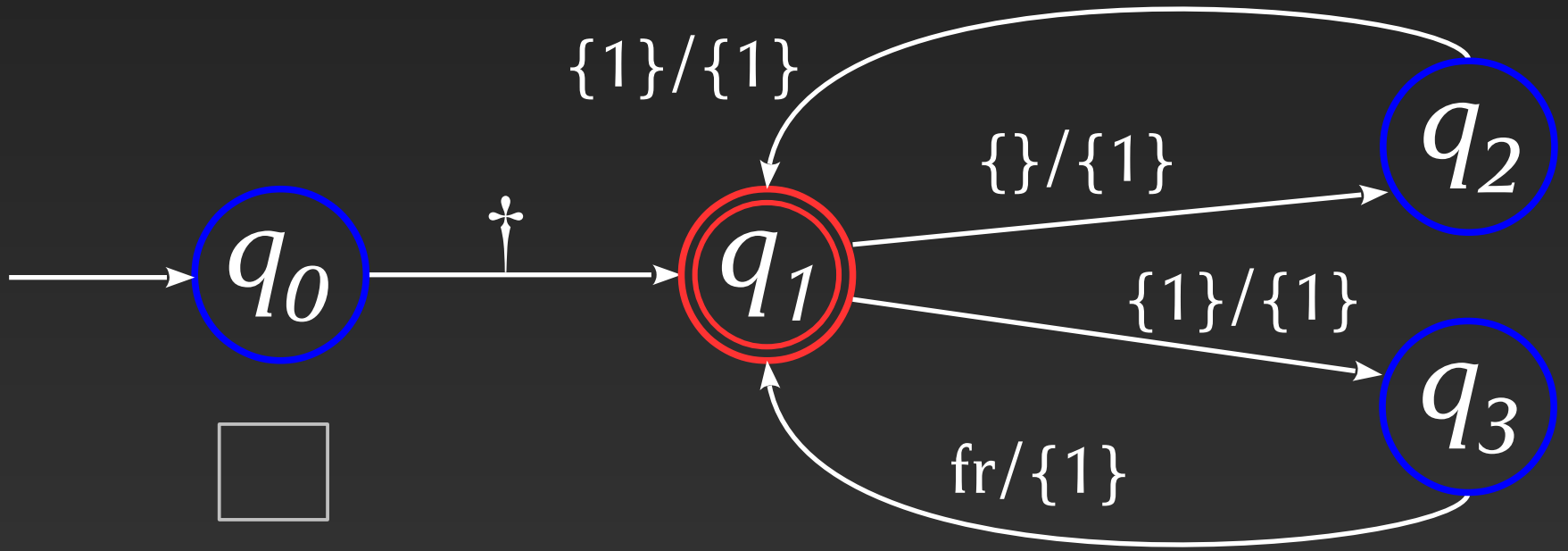


Fresh-register automata

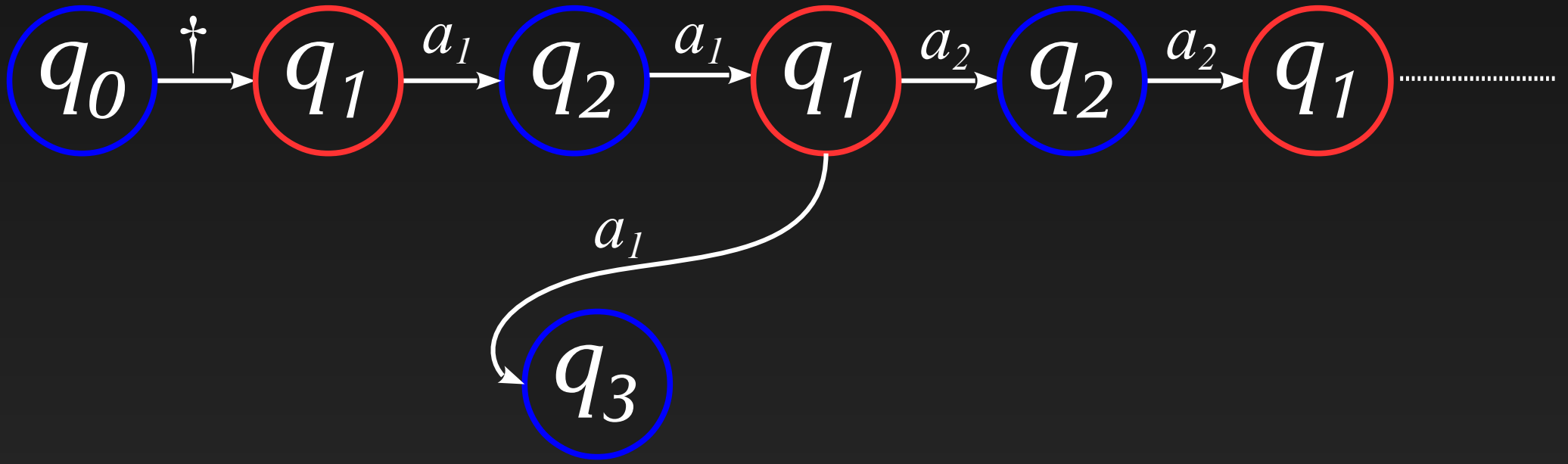


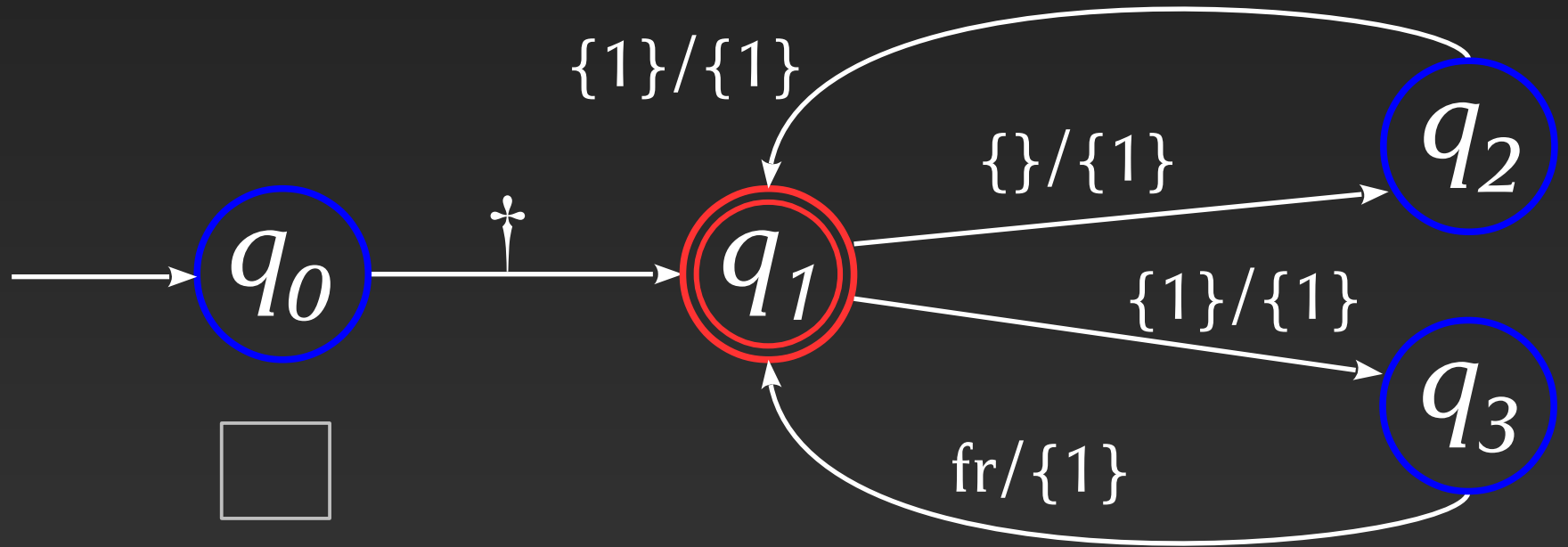
Fresh-register automata



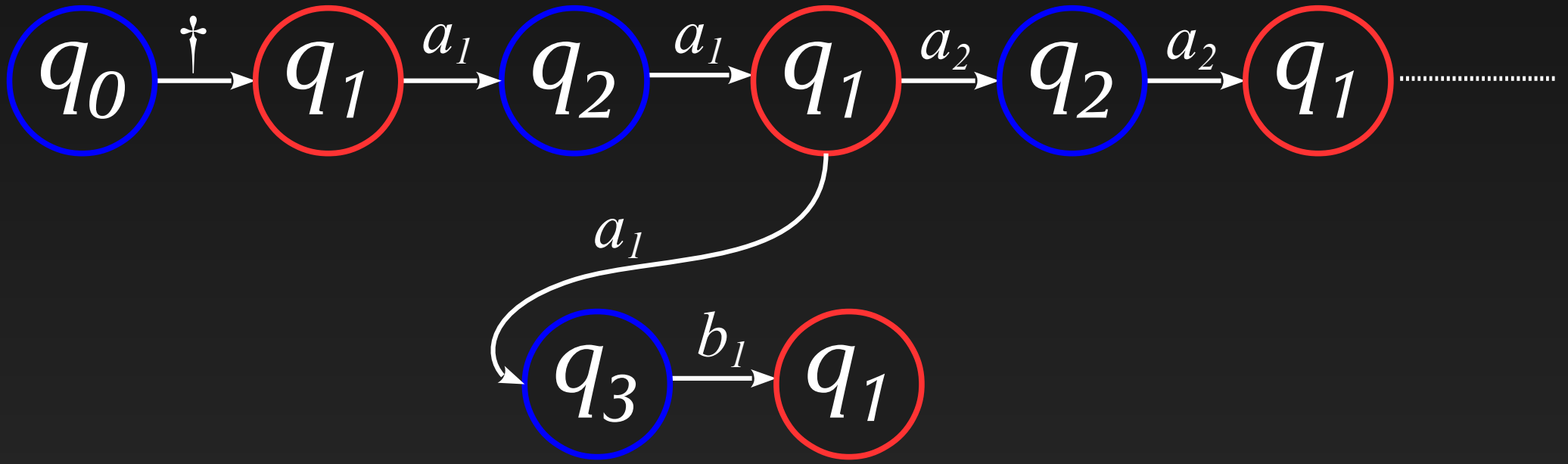


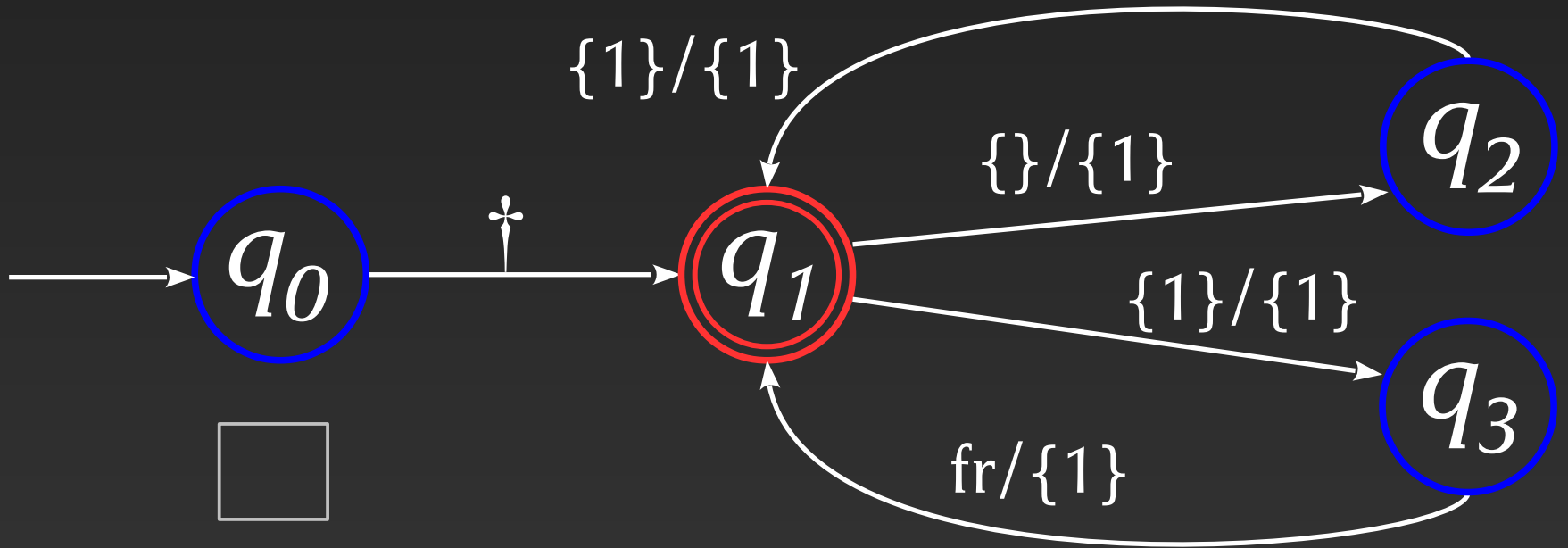
Fresh-register automata



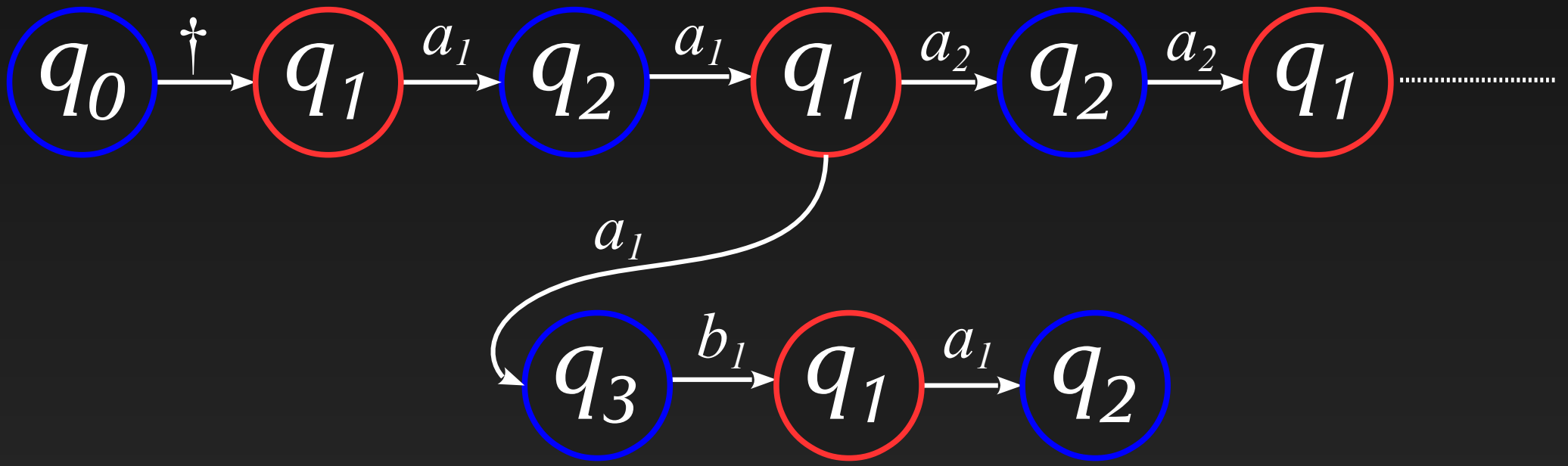


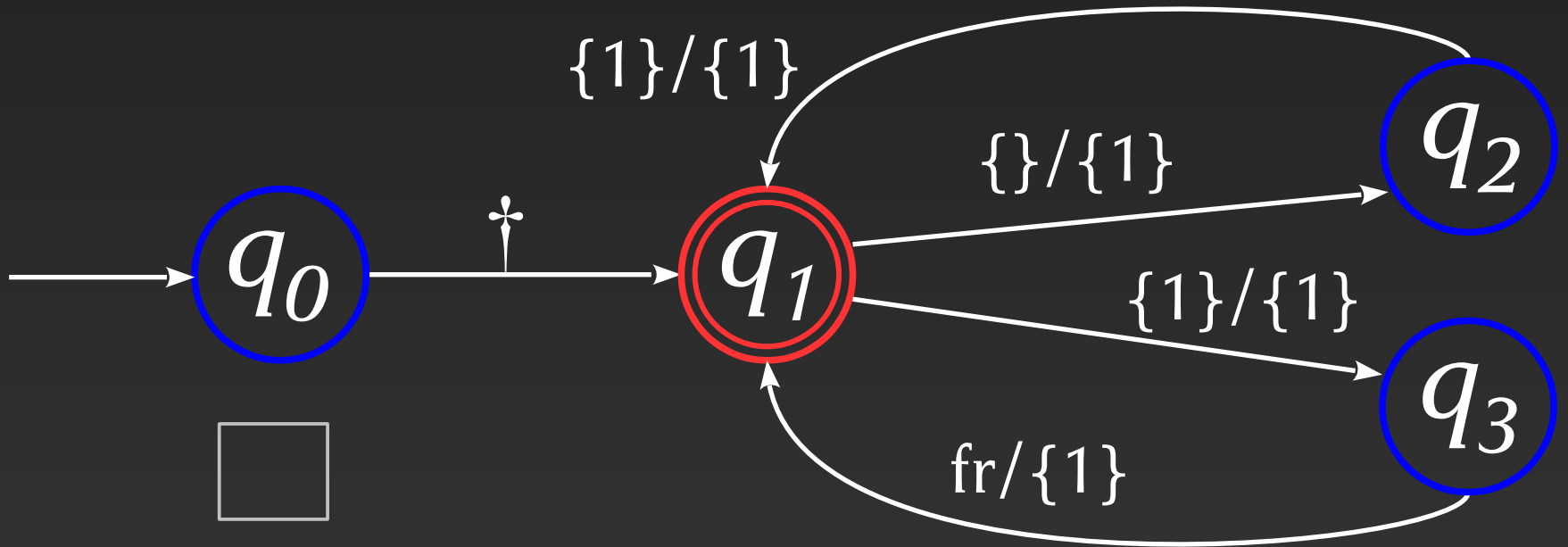
Fresh-register automata



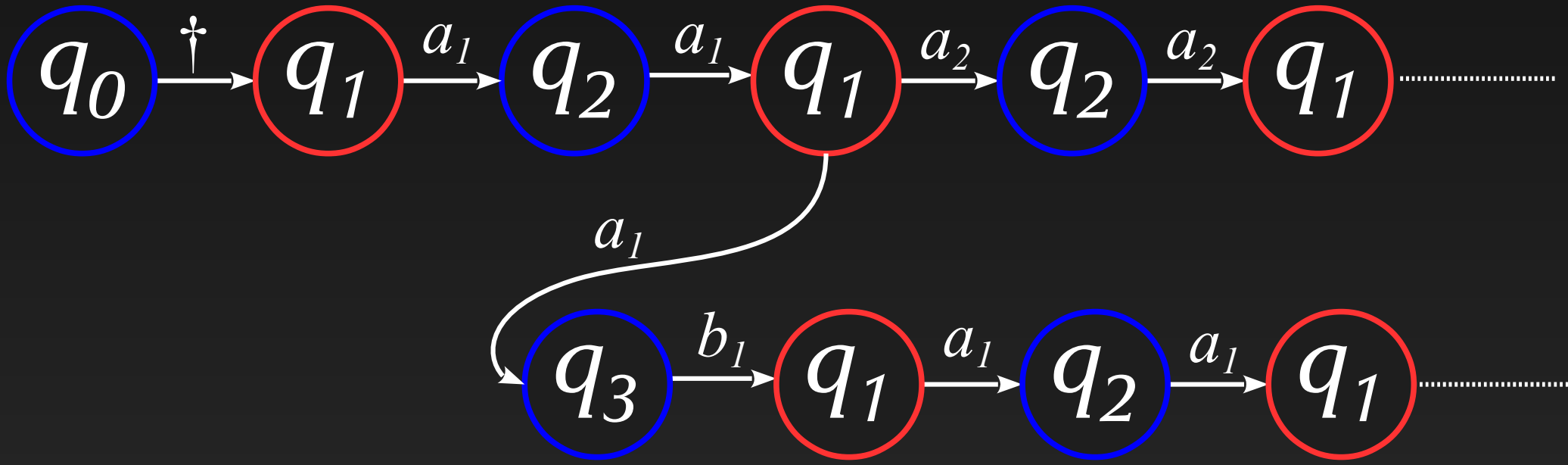


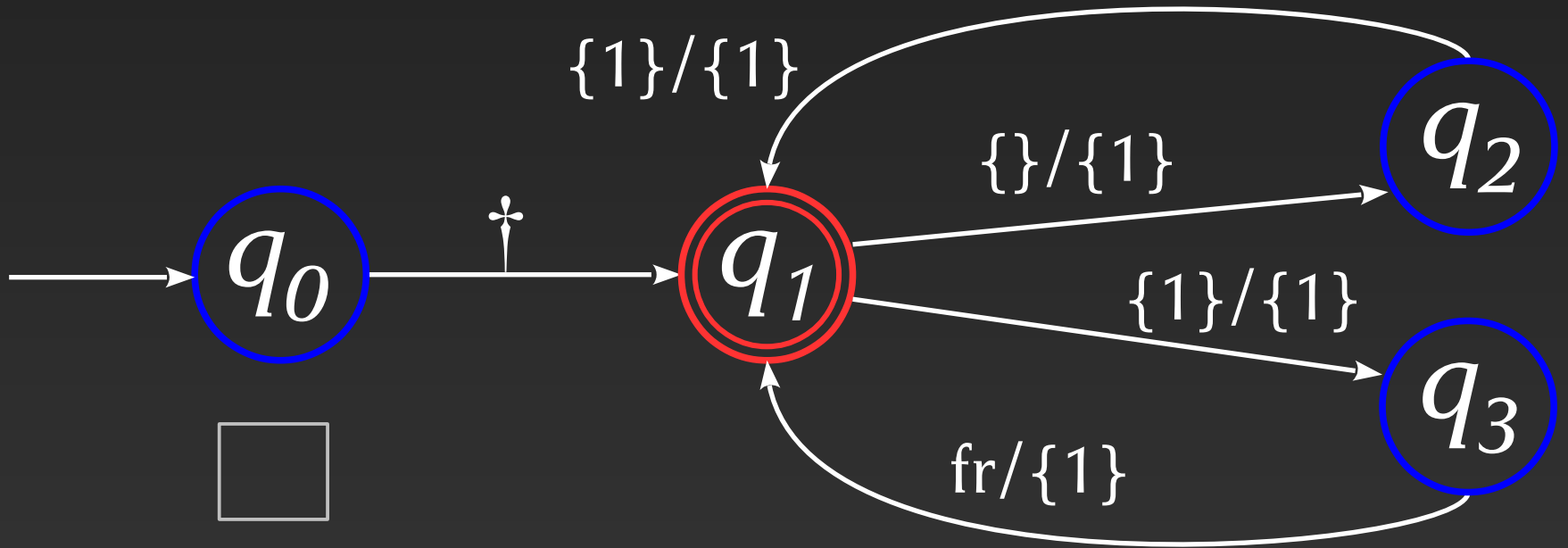
Fresh-register automata



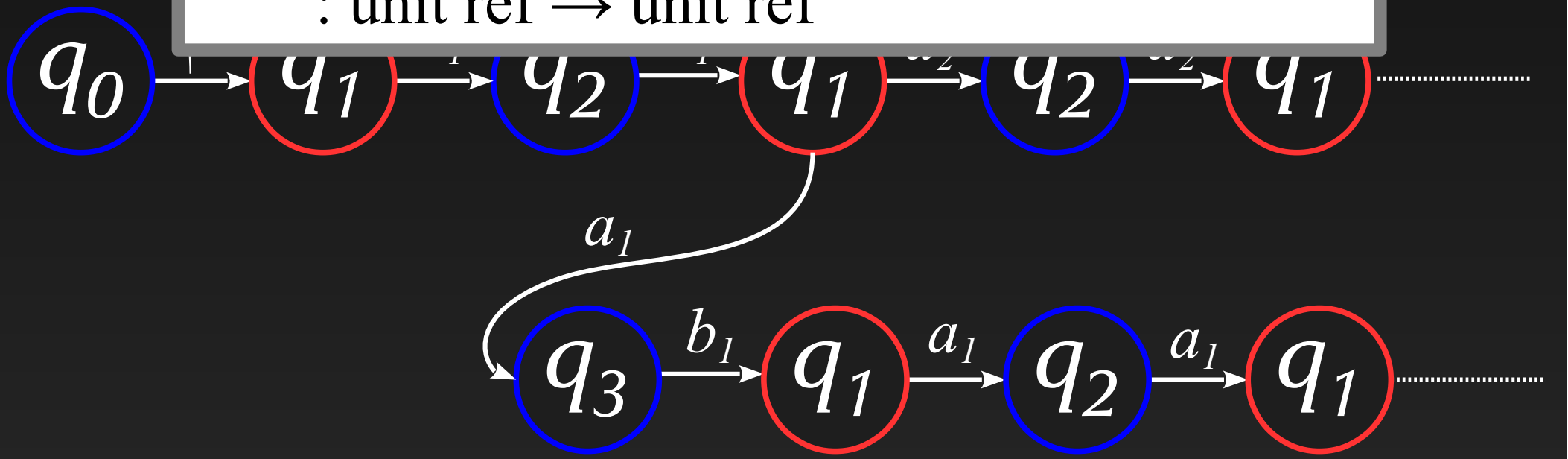


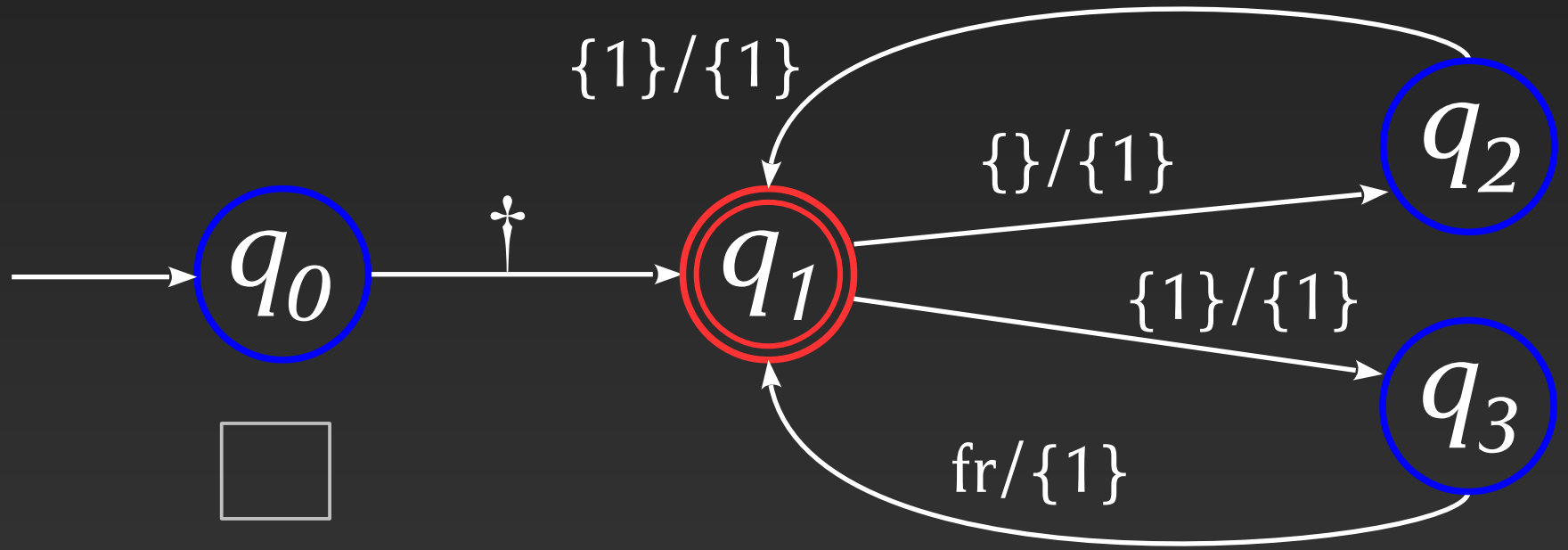
Fresh-register automata





```
let y = ref(ref()) in
    λx. y := ( if x==!y then ref() else x ); !y
    : unit ref → unit ref
```





Properties of FRAs

- Cleanly extend Register Automata (RA's)
- Closed under union and intersection;
  not under complementation, concatenation, _*
- Universality undecidable (from RA's)
- Emptiness decidable (from RA's)
- Bisimilarity decidable

Algorithmic nominal game semantics

$$P \cong P' \Leftrightarrow A(P) \sim A(P')$$

Representation of strategies as FRA's

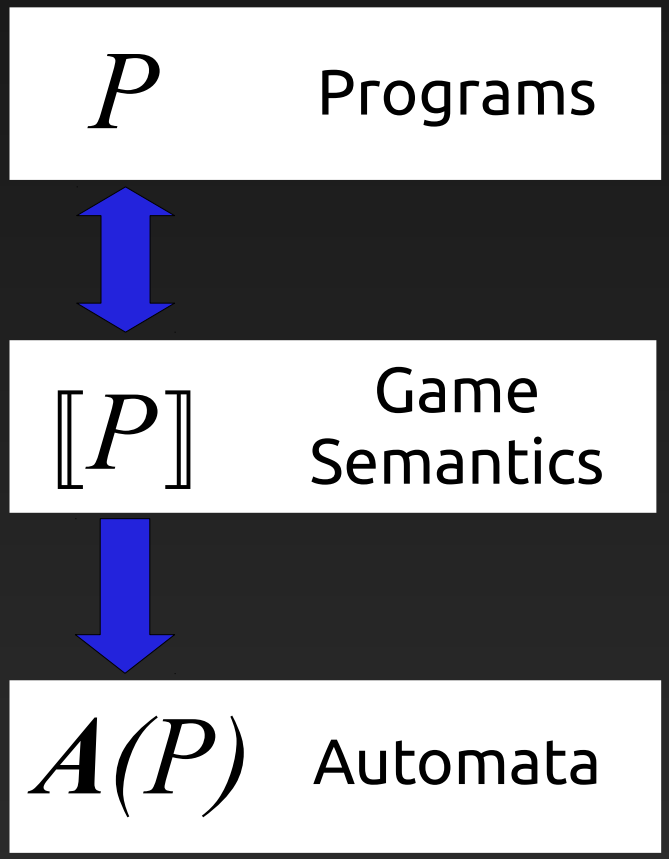
- with “data words”
- pushdown variants

pairs (f, a):
- f from finite alph.
- a a name

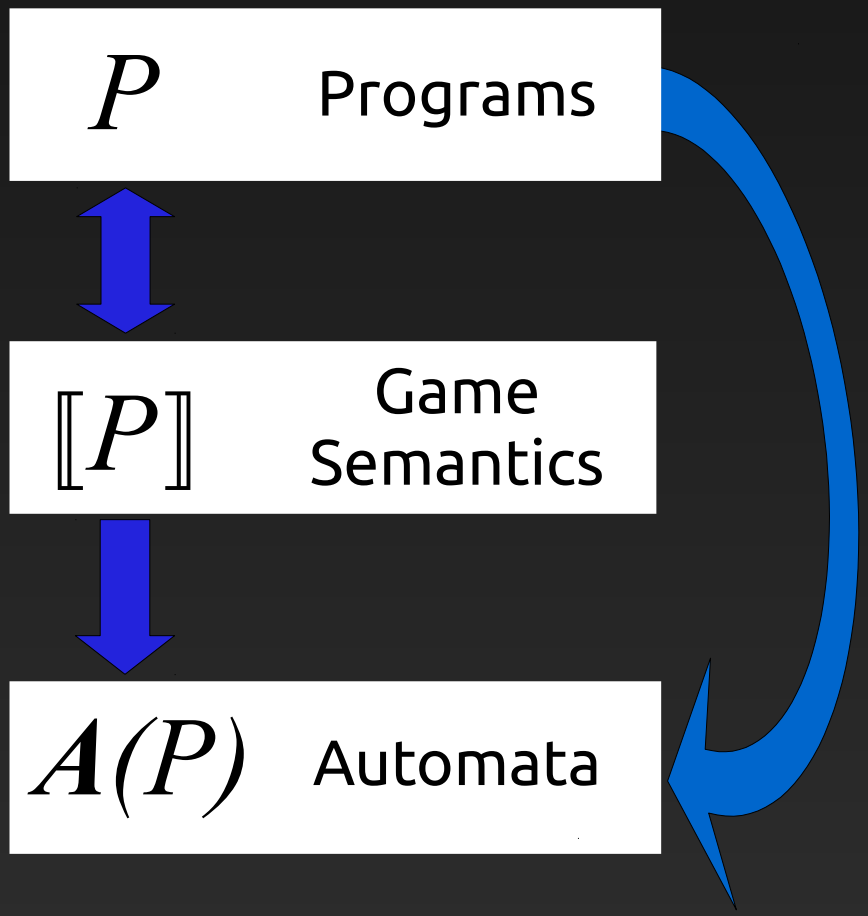
stack stores constants and register “snapshots”

`dblp:` Murawski, Ramsay, T.

From programs to automata



From programs to automata



Algorithmic results

For Ground ML [Murawski & T, ICALP'12]

- Decide equivalence for terms in contexts $\Theta_L \vdash \Theta_R$

$$\Theta_L ::= \beta \mid \Theta_R \to \Theta_L \qquad \Theta_R ::= \beta \mid \Theta_1 \to \beta$$

- This is a full characterisation:

  in higher-order contexts, strategies can be used to encode queue machines

"Weaker" results for Int ML [Murawski & T, ESOP'11]

Other nominal automata in semantics

- History-Dependent Automata
  - finitary π-calculus, symmetries, minimisation

`dblp:` Montanari, Pistore, Ciancia, Ferrari, Tuosto, …

- Automata for languages with scoping

Distefano *etal*

  - Dynamic (de)allocation of registers

`dblp:` Kurz, Suzuki, Tuosto

- History-Register Automata
  - Registers → histories

`dblp:` Grigore, T.

- Automata over infinite alphabets (~ XML)

`dblp:` Bojanczyk, Demri, Figueira, Lasota, Lazic, Segoufin, …

Nominal automata in program verification

- Algorithmic game semantics
- Pushdown analyses (fresh RA's!)
  - Pointers
  - Concurrent processes
  - Reachability, MSO-decidability

`dblp:` Atig, Bouajjani, Fratani, Qadeer, Bollig *etal*

- Runtime Java verification
  - TOPL (register automata)

`dblp:` Grigore *etal*

What's next

- More ML effects out there!
  - *one model to rule them all*
- More languages out there! (e.g. Java)
- Verification
  - Expand algorithmic approach to gen. model checking
  - Program logics from games

What's next

thanks

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