Reachability in Pushdown Register Automata

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What this talk is about

This talk is about automata over infinite alphabets

In particular, we examine automata with two store mechanisms:

finitely many registers and a pushdown stack

We examine their reachability properties and establish complexity bounds

Why automata over *infinite* alphabets?

```
public void foo() {
  // Create new list
  List x = new ArrayList();

  x.add(1); x.add(2);
  Iterator i = x.iterator();
  Iterator j = x.iterator();
  i.next(); i.remove(); j.next();
}
```

finite alphabet is not a satisfactory abstraction here

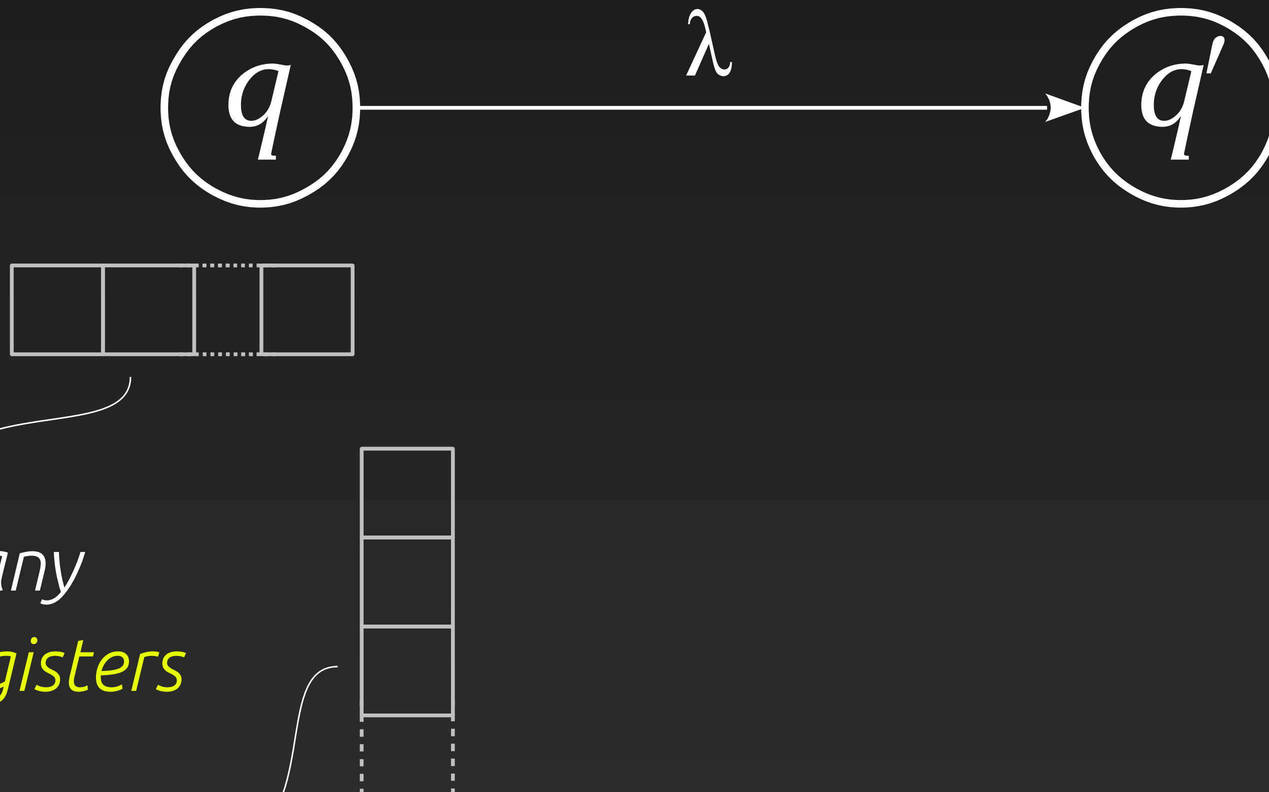
Let $\Sigma = \{a_1, a_2, \ldots, a_n, \ldots\}$ be an infinite alphabet of names

can only be compared for equality

[Cheng & Kaminski '98, Segoufin '06]

Pushdown Register Automata *(PDRA)*

Let $\Sigma = \{a_1, a_2, \ldots, a_n, \ldots\}$ be an infinite alphabet of names



finitely many (say R) registers

pushdown stack

registers & stack store names

[Cheng & Kaminski '98, Segoufin '06]

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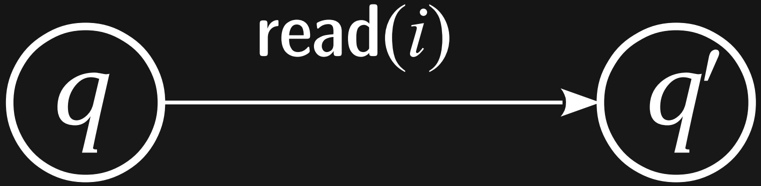
Label λ of the form:

- **read**(i), $i \in \{1,\ldots,R\}$
- **refresh**(i), $i \in \{1,\ldots,R\}$
- **push**(i), $i \in \{1,\ldots,R\}$
- **pop**(i), $i \in \{1,\ldots,R\}$
- **pop-fresh**

Transitions:



Transitions:

$$q \xrightarrow{\mathbf{read}(i)} q'$$

$$q \xrightarrow{\mathbf{read}(2)} q'$$

$$\begin{array}{|c|c|c|}\hline a & g & b \\ \hline\end{array}$$

$$\begin{array}{|c|}\hline e \\ \hline a \\ \hline c \\ \hline\end{array}$$

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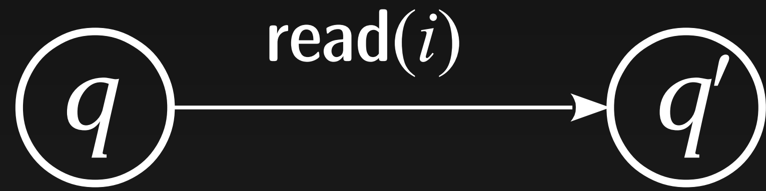
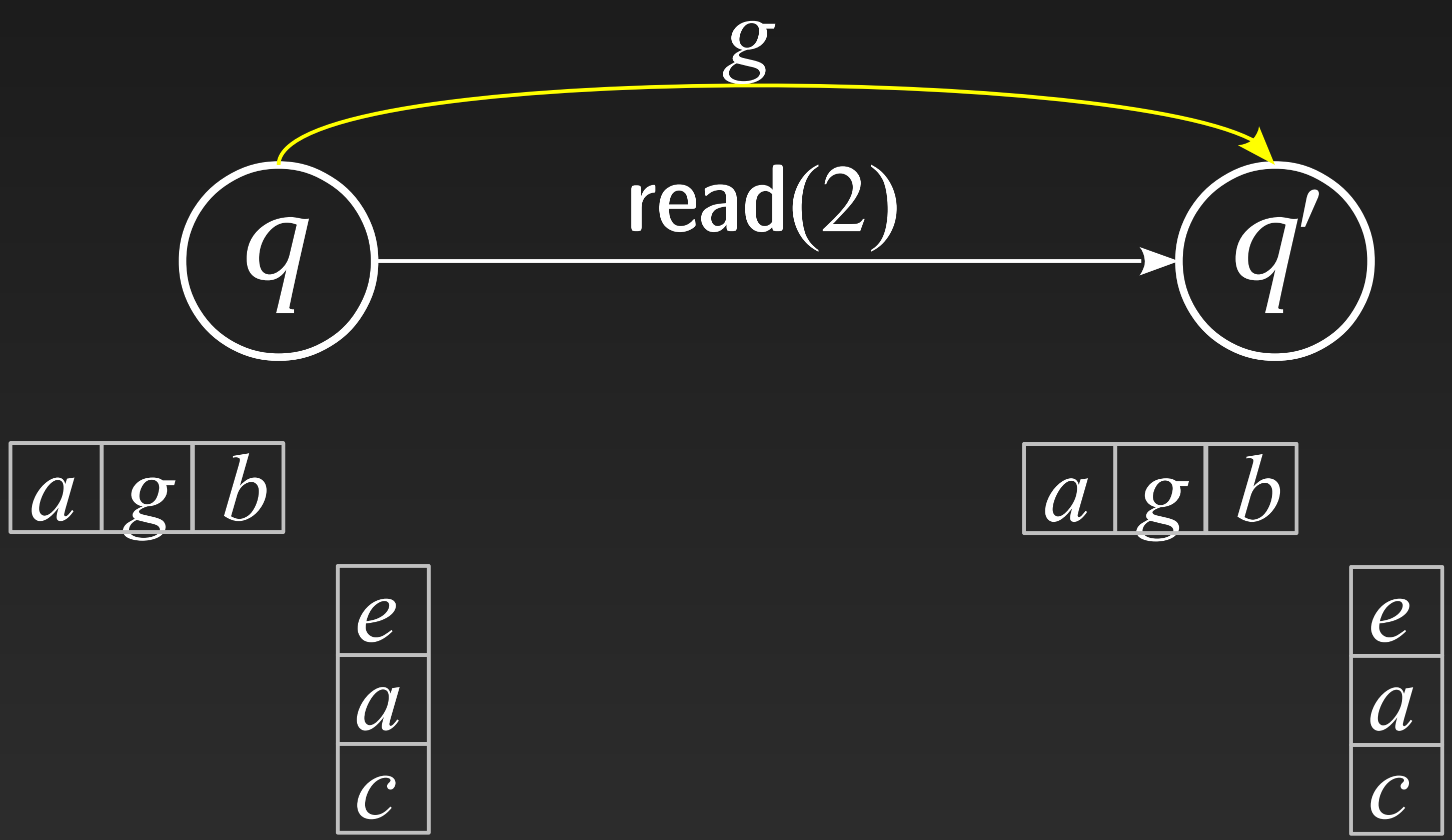




Transitions:

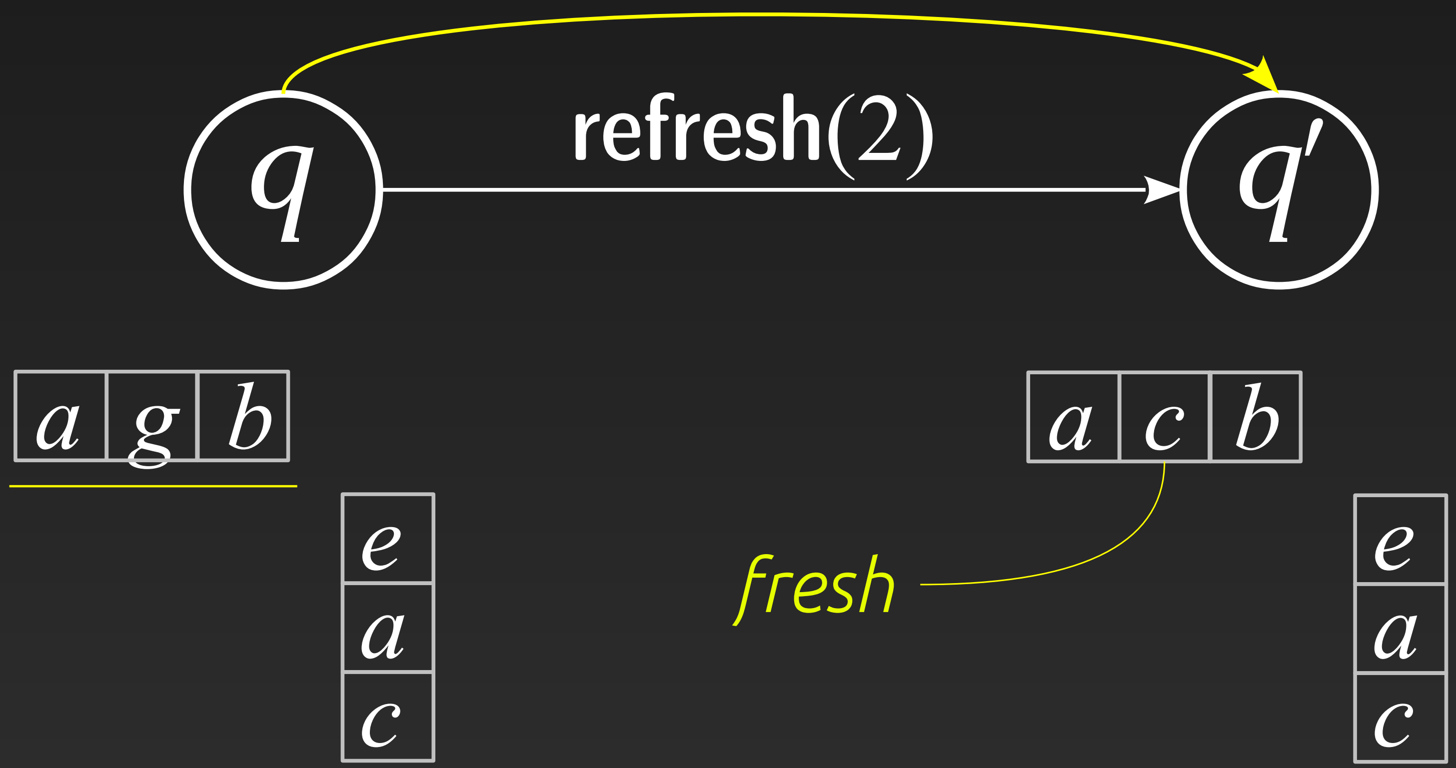
$$q \xrightarrow{\textbf{refresh}(i)} q'$$

$$q \xrightarrow{\textbf{refresh}(2)} q'$$

$$\boxed{a}\boxed{g}\boxed{b}$$

$$\begin{array}{|c|}\hline e \\ \hline a \\ \hline g \\ \hline \end{array}$$

Transitions:



Transitions:

$$q \xrightarrow{\mathsf{push}(i)} q'$$

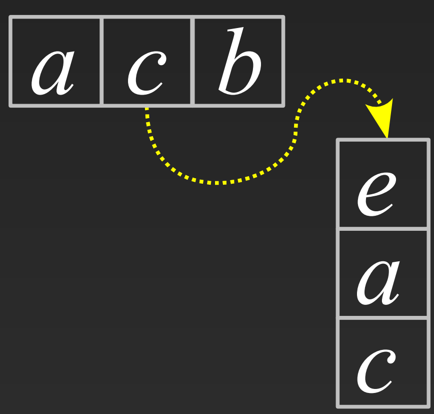
$$q \xrightarrow{\mathsf{push}(2)} q'$$

$$\boxed{a}\boxed{c}\boxed{b}$$

$$\begin{array}{|c|}\hline e \\ \hline a \\ \hline c \\ \hline \end{array}$$

Transitions: $q \xrightarrow{\textsf{push}(i)} q'$

$$q \xrightarrow{\textsf{push}(2)} q'$$



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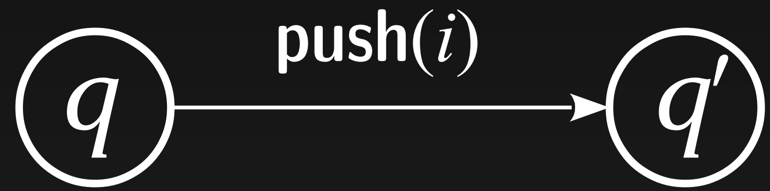

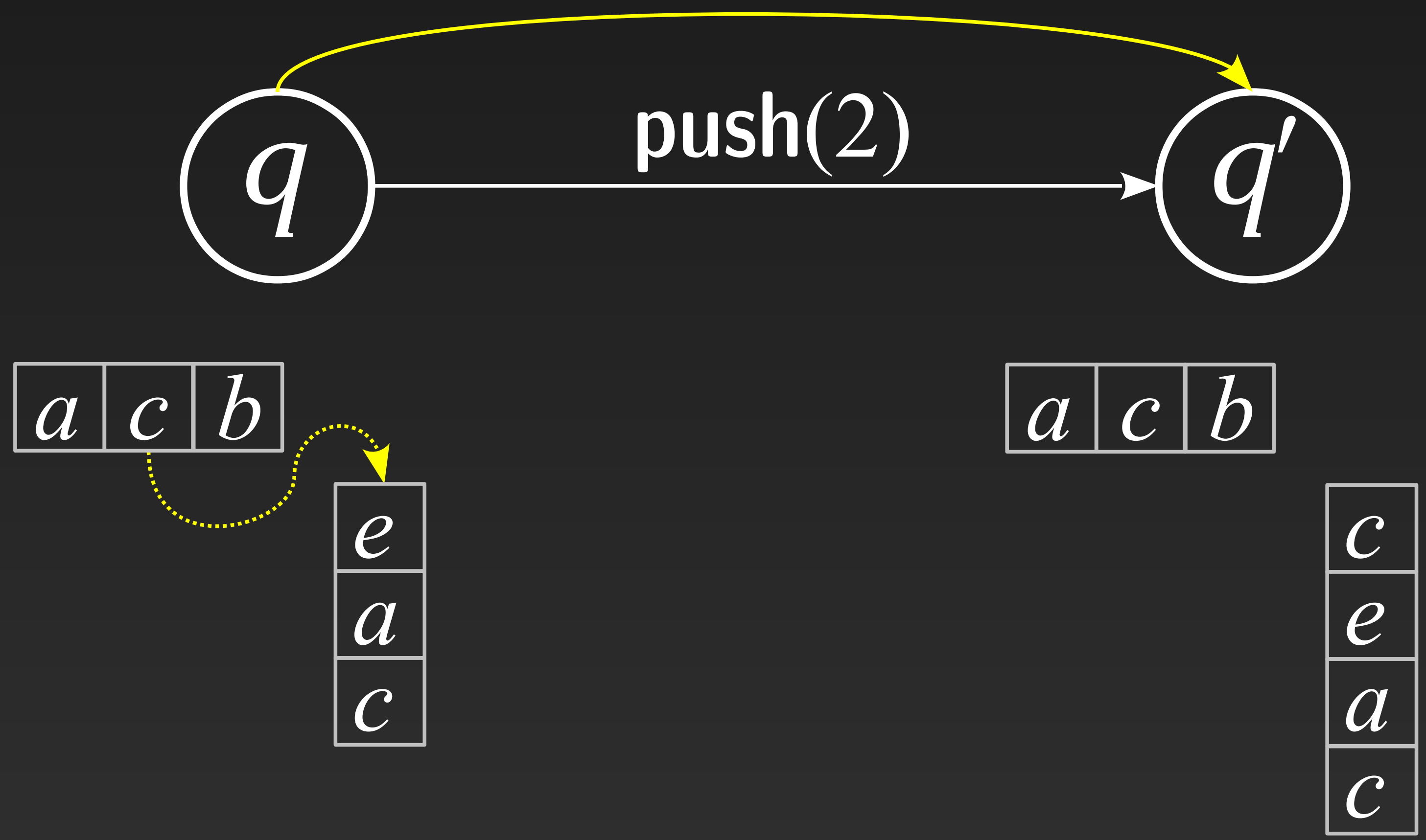

Transitions: $q \xrightarrow{\textbf{pop}(i)} q'$

$$q \xrightarrow{\textbf{pop}(2)} q'$$

a	c	b

c
e
a
c

Transitions:

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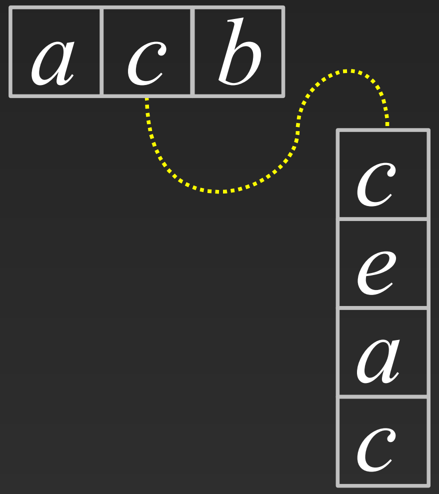

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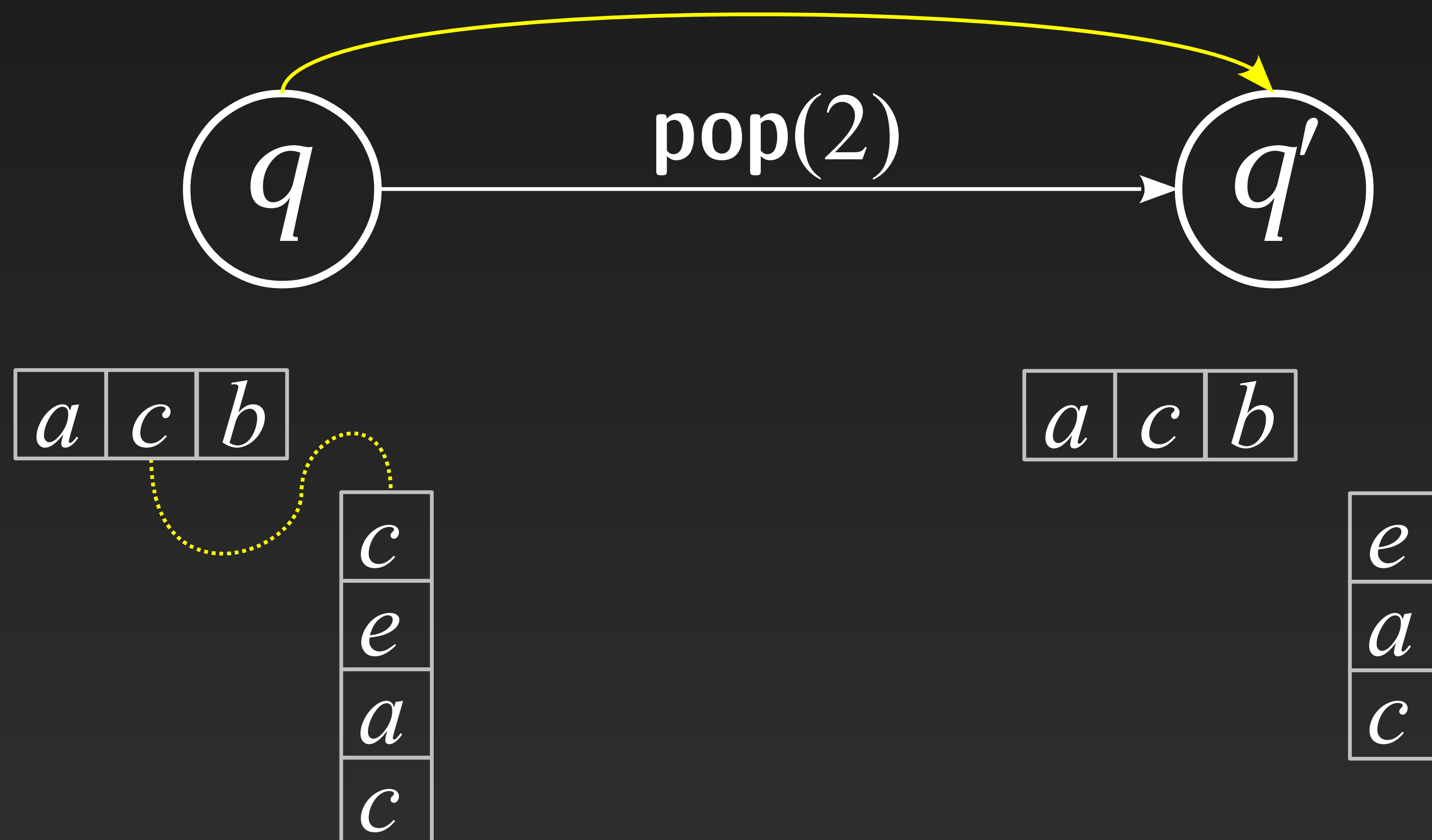

Transitions:

$$q \xrightarrow{\textbf{pop-fresh}} q'$$

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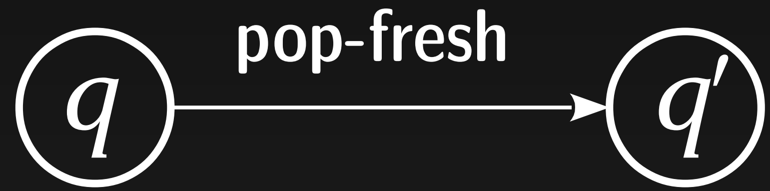
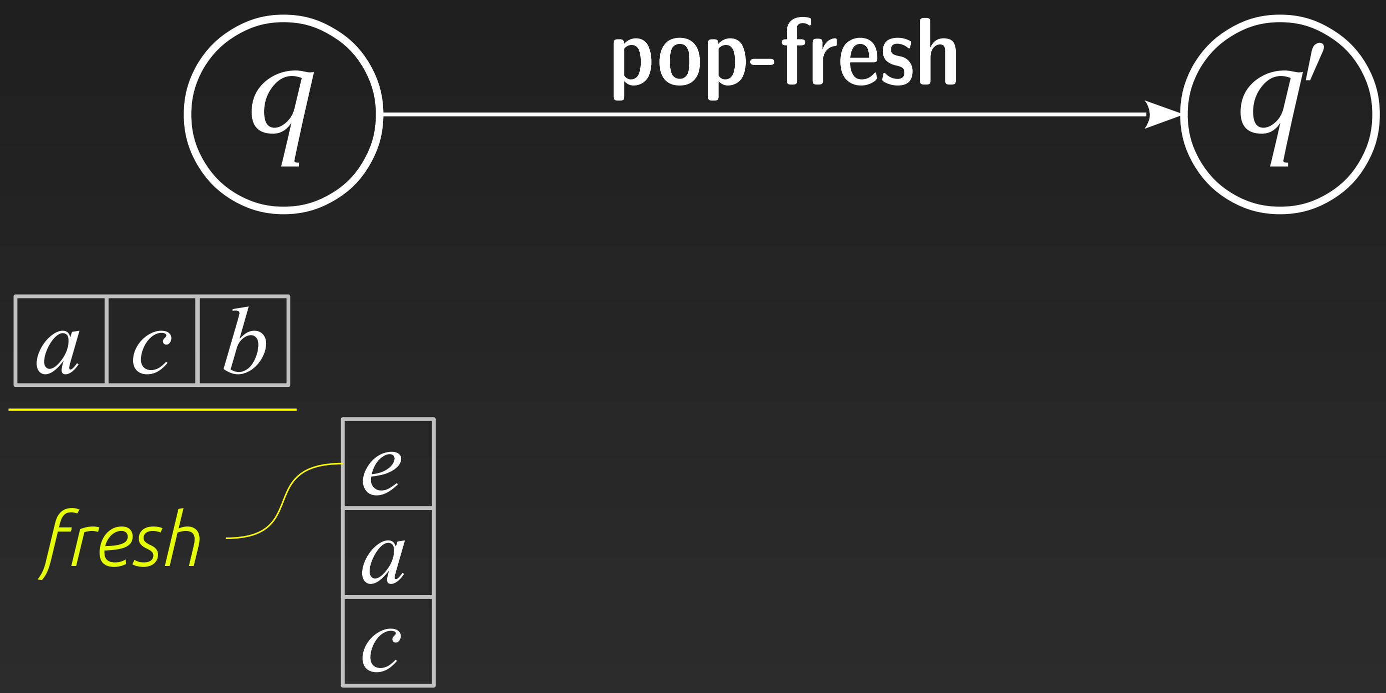
a	c	b

e
a
c

Transitions:





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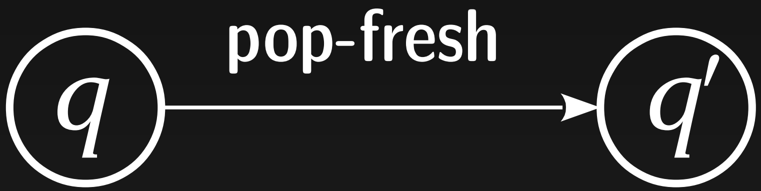
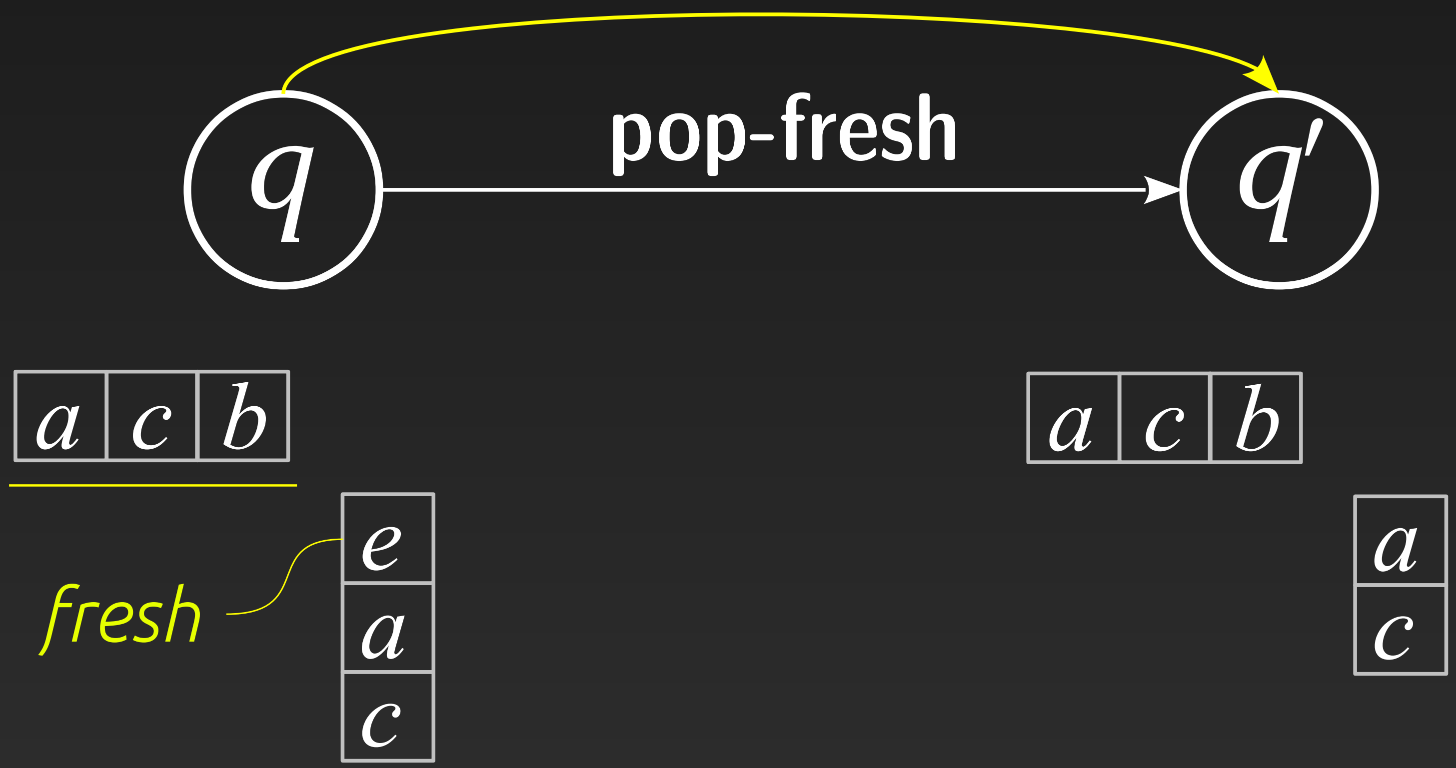
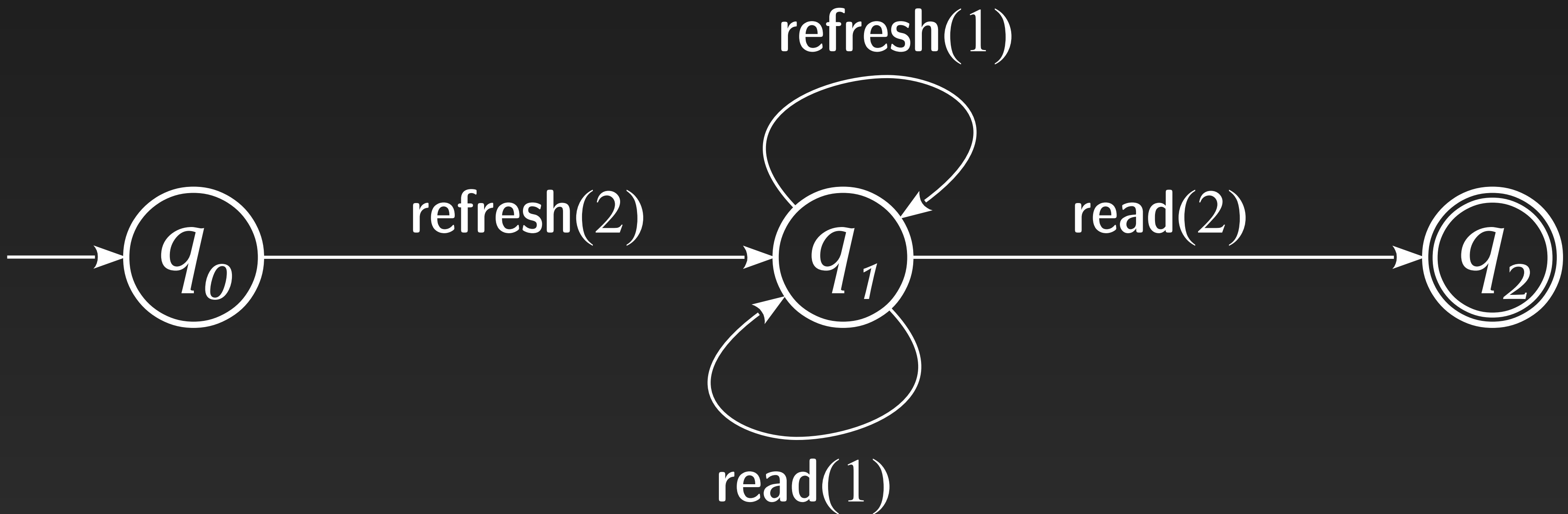




Example

$$L = \{ a_1 a_2 \ldots a_n b \in \Sigma^* \mid n \geq 0, \forall i \leq n.\ a_i \neq b \}$$

(all strings where last name is distinct from all previous ones)



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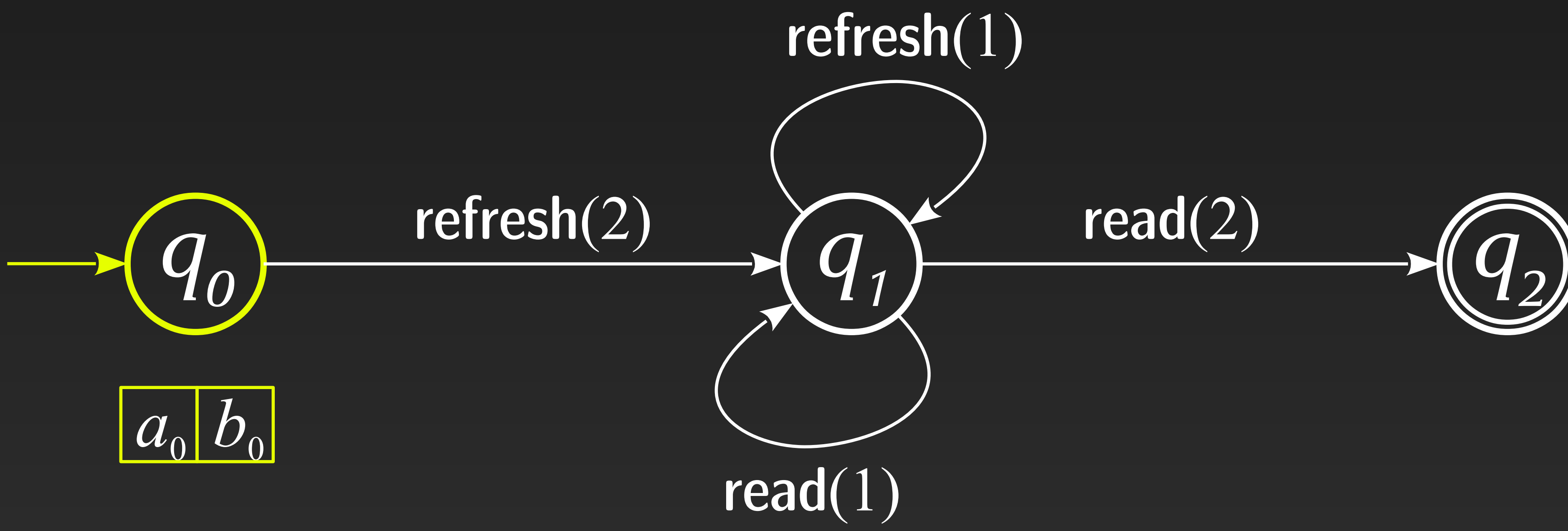

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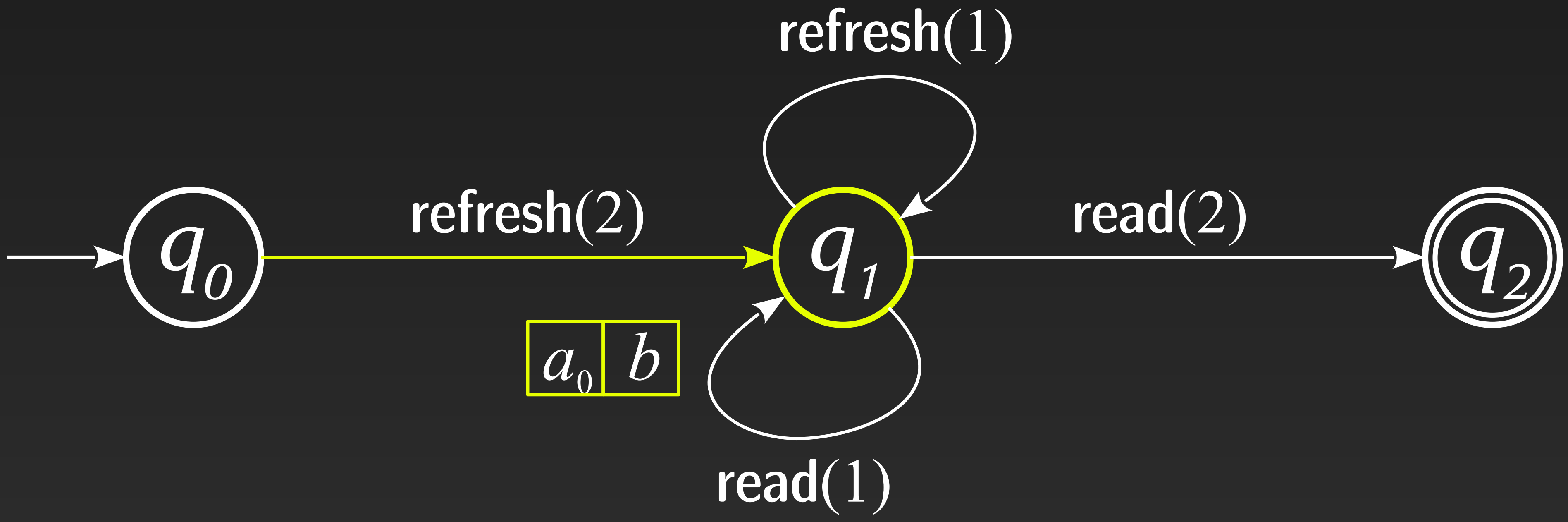

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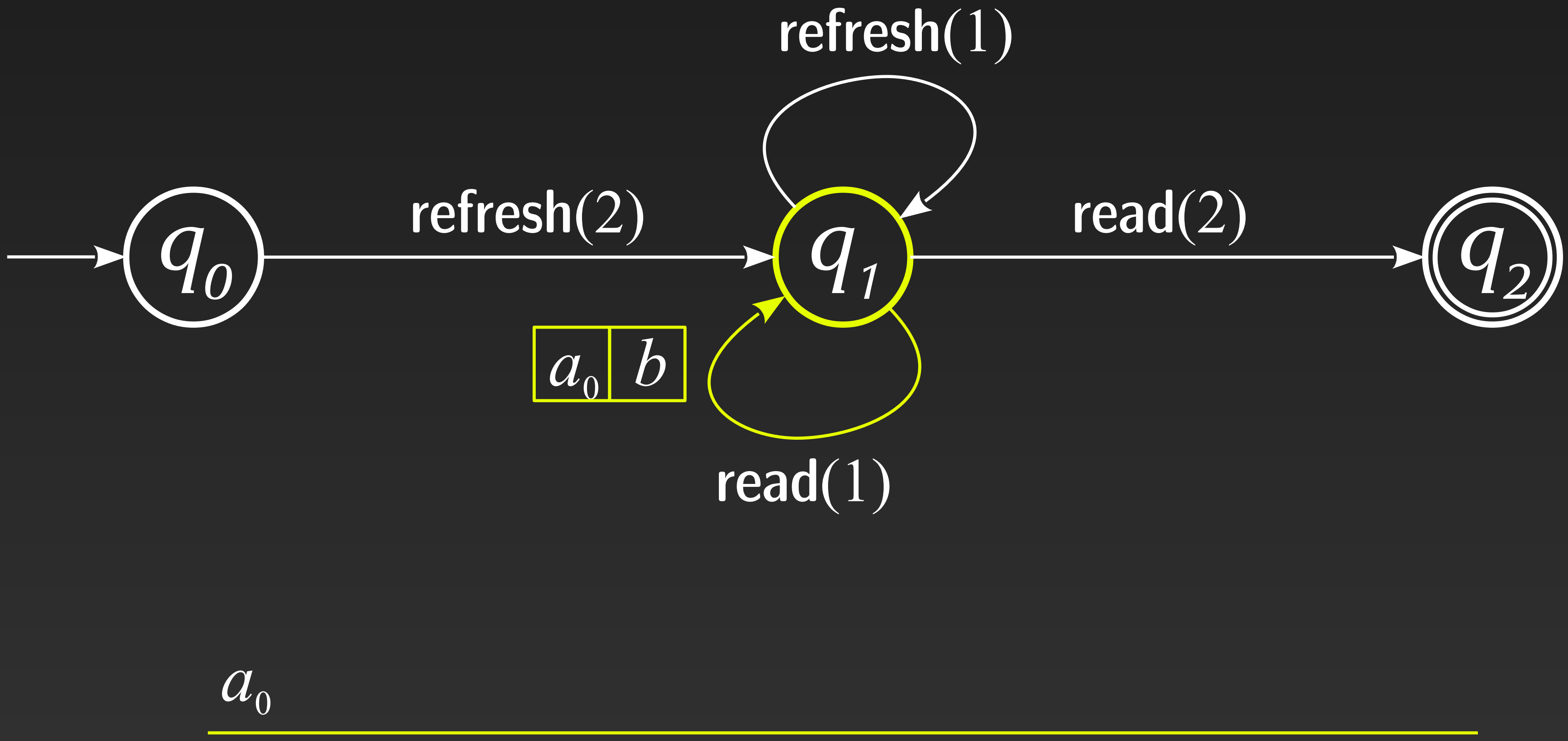

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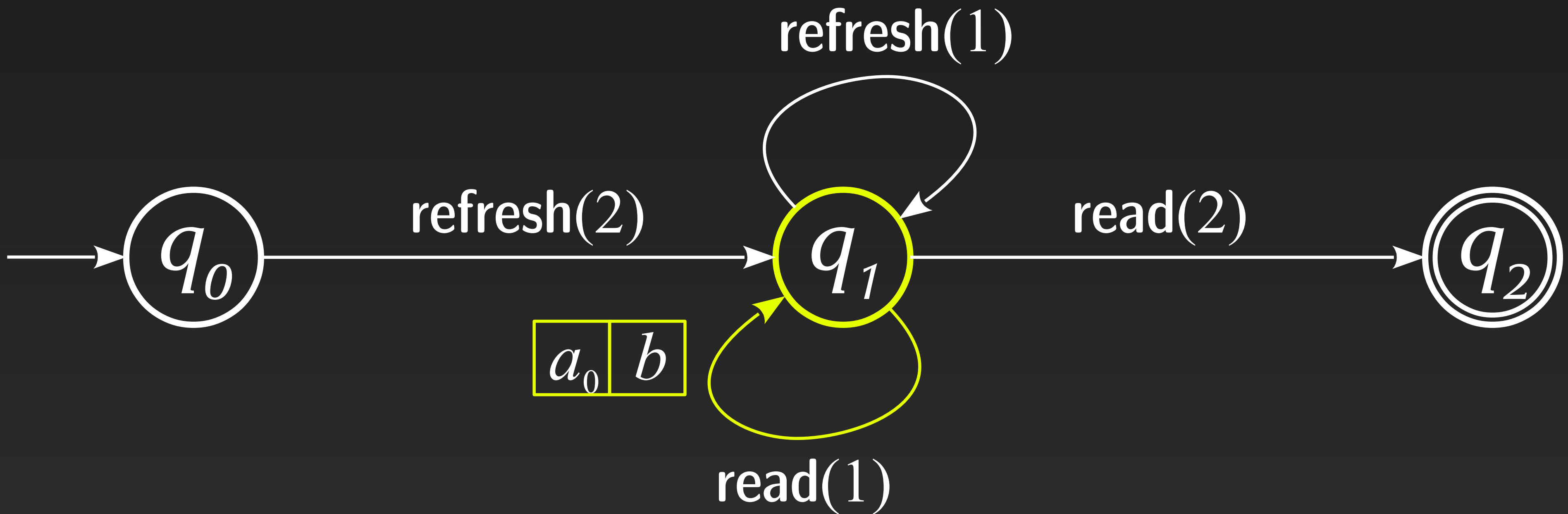

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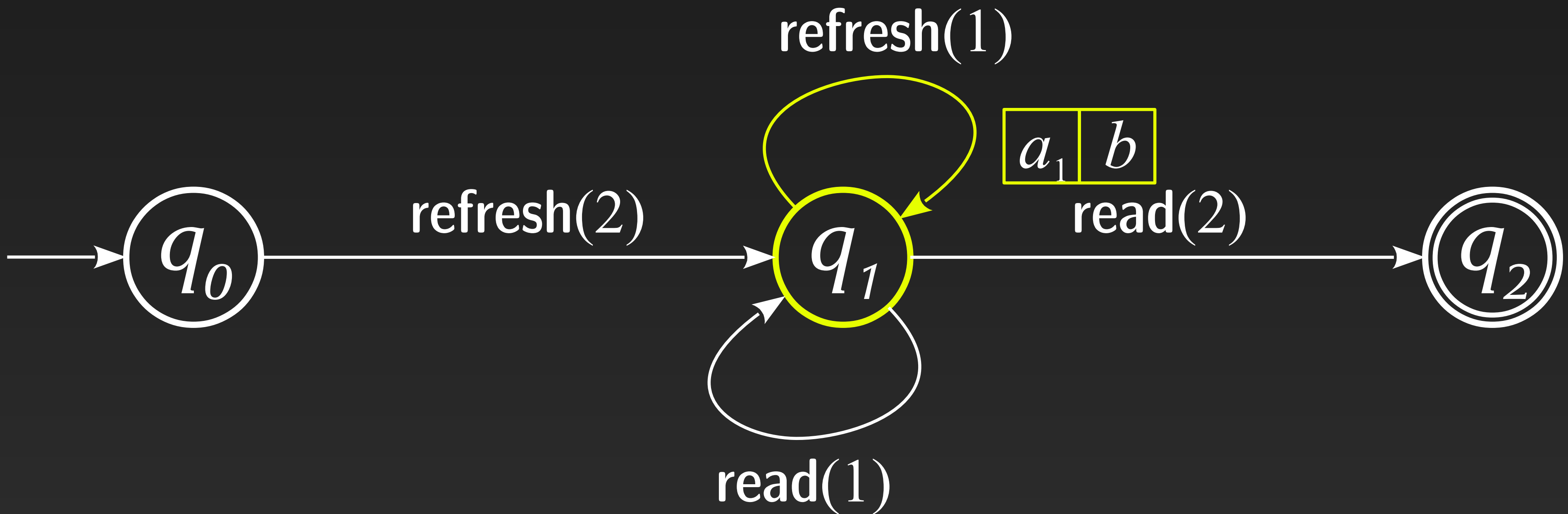

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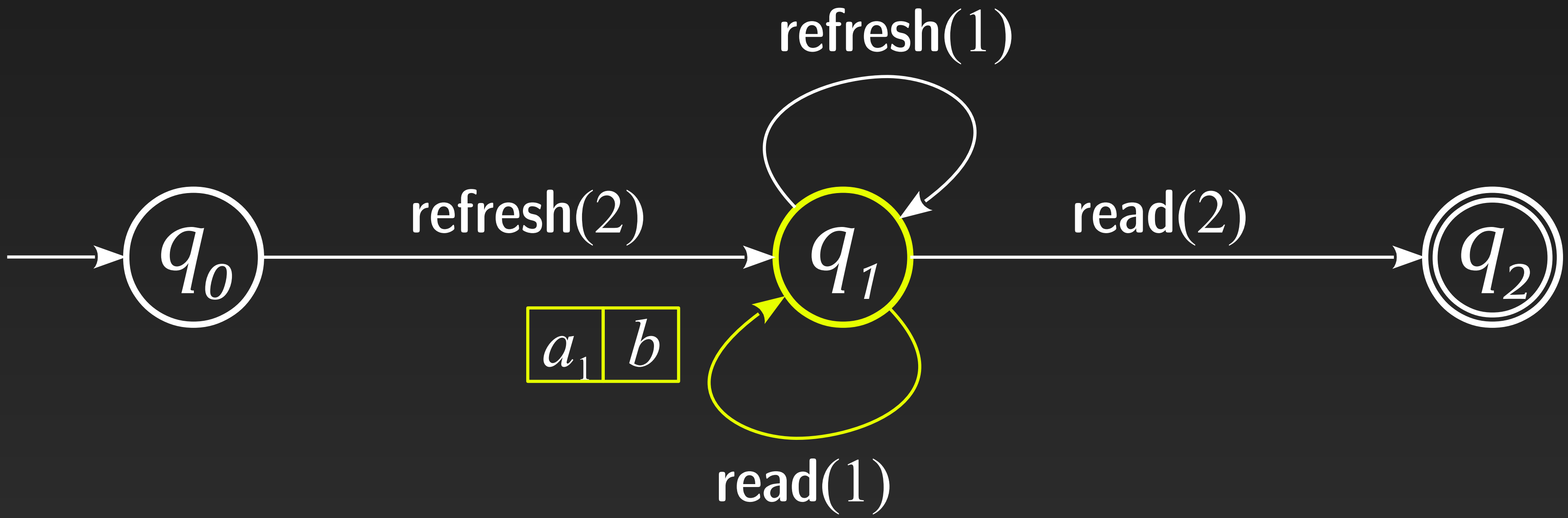

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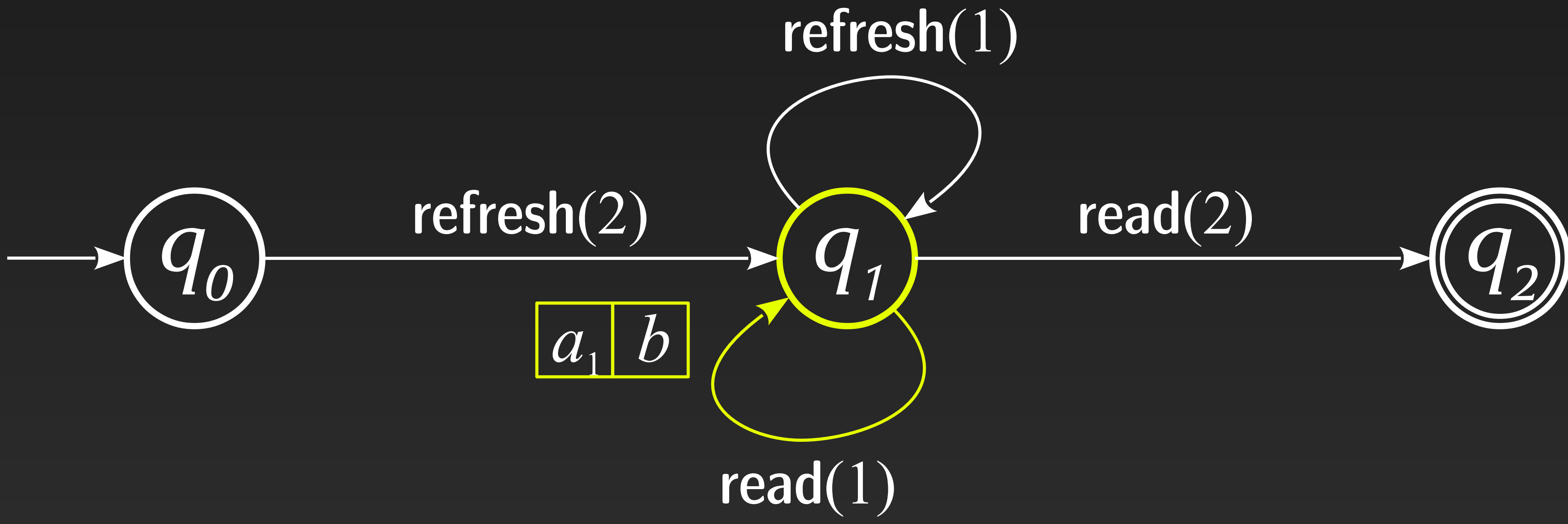

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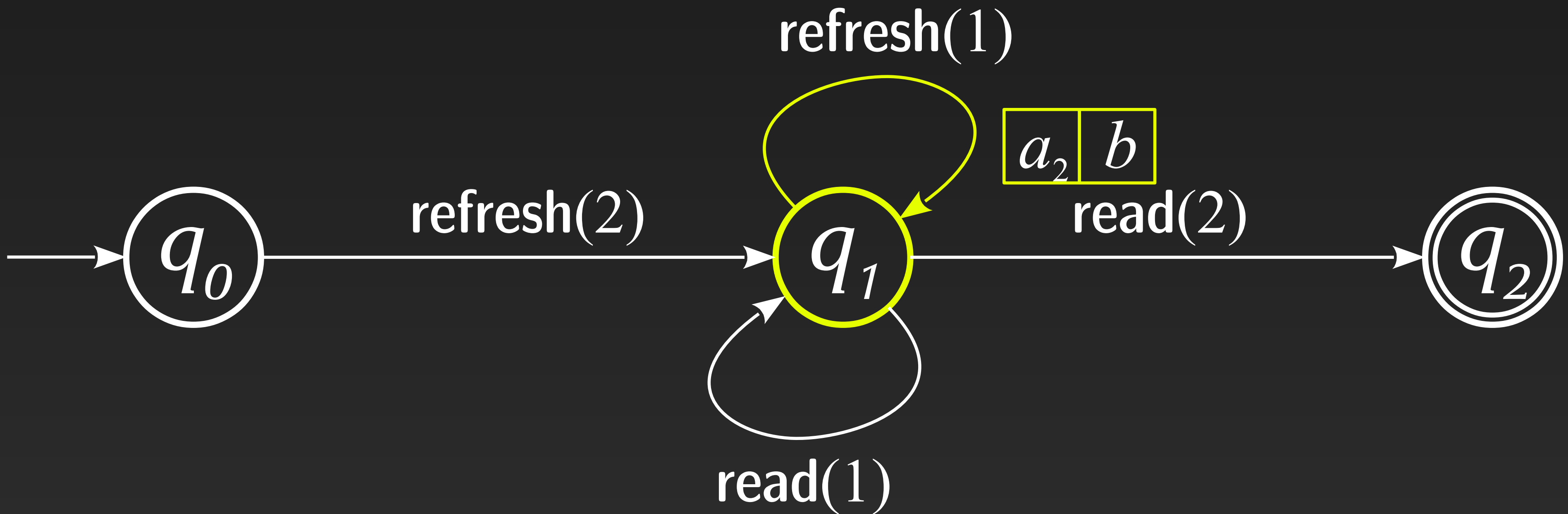

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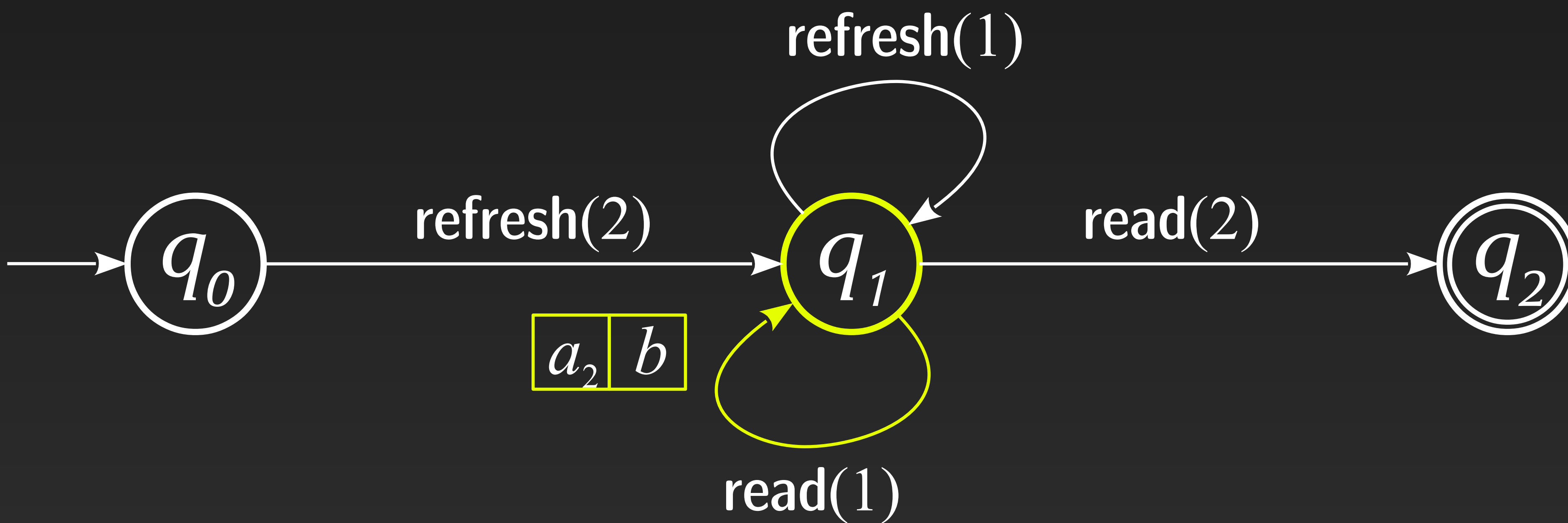

$a_0\, a_0\, a_1\, a_1\, a_1\, a_2$

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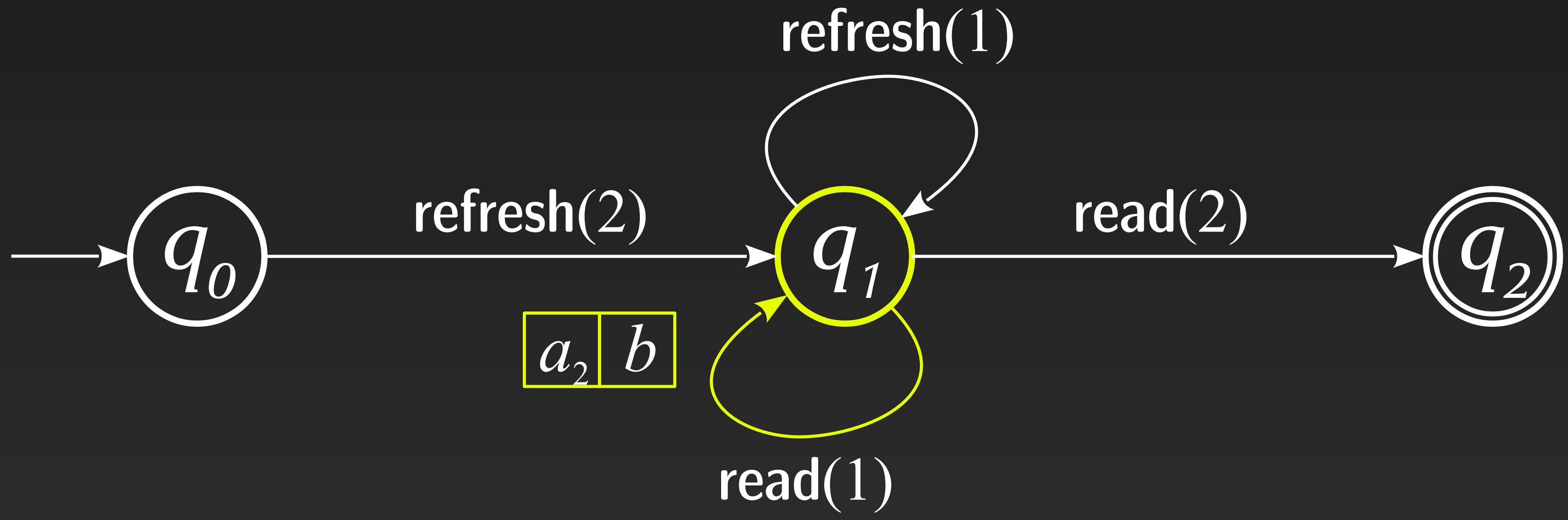

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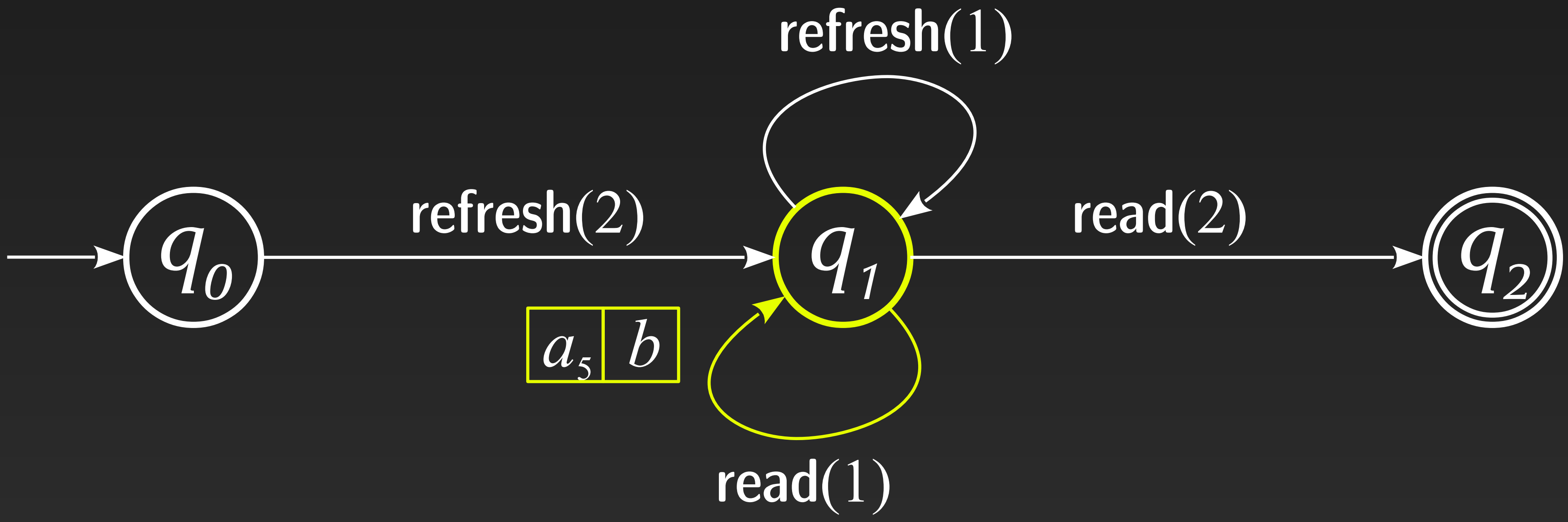

$a_0\, a_0\, a_1\, a_1\, a_1\, a_2\, a_2\, a_2\, a_2\, a_2\, a_3\, a_3\, a_4\, a_4\, a_5\, a_5\, a_5\, a_5\, a_5\, a_5$

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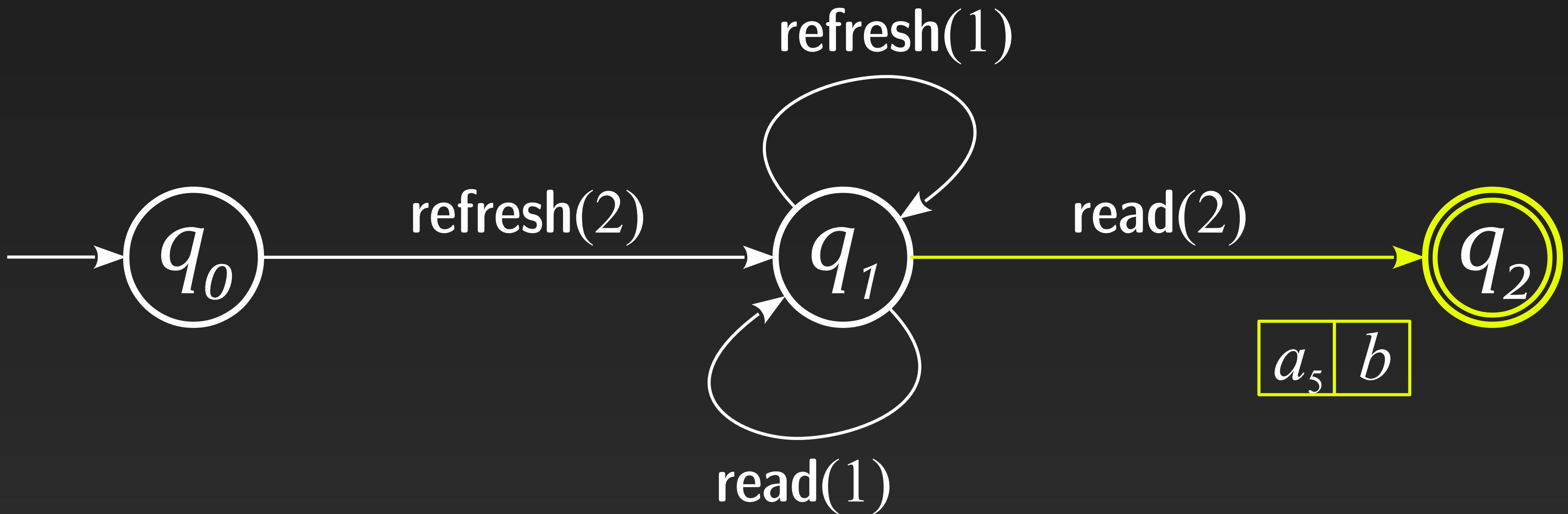

$$a_0\, a_0\, a_1\, a_1\, a_1\, a_2\, a_2\, a_2\, a_2\, a_2\, a_3\, a_3\, a_4\, a_4\, a_5\, a_5\, a_5\, a_5\, a_5\, a_5\, b$$

Limited distinguish-ability

$$L = \{ a_1 a_2 \ldots a_n \in \Sigma^* \mid n \geq 0, \forall i{\neq}j.\ a_i{\neq}a_j \}$$

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A *PDRS (PDR System)* is a PDRA without **read** transitions

Lemma: Let S be a PDRS with R-many registers.
For any pair of states q_1 and q_2, if there is a run between them (from empty stack to empty stack) then there is one involving at most $3R$ names.

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Conversely, there is a PDRS with R registers whose runs to a designated state involve exactly $3R$ names.

Reachability

R-PRDS Reach: *Given a PDRS S with R registers and a state q, is there a run of S to q?*

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Theorem: R-PRDS Reach is EXPTIME-complete.

For EXPTIME solvability:

- By previous Lemma, $3R$ names suffice: $\Sigma' = \{a_1, \ldots, a_{3R}\}$
- so, registers can be encoded inside states: $Q' = Q \times R^{3R}$
- and we reduce to PDA reachability (PTIME)

EXPTIME-hardness

For hardness, the argument is harder...

- We reduce from PSPACE Turing machines with stack
- crux of the reduction is the simulation of the tape
    - if we allowed name repetitions in registers then easy:

$$\boxed{0}\boxed{1}\boxed{0}\boxed{0}\boxed{1} \quad \mapsto \quad \boxed{a_1}\boxed{a_1}\boxed{a_2}\boxed{a_2'}\boxed{a_3}\boxed{a_3}\boxed{a_4}\boxed{a_4}\boxed{a_5}\boxed{a_5'}$$

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  - now, instead, we use *mask encodings* + the stack:

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hygiene to ensure that the masks are soundly applied and the stack is not messed up…

EXPTIME-hardness

without the stack, repetitions give a huge complexity gap!

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More results!

Global reachability: For a PDRS S, capture all configurations from which S can reach a specified set of configurations

Theorem: Register automata (i.e. PDRA but no stack) capture configurations that can reach "regular" sets.

- *cf. "saturation" technique of Bouajjani, Esparza and Maler*

Higher-order PDRS: Extensions with stacks of stacks

Theorem: Reachability is undecidable at order 2.

- *reduce from Pebble automata language emptiness*

Concluding

Reachability analysis for infinite-alphabet systems with:

- finitely many registers & a pushdown stack

paper appears at MFCS 2014

Further directions include:

- Implementation & use in automata-base verification
  - e.g *algorithmic game semantics*
- Bisimulation analysis
- dPDRA equivalence

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