Game Semantics for Nominal Exceptions

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What this talk is about

Fully abstract denotational models for higher-order languages with exceptions

The models are constructed in game semantics

They are the first such models for nominal exceptions

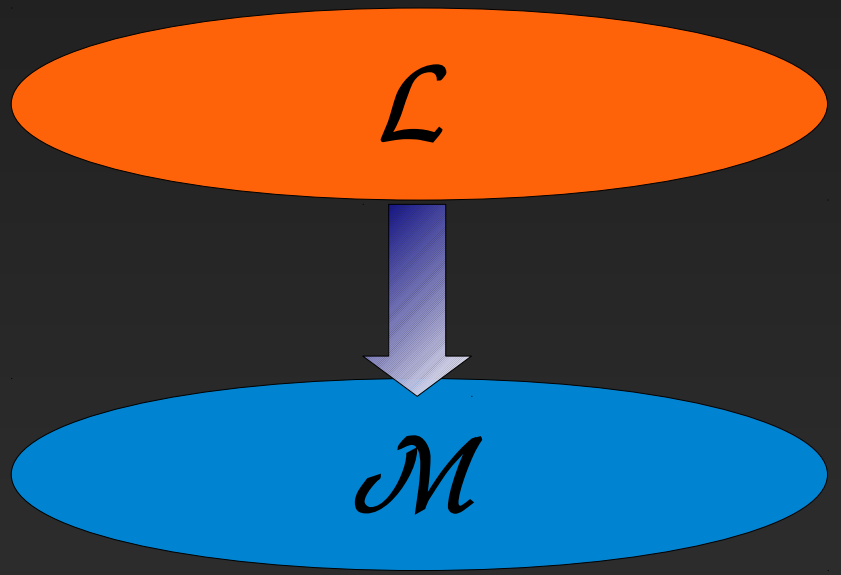
Denotational Semantics

Translate programs into a formal model (of 'functions'):

- abstract mathematical description
- free-of-syntax meaning

program: P

denotation: $[\![P]\!]$



Power of denotations

- Abstract away from implementation details
- Compositional translation:
  - modelling of components in isolation
  - modularity

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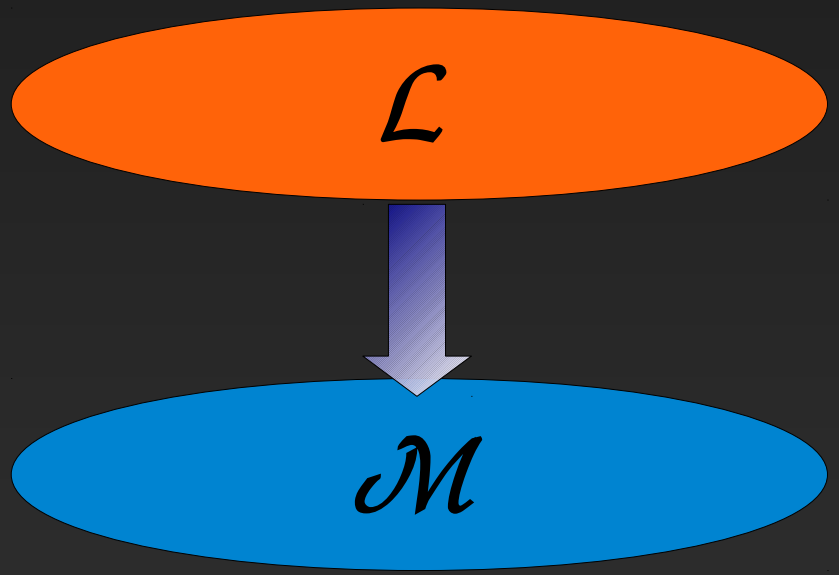


denotation: $[\![P]\!]$

Useful for understanding & analysing programs

Full Abstraction

Ideally, a denotational model exactly describes the meaning, i.e. the observable behaviour, of programs

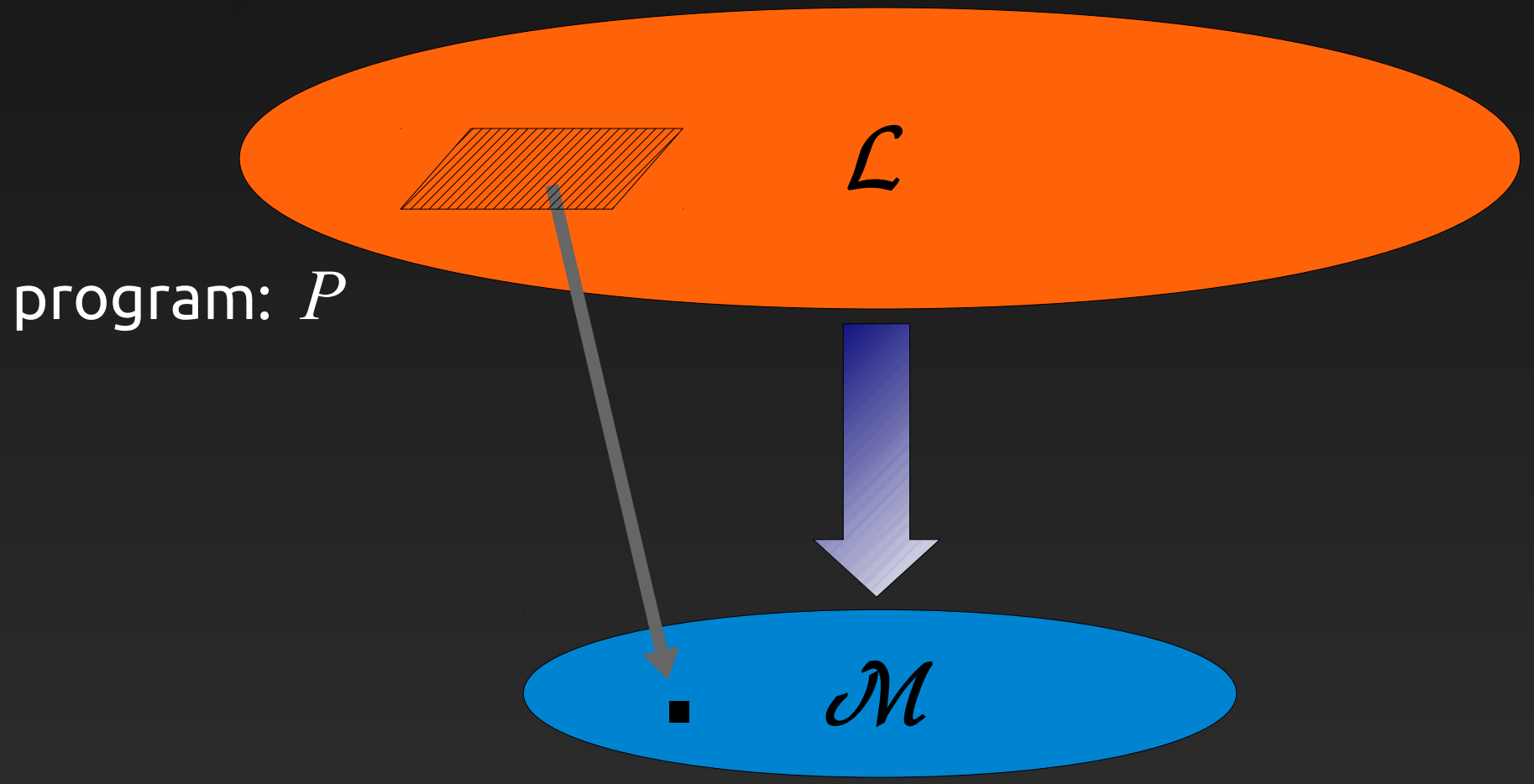
- Full abstraction (equational):

$$P \cong P' \iff [\![P]\!] = [\![P']\!]$$

- Full abstraction (inequational):

$$P \sqsubseteq P' \iff [\![P]\!] \subseteq [\![P']\!]$$

Full abstraction pictorially



The quest for full abstraction

1977 [Milner, Plotkin]:

- Formulation of the problem
- Functions cannot model sequentiality

1980-90's:

- Function stability [Berry, Bucciarelli, Erhard]
- Sequential algorithms [Berry, Currien]

1993 [AJM, HO/N *]: Full abstraction for PCF

- 'Functions' with operational content (*games*)

* Abramsky, Jagadeesan, Malacaria; Hyland, Ong; Nickau

Game semantics so far

From PCF towards general HO-programs:

- Variants of Idealized Algol:
  - non-determinism, probabilities, concurrency, polymorphism, exceptions, …
- Nominal game semantics (2004-):
  - *use names for dynamic resource generation*

    → fragments of ML, CML, C, Java

The need for names

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Exceptions as *pairs:*

$$\texttt{exn} = (\texttt{unit} \to \texttt{emp})\ \texttt{x}\ ((\texttt{unit} \to \texttt{emp}) \to \texttt{unit})$$

$$\Longrightarrow\ (1 \to 0)\texttt{x}((1 \to 0) \to 1)$$

- Theoretically attractive
- but: $\texttt{mkexn}(R, H)$, all R, H (*bad exceptions*)

The need for names

Exceptions as *pairs:*

$\texttt{exn} = (\texttt{unit} \to \texttt{emp}) \times ((\texttt{unit} \to \texttt{emp}) \to \texttt{unit})$

$\Longrightarrow \; (1 \to 0) \times ((1 \to 0) \to 1)$

- Theoretically attractive
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Exceptions as *names:*

$\texttt{exn} = \texttt{base type}$

$\Longrightarrow \; \mathcal{E}$ (exception names)

- Notion of *resource (name):*
  - atomic values
  - infinitely many
  - comparable for equality

The use of names in games allows us to pass from 'idealised' resources to 'realistic' ones

ExnML: HO-programs with exceptions

Types

$$\beta ::= \text{unit} \mid \text{int} \mid \text{exn} \mid \text{ref}\ \beta$$

$$\theta ::= \beta \mid \theta \rightarrow \theta$$

ExnML: HO-programs with exceptions

Types $\beta ::= \text{unit} \mid \text{int} \mid \text{exn} \mid \text{ref}\ \beta$

$\theta ::= \beta \mid \theta \to \theta$

Terms $M ::= () \mid 0 \mid \ldots \mid \Omega \mid x \mid \lambda x^{\theta}.M \mid MM$

$\mid \text{if0}\ MMM \mid M \oplus M$

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$\mid \text{if0}\ MMM \mid M \oplus M$

$\mid l \mid e \mid M = M$

$\mid \text{exn}() \mid \text{raise}\ M \mid M\ \text{handle}\ x \Rightarrow M$

$\mid \text{ref}_{\beta}(M) \mid !M \mid M := M$

location names: $l \in \mathcal{L}_{\theta}$

exception names: $e \in \mathcal{E}$

ExnML: operational semantics

$$\Sigma, M \longrightarrow \Sigma', M'$$

Σ stores exception names and location names&values

$$\Sigma, \mathsf{ref}_\beta(v) \longrightarrow \Sigma \uplus \{(l, v)\}, l \qquad \Sigma, l = l' \longrightarrow \Sigma, 0/1$$

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Nominal exceptions in OCaML

```
# exception MyExn
  let e1 = MyExn
  let e2 = MyExn
  let foo x = raise x;;

# try foo(e1) with x -> (x==e1, x==e2);;
```

Similar examples for Java and SML

ExnML example

M_1 : let $y = \mathsf{exn}()$ in $\lambda x^{\mathsf{unit}}.\ \mathsf{raise}\ y$: $\mathsf{unit} \to \mathsf{unit}$

M_2 : $\lambda x^{\mathsf{unit}}.\ \mathsf{raise}(\mathsf{exn}())$: $\mathsf{unit} \to \mathsf{unit}$

ExnML example

M_1 : let $y = \text{exn}()$ in $\lambda x^{\text{unit}}.$ raise y : unit $\to$ unit

M_2 : $\lambda x^{\text{unit}}.$ raise $(\text{exn}())$: unit $\to$ unit

$$M_1 \ncong M_2$$

let $g = [\_]$ in
  let $f = \lambda z^{\text{unit}}.\ g()$ handle x => x in
    $f() = f()$

Game semantics

Computation = a 2-player game between:

- *Opponent* (the environment, O)
- *Proponent* (the program, P)

Qualitative games (≠ Game Theory)

Game semantics

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- *Opponent* (the environment, O)
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Qualitative games (≠ Game Theory)

Computations = *plays* of a specified game

Programs = *strategies* for P

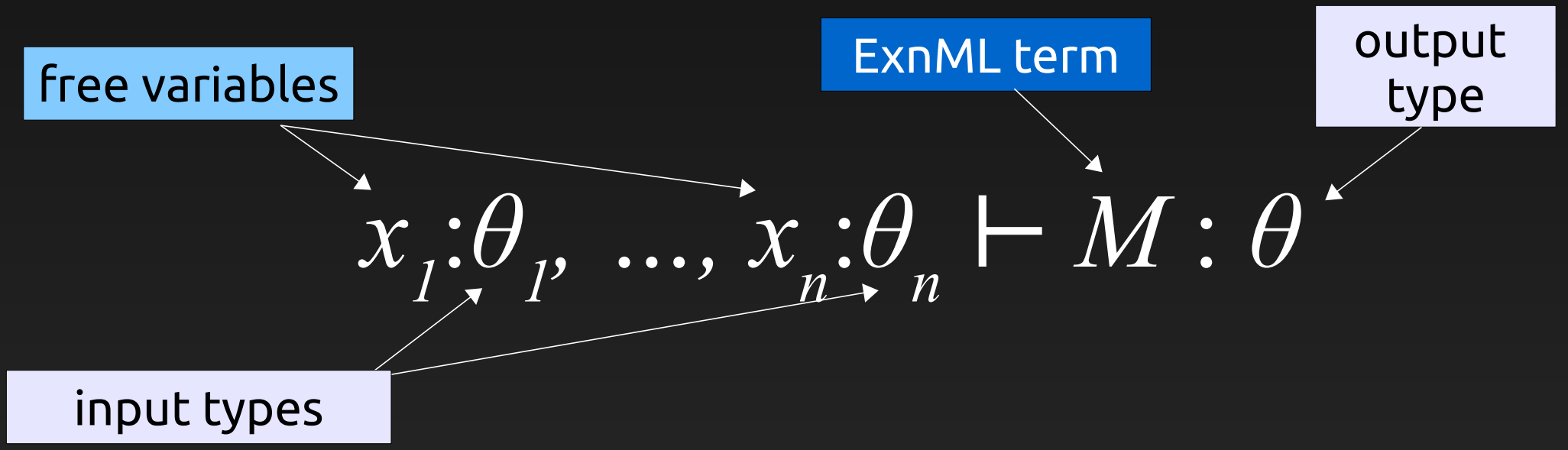
Strategies compose → *categories* of games

Games in detail

$$x_1{:}\theta_1,\ \ldots,\ x_n{:}\theta_n \vdash M : \theta$$

$$[\![M]\!] :\ [\![\theta_1,\ \ldots,\ \theta_n]\!] \longrightarrow\ [\![\theta]\!]$$

Games in detail



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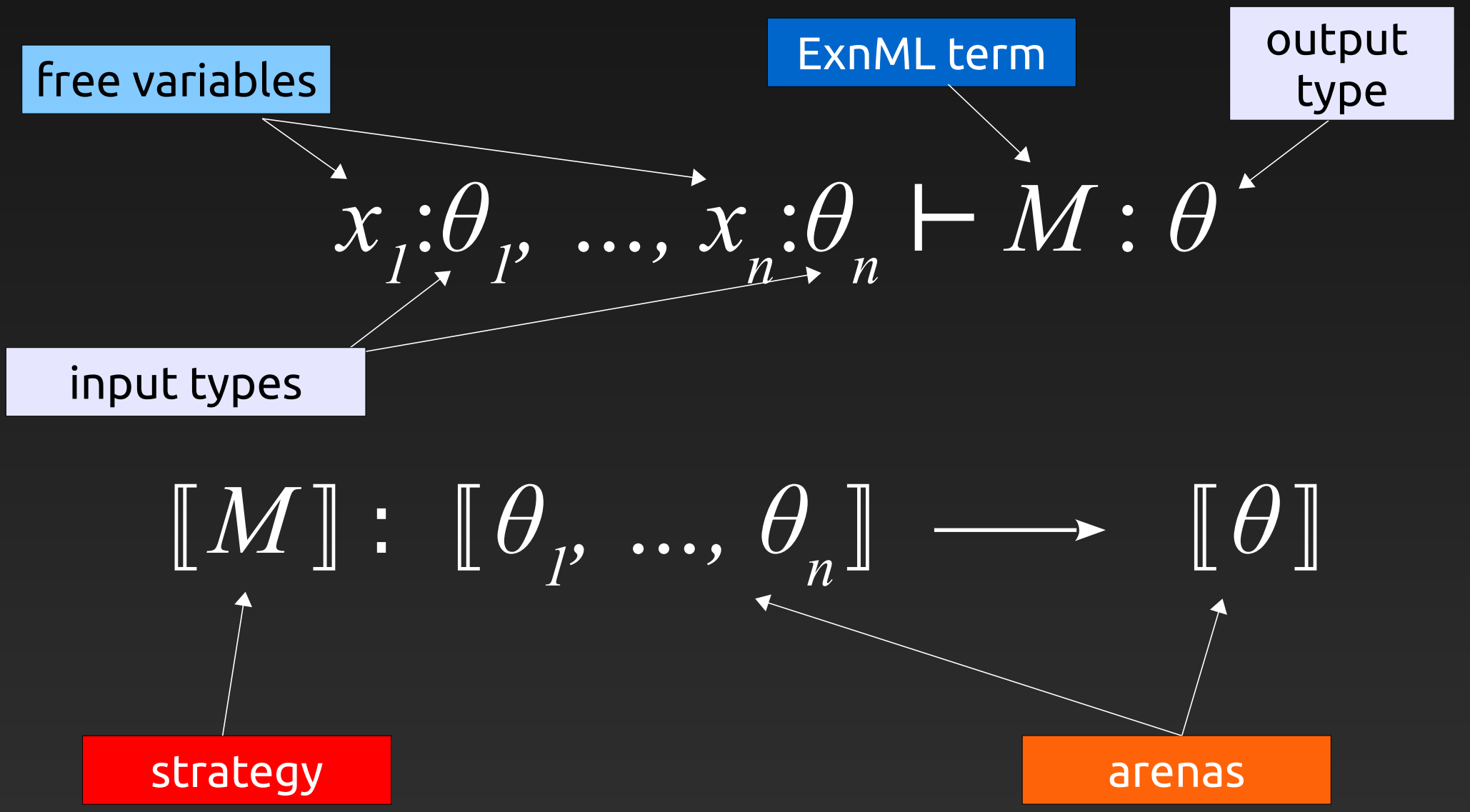
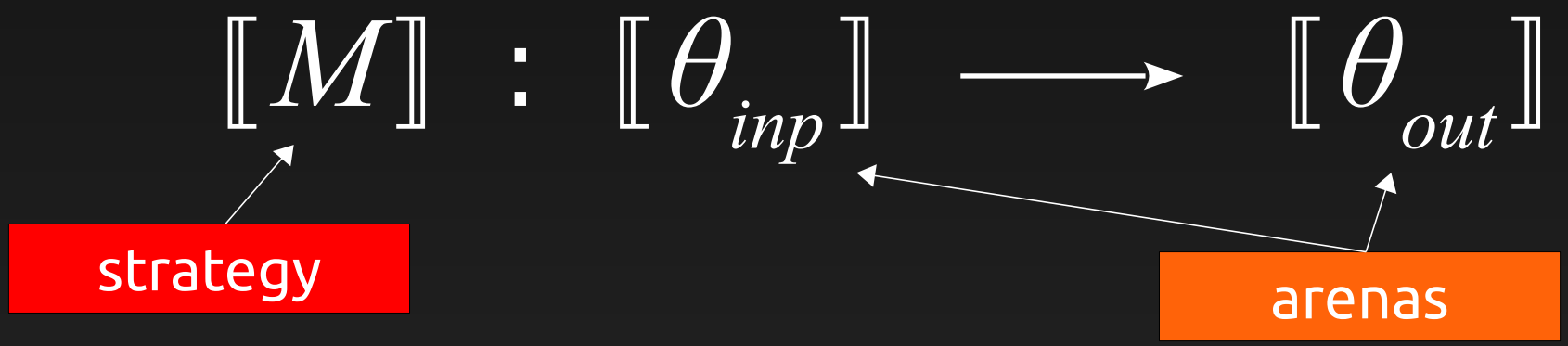


Arenas, moves



Arenas: sets of *moves* with:

- assigned polarities: *O*/*P* and *Q*/*A*
- dependence (justification) pointers
- names (locations and unraised exceptions)
- exceptional answers $e!$ for *O* and *P*

Arenas, moves

$$[\![M]\!] : [\![\theta_{inp}]\!] \longrightarrow [\![\theta_{out}]\!]$$

strategy

arenas

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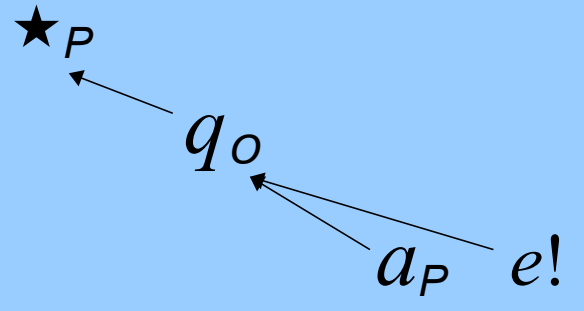
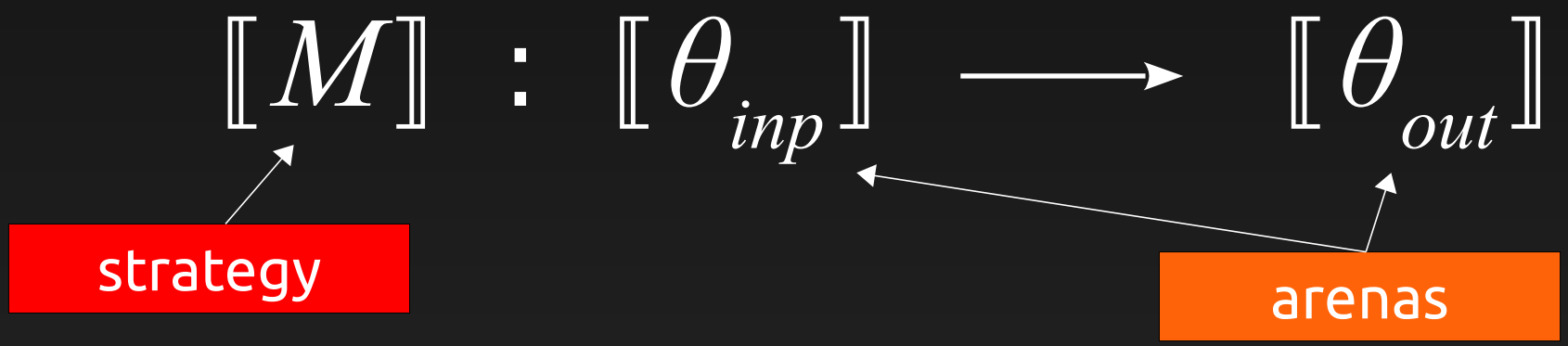
$[\![\text{unit} \to \text{unit}]\!]$:



Strategies, plays



Strategies : sets of *plays*, i.e. sequences of moves-with-store, satisfying:

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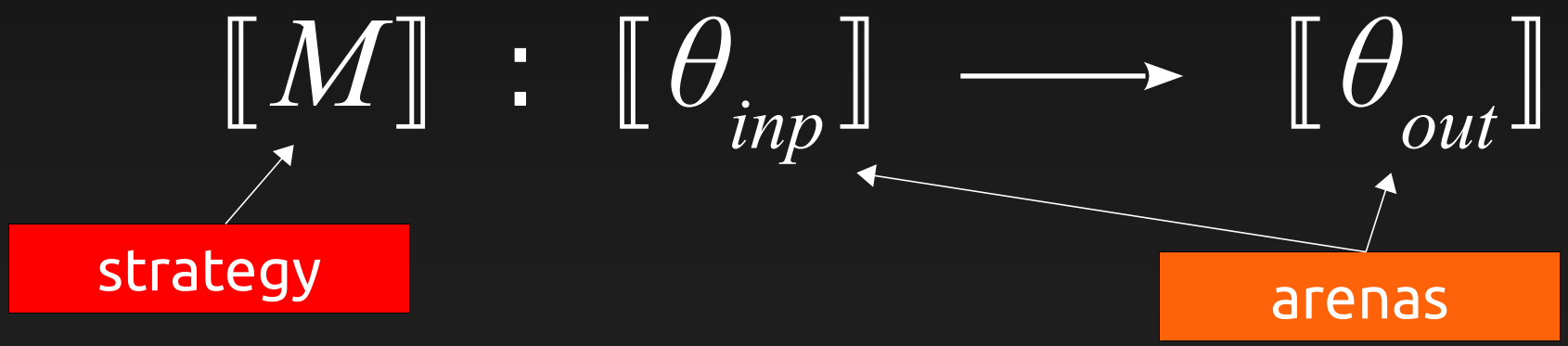
Strategies, plays



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Composition : via interaction, respecting name privacy

ExnML example: game semantics

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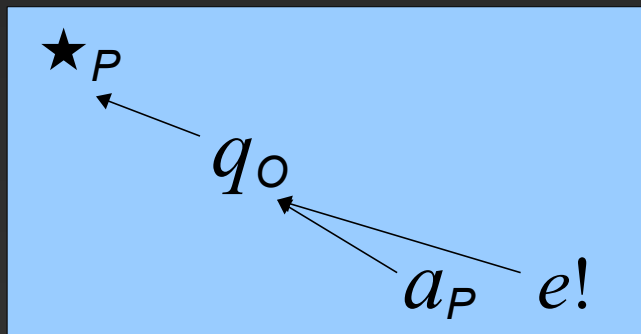

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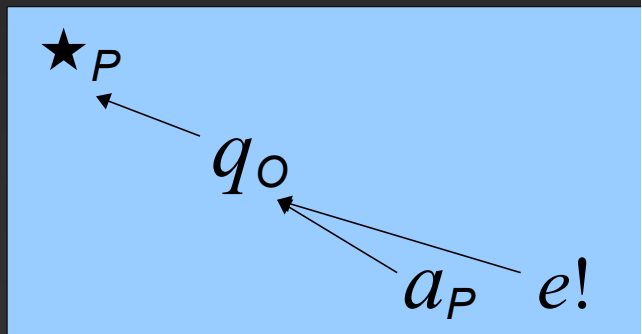

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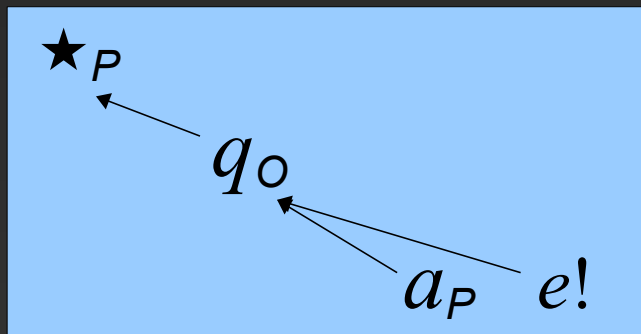

Full abstraction for ExnML

Lemma. The game model is sound

Lemma. Every finitary strategy is ExnML-definable

Proof goes by factorising steps which produce strategies with increasingly less power on exceptions

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Lemma. Every finitary strategy is ExnML-definable

Proof goes by factorising steps which produce strategies with increasingly less power on exceptions

Theorem. The game model is fully abstract

$$P \sqsubseteq P' \iff [\![P]\!] \subseteq [\![P']\!]$$

ExnML$: idealized exceptions

Shouldn't private exceptions always escape their context?

ExnML$: idealized exceptions

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M handle M -> M

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Games for idealised exceptions

Strategies are restricted by *propagation conditions* for private exceptions:

- if O plays $e!$ for some fresh e then
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These restrictions give a fully abstract model for $\mathrm{ExnML}_{\$}$

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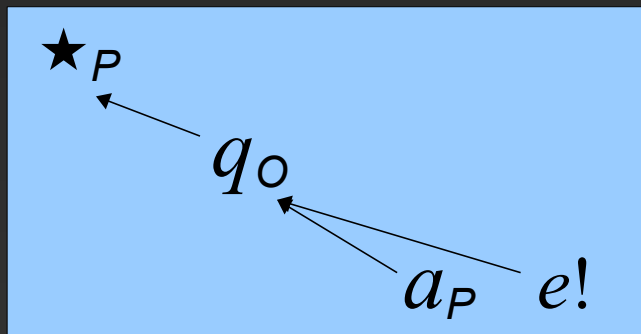

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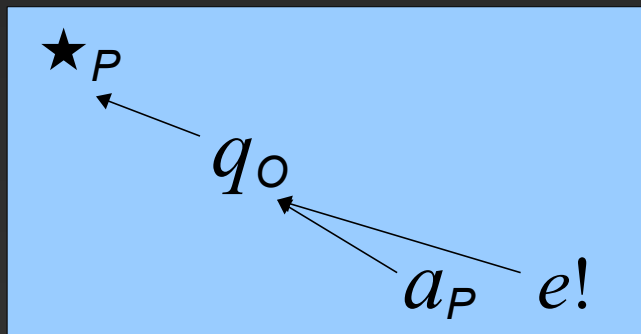

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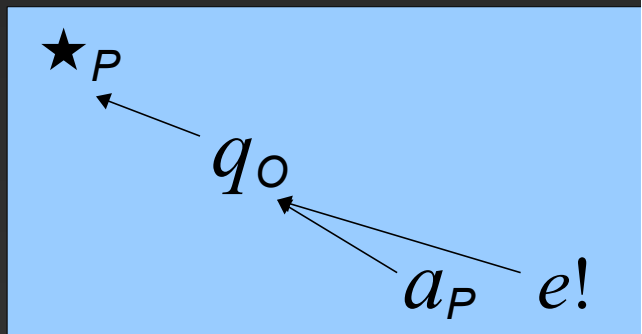
$[\![M_1]\!]_{\$} = [\![M_2]\!]_{\$} = \star$

P

$\Sigma_0 = \{\}$

$M_1 \cong M_2$



Related and further work

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- Laird's models of “bad exceptions” ('98, '01, '13)
- Quotiented nominal model in Tz'08

Future work

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