Game Semantic Analysis of Equivalence in Interface Middleweight Java

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what this talk is about

We examine contextual equivalence of Java programs

We work in an fragment of Middleweight Java

- open code modelled by use of interfaces
- denotational approach based on game semantics

Full characterisation based on type disciplines:

- queue machine encodings (negative cases)
- pushdown register automata (algorithms)

Program equivalence

$$M \cong M'$$

same observable behaviour in every context

for all closing contexts C:

$$(C[M],\phi) \rightarrow^* (\mathsf{skip},S) \iff (C[M'],\phi) \rightarrow^* (\mathsf{skip},S')$$

- subsumes standard verification properties
- note: quantification over *every* context
- several methods for proving given equivalences yet no general procedures

Interface Middleweight Java (IMJ)

Types $\quad \theta ::= \text{void} \mid \text{int} \mid \mathcal{I}$

Interface definitions

$$\Theta ::= \phi \mid (\mathsf{f} : \theta), \Theta \mid (\mathsf{m} : \overline{\theta} \rightarrow \theta), \Theta$$

Interface tables

$$\Delta ::= \phi \mid (\mathcal{I} : \Theta), \Delta \mid (\mathcal{I}\langle\mathcal{I}\rangle : \Theta), \Delta$$

Object calculus based on MJ [Bierman, Parkinson, Pitts]

- Objects, inheritance, casting, interfaces

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interface ident.

field identifier

method identif.

Object calculus based on MJ [Bierman, Parkinson, Pitts]

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Interface Middleweight Java (IMJ)

Terms

$$M ::= \text{skip} \mid a \mid \text{null} \mid x \mid i \mid M \oplus M \mid \text{if } M\ M\ M$$
$$\mid \text{let } x = M \text{ in } M \mid M = M \mid (\mathcal{I})M \mid \text{while } M\ M$$
$$\mid \text{new}(x : \mathcal{I};\ \mathcal{M}) \mid M.\text{f} \mid M.\text{f} := M \mid M.\text{m}(\overline{M})$$

Method implementations $\mathcal{M} ::= \phi \mid (\text{m} : \lambda\overline{x}.M),\ \mathcal{M}$

Interface Middleweight Java (IMJ)

$\mathcal{M} = \{ \mathsf{m}_i : \lambda \overline{x}_i.M_i \mid 1 \le i \le n \}$

$$\frac{\Delta|\Gamma \vdash M : \mathsf{int} \quad \Delta|\Gamma \vdash M', M'' : \theta}{\Delta|\Gamma \vdash \mathsf{if}\ M\ \mathsf{then}\ M'\ \mathsf{else}\ M'' : \theta}$$

$$\frac{\bigwedge_{i=1}^{n}(\Delta|\Gamma \uplus \{\vec{x}_i : \vec{\theta}_i\} \vdash M_i : \theta_i)}{\Delta|\Gamma \vdash \mathcal{M} : \{\mathsf{m}_i : \vec{\theta}_i \to \theta_i \mid 1 \le i \le n\}}$$

$$\frac{\Delta|\Gamma \vdash M, M' : I}{\Delta|\Gamma \vdash M = M' : \mathsf{int}} \qquad \frac{\Delta|\Gamma \vdash M : \mathsf{void} \quad \Delta|\Gamma \vdash M' : \theta}{\Delta|\Gamma \vdash M; M' : \theta} \qquad \frac{\Delta|\Gamma \vdash M : I \quad \Delta|\Gamma \vdash M' : \theta}{\Delta|\Gamma \vdash M.\mathsf{f} := M' : \mathsf{void}}\ \Delta(I).\mathsf{f}=\theta$$

$$\frac{\Delta|\Gamma \vdash M : I}{\Delta|\Gamma \vdash M.\mathsf{f} : \theta}\ \Delta(I).\mathsf{f}=\theta \qquad \frac{\Delta|\Gamma \vdash M : I'}{\Delta|\Gamma \vdash (I)M : I}\ \begin{array}{l}\Delta \vdash I \le I' \\ \vee I' \le I\end{array}$$

$$\frac{\Delta|\Gamma, x : I \vdash \mathcal{M} : \Theta}{\Delta|\Gamma \vdash \mathsf{new}\{\, x : I; \mathcal{M}\,\} : I}\ \Delta(I) \restriction Meths = \Theta$$

$$\frac{\Delta|\Gamma \vdash M : I \quad \bigwedge_{i=1}^{n}(\Delta|\Gamma \vdash M_i : \theta_i)}{\Delta|\Gamma \vdash M.\mathsf{m}(M_1, \cdots, M_n) : \theta}\ \Delta(I).\mathsf{m}=\vec{\theta}\to\theta \qquad \frac{\Delta|\Gamma \vdash M : \theta' \quad \Delta|\Gamma, x : \theta' \vdash M' : \theta}{\Delta|\Gamma \vdash \mathsf{let}\ x = M\ \mathsf{in}\ M' : \theta}$$

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IMJ: operational semantics

$$S,\ M \ \longrightarrow\ S',\ M'$$

S stores object names + their types and values

Obj : set of obj. names

$$S,\ \text{let}\ x = v\ \text{in}\ M \ \longrightarrow\ S,\ M[v/x]$$

$$S,\ a = a' \ \longrightarrow\ S,\ 0/1$$

$a, a' \in \text{Obj}$

$$S,\ \text{new}(x : \mathcal{I};\ M) \ \longrightarrow\ S \uplus \{(a,\ \mathcal{I},\ (V_{\mathcal{I}},\ M[a/x])\},\ a$$

$V_{\mathcal{I}}$: default field values

Program equivalence – our plan

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termination → inequivalence

we consider a finitary restriction IMJ_{fin} and first identify the fragment decidable for termination

we use game semantics and types to characterise the fragment of IMJ_{fin} decidable for equivalence

Termination

M *terminates* if:
$(M,\phi) \rightarrow^* (\text{skip},S)$

Two distinct sources of undecidability:

I. recursive methods

$\text{new}(x : \mathcal{I};\ \text{main: } \lambda y.\ \{\ldots\ x.\text{main}(..)\ \ldots\ \})$

II. objects with methods can be stored

$\text{let } a = \text{new}(x : \mathcal{I};\ \text{main: } \lambda y.\ \{\ldots\ \})\ \text{ in}$
$\text{let } b = \text{new}(x' : \mathcal{I}';\ )\ \text{ in }\ b.\text{var} := a$

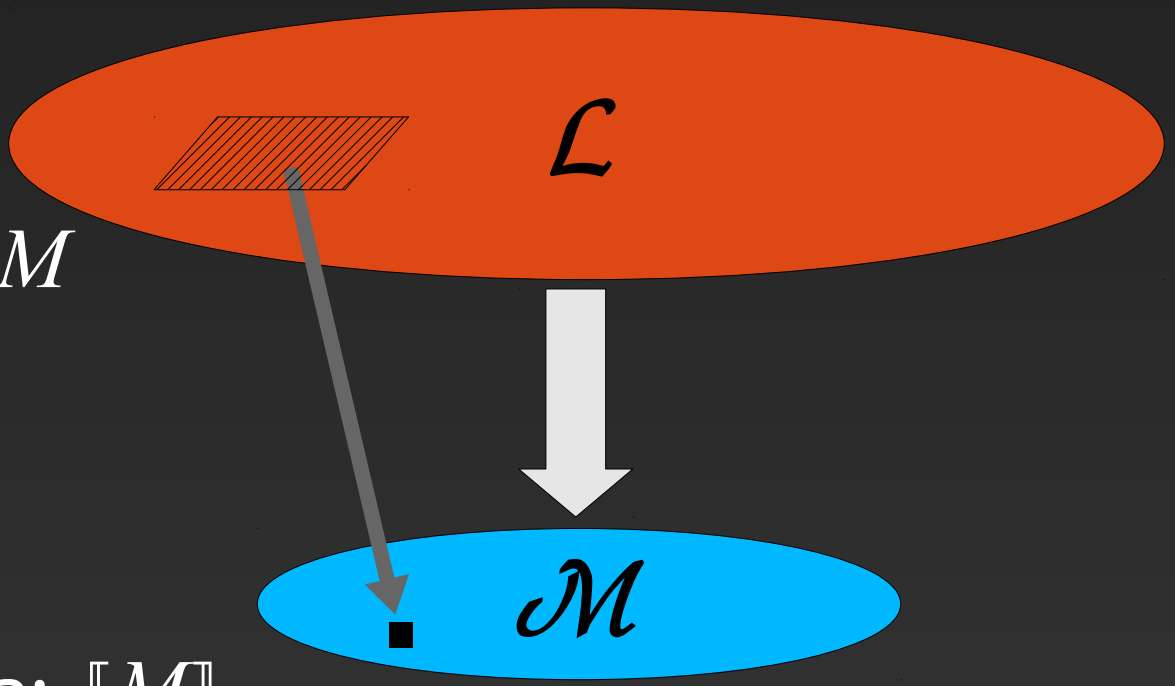
Theorem: Termination is undecidable for IMJ_{fin} terms that may feature I or II.
Conversely, given an IMJ_{fin} term M that does not feature I or II, we can always decide whether it terminates.

Games for program equivalence

We focus on: $\mathrm{IMJ}_{fin} - \{\mathrm{I},\mathrm{II}\}$

and employ the fully abstract game semantics for IMJ:

$$M \cong M' \Leftrightarrow [\![M]\!] = [\![M']\!]$$



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Game semantics

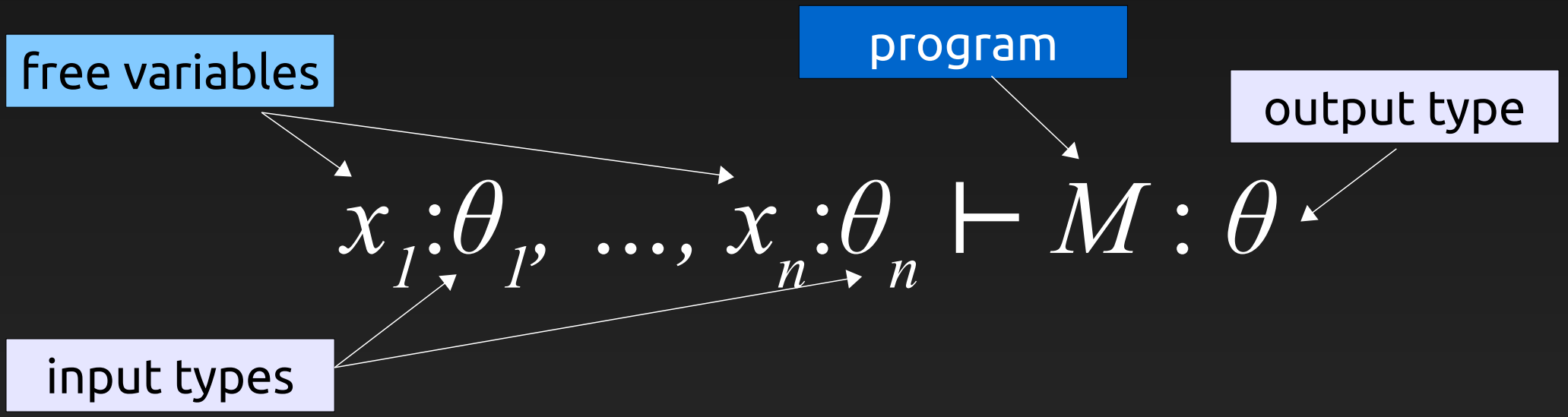
We can model the execution of open programs as a 2-player game between:

- *Opponent* (the environment, O)
- *Proponent* (the program, P)

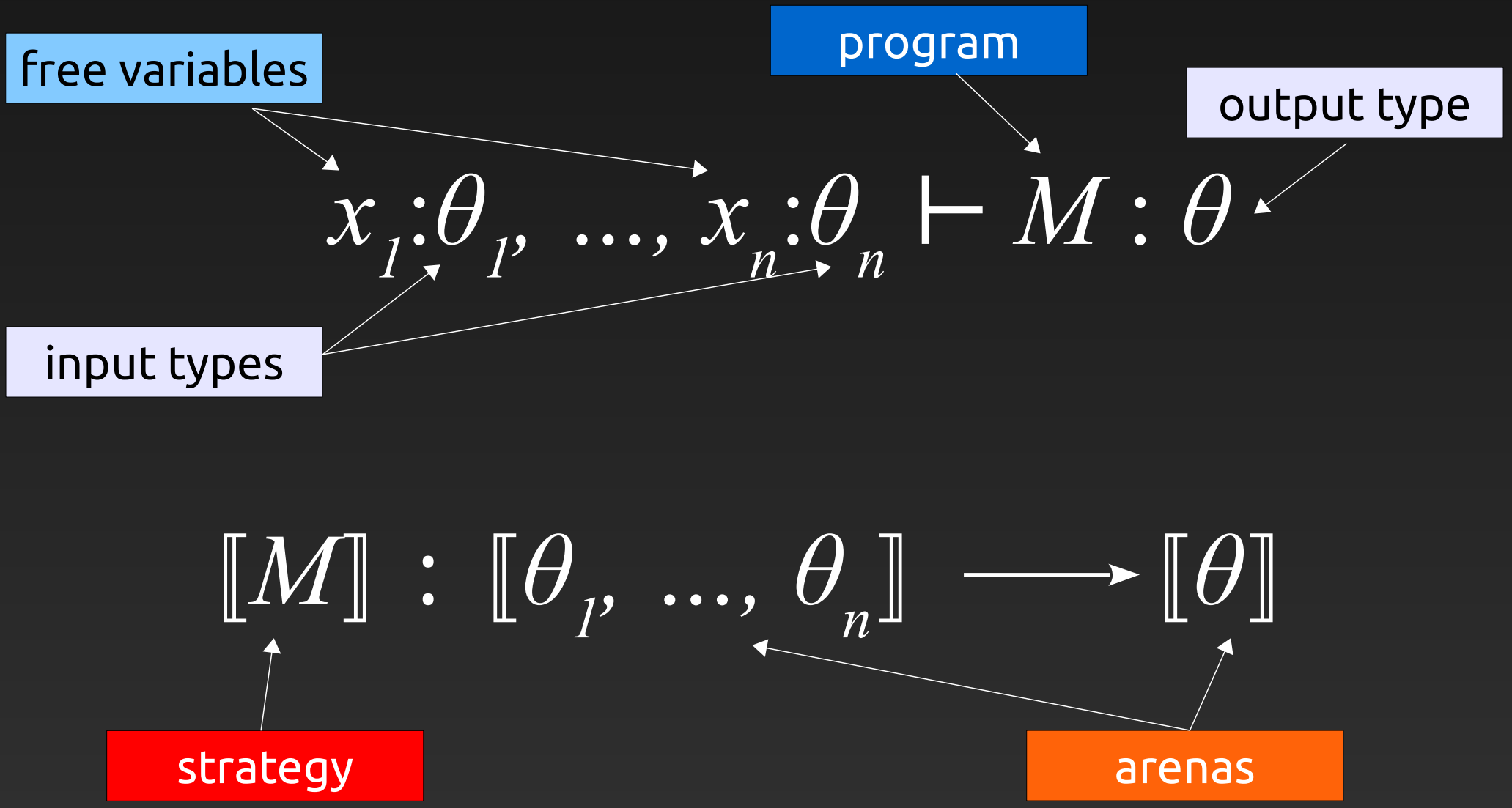
→ think of these as open program traces

Programs = sets of traces (for P), called *strategies*

Game apparatus



Game apparatus



Games for IMJ programs

Program interactions are modelled by sequences of moves-with-store, written m^{Σ}, where:

- move m is either:
  - a value/arena move
  - an object method call/return

call c.set(c'), ret q.get(), …

- stores are of the form:

$$\Sigma = \{ n \mapsto (\mathsf{Emp}, \emptyset),\ a \mapsto (\mathsf{Ptr}, \mathsf{val} = a),\ p \mapsto (\mathsf{Pt2}, \mathsf{x} = 1, \mathsf{y} = 0),\ c \mapsto (\mathsf{Ptr'}, \emptyset),\ q \mapsto (\mathsf{Pt2'}, \emptyset),\ \ldots \}$$

Ptr' : (get : Ptr' → Ptr', set : Ptr' → Ptr')
Pt2' : (get : void → (int,int), set : (int,int) → void)

[Murawski & T. '14]

IMJ example: game semantics

M_1 : let u = new(Var_{Emp}) in
new(M_1) : Cell

M_1 : get: $\lambda().\ u.\mathrm{val}$,
set: $\lambda y.\ u.\mathrm{val} := y$

Δ = Empty: $\emptyset$,
Cell: (get: void → Empty,
set: Empty → void),
Var_{Emp}: (val: Empty),
Var_{Int}: (val: int)

$$[\![M_1]\!] = * \; n^{\Sigma_0} \; (\; \underset{O}{\text{call } n.\text{get}()^{\Sigma_0}} \; \underset{P}{\text{ret } n.\text{get}(\text{nul})^{\Sigma_0}} \;)^* \; \underset{O}{\text{call } n.\text{set}(n_1)^{\Sigma_1}} \; \underset{P}{\text{ret } n.\text{set}()^{\Sigma_1}} \; (\; \text{call } n.\text{get}()^{\Sigma_1} \; \text{ret } n.\text{get}(n_1)^{\Sigma_1} \;)^* \; \text{call } n.\text{set}(n_2)^{\Sigma_2} \; \text{ret } n.\text{set}()^{\Sigma_2} \; \ldots$$

$$\Sigma_i = \{\; (n : \text{Cell},) \;\} \cup \{\; (n_j : \text{Empty},) \mid 1 \le j \le i \;\}$$

* [Koutavas & Wand' 07]

IMJ example: game semantics

M_2 : let b = new(Var_{Int}) in
let u_1 = new(Var_{Emp}) in
let u_2 = new(Var_{Emp}) in
new(M_2) : Cell

M_2 : get: $\lambda()$. if b.val
then b.val := 0; u_1.val
else b.val := 1; u_2.val,
set: λy. u_1.val := y;
u_2.val := y

O **P** **O** **P**

$[\![M_1]\!]$ = $*$ n^{Σ_0} (call n.get()$^{\Sigma_0}$ ret n.get(nul)$^{\Sigma_0}$)*
call n.set(n_1)$^{\Sigma_1}$ ret n.set()$^{\Sigma_1}$
(call n.get()$^{\Sigma_1}$ ret n.get(n_1)$^{\Sigma_1}$)*
call n.set(n_2)$^{\Sigma_2}$ ret n.set()$^{\Sigma_2}$ ... $= [\![M_2]\!]$

$\Sigma_i = \{ (n : \mathrm{Cell},) \} \cup \{ (n_j : \mathrm{Empty},) \mid 1 \leq j \leq i \}$

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One source of Undecidability

P passes objects of type $\mathcal{I}$, and $\mathcal{I}$ contains methods

$\mathcal{I}$: (step: void$\rightarrow$void)
N : (val: int)

(run: $\mathcal{I}\rightarrow$void) $\vdash$ void

we can encode computations of queue machines!

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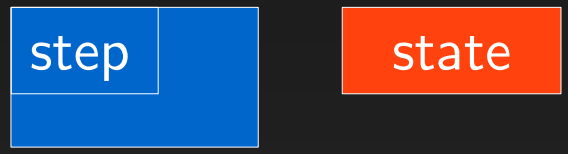

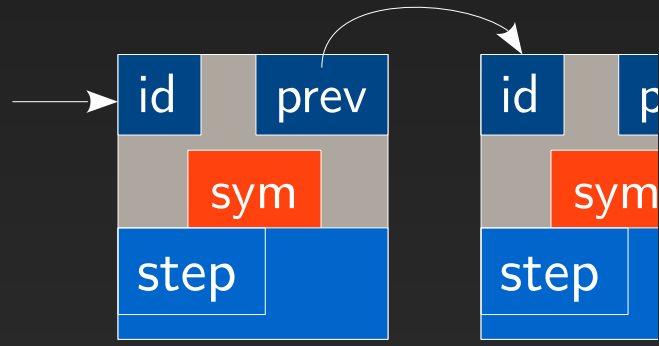

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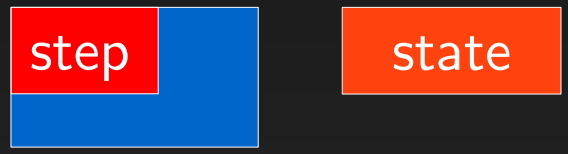

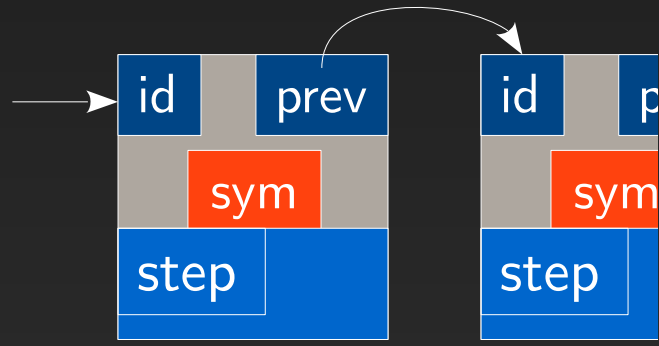

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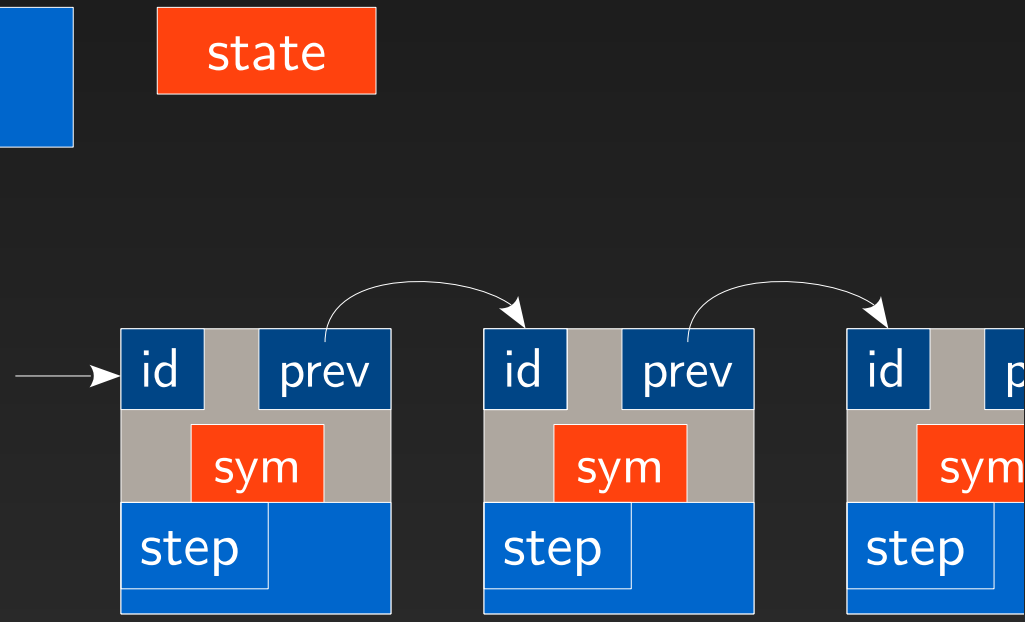

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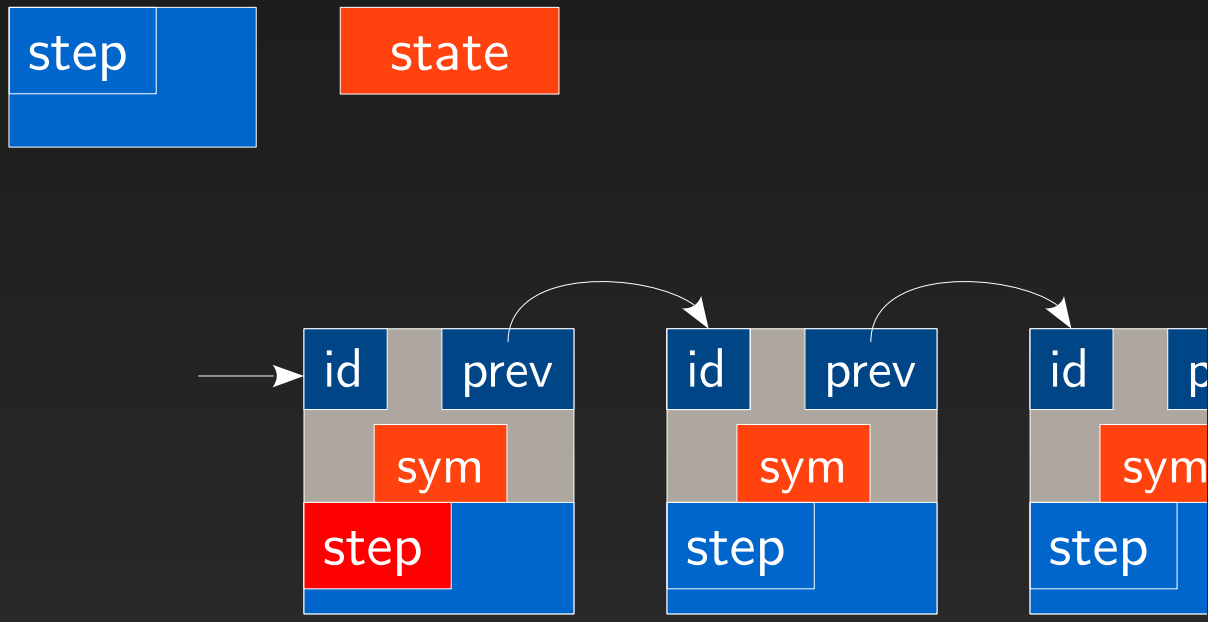

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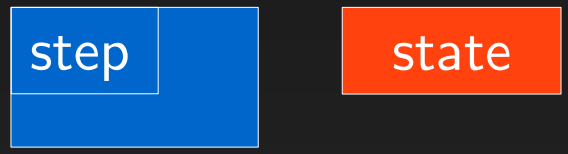

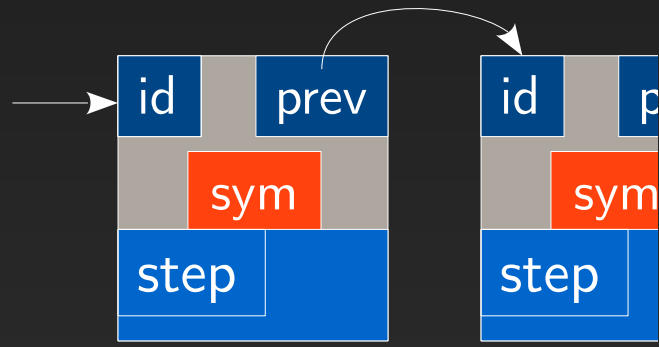

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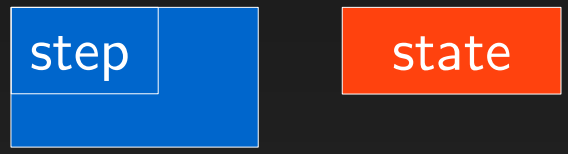

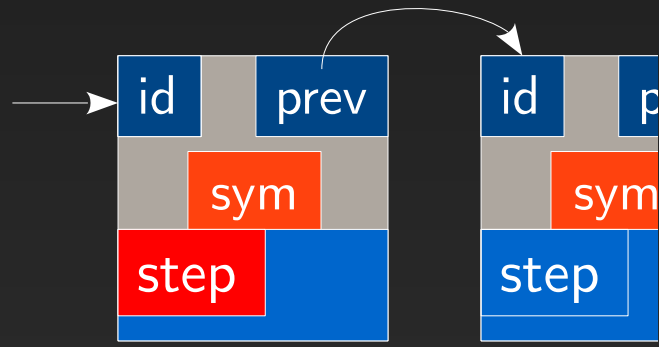

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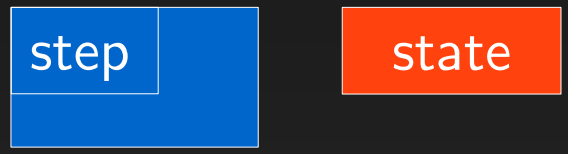

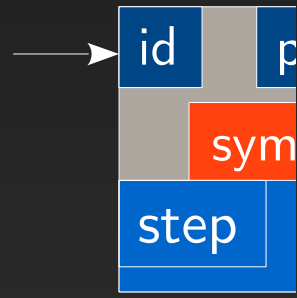

Decidable types – IMJ*

$x : \mathsf{L} \vdash M : \mathsf{R}$

$$\mathsf{G} \;::=\; \mathsf{void} \mid \mathsf{int} \mid (\mathsf{f} : \mathsf{G})$$

$$\mathsf{L} \;::=\; \mathsf{void} \mid \mathsf{int} \mid (\mathsf{f} : \mathsf{G},\ \mathsf{m} : \mathsf{G} \to \mathsf{L})$$

$$\mathsf{R} \;::=\; \mathsf{void} \mid \mathsf{int} \mid (\mathsf{f} : \mathsf{G},\ \mathsf{m} : \mathsf{L} \to \mathsf{G})$$

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From games to automata

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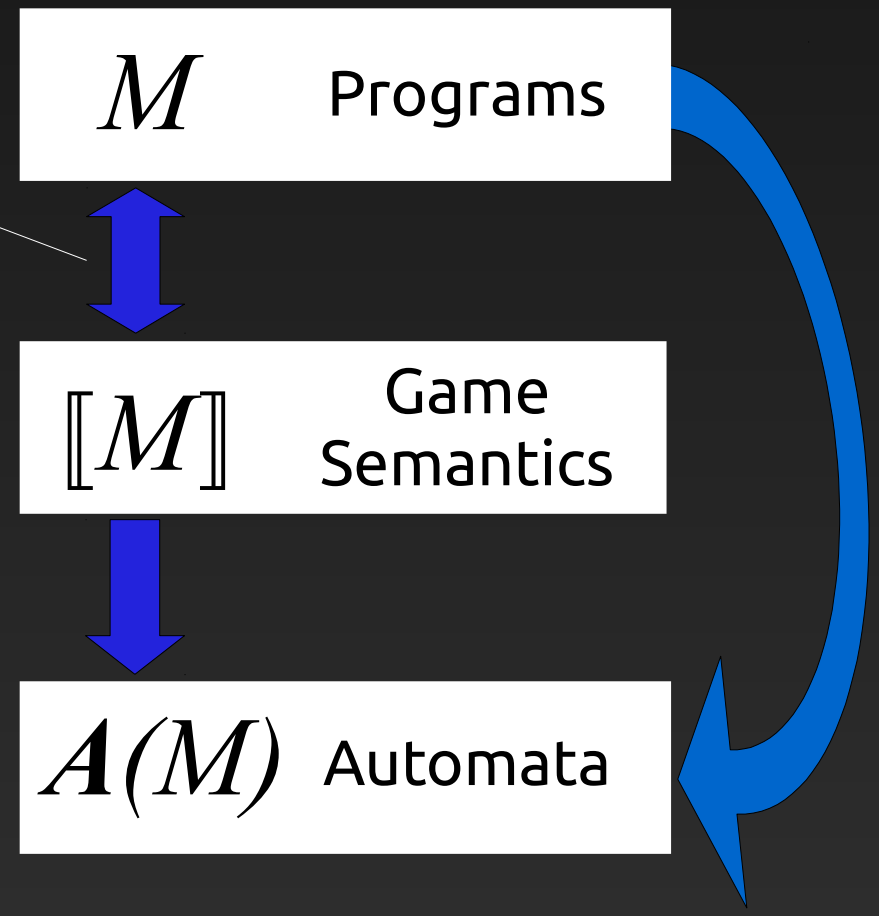

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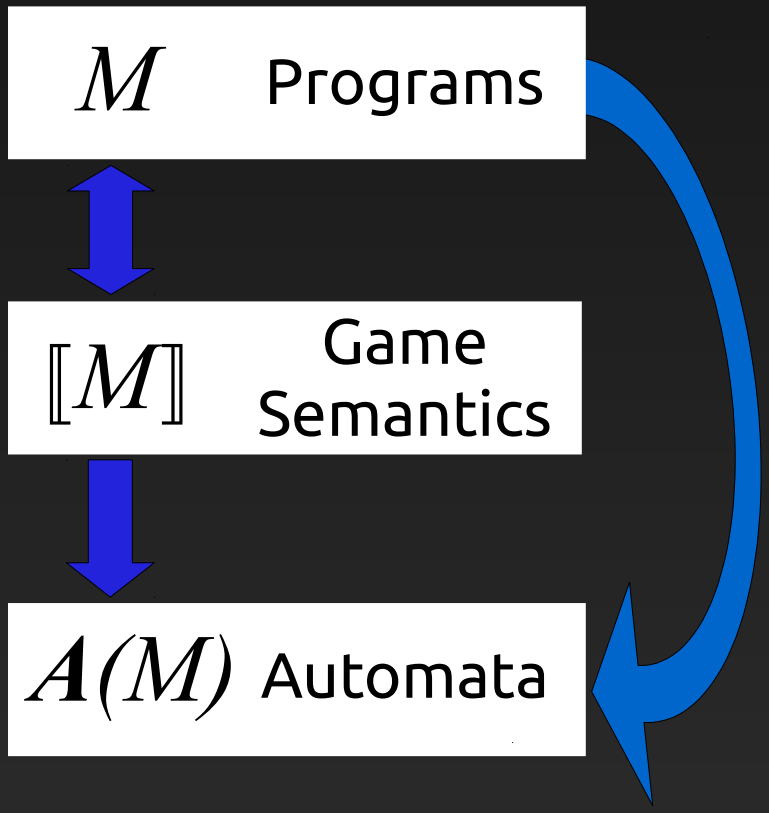

Translation done in two steps:

- terms reduced to canonical forms
- canonical terms are compositionally transformed into automata

We work with pushdown automata:

- over infinite alphabets
- visibly pushdown & deterministic

$$M \cong M' \Leftrightarrow [\![M]\!] = [\![M']\!]$$
$$\Leftrightarrow A \sim A' \Leftrightarrow A \otimes A' = 0$$

Implemented in Coneqct!

Wrapping up

Contextual equivalence

- captured accurately by game semantics
- decidability characterised via type disciplines
- automated decision via FPDRAs → Coneqct

Further on:

- sound methods for the whole of IMJ
- general model checking
- logics (over infinite alphabets)